

# Computer algebra independent integration tests

5-Inverse-trig-functions/5.1-Inverse-sine/5.1.2-d-x-<sup>m</sup>-a+b-arcsin-c-x-<sup>n</sup>

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July 22, 2021      Compiled on July 22, 2021 at 12:55am

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3.185	$\int (a+b \sin^{-1}(cx))^{5/2} dx$	613
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3.187	$\int \frac{(a+b \sin^{-1}(cx))^{5/2}}{x^2} dx$	619
3.188	$\int \frac{1}{\sqrt{a+b \sin^{-1}(cx)}} dx$	621
3.189	$\int \frac{x}{\sqrt{a+b \sin^{-1}(cx)}} dx$	625
3.190	$\int \frac{1}{\sqrt{a+b \sin^{-1}(cx)}} dx$	629
3.191	$\int \frac{1}{x \sqrt{a+b \sin^{-1}(cx)}} dx$	632
3.192	$\int \frac{1}{x^2 \sqrt{a+b \sin^{-1}(cx)}} dx$	634
3.193	$\int \frac{x^2}{(a+b \sin^{-1}(cx))^{3/2}} dx$	636

3.194	$\int \frac{x}{(a+b \sin^{-1}(cx))^{3/2}} dx$	640
3.195	$\int \frac{1}{(a+b \sin^{-1}(cx))^{3/2}} dx$	643
3.196	$\int \frac{1}{x(a+b \sin^{-1}(cx))^{3/2}} dx$	646
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3.201	$\int \frac{1}{x(a+b \sin^{-1}(cx))^{5/2}} dx$	663
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3.215	$\int (dx)^{3/2} (a + b \sin^{-1}(cx))^3 dx$	705
3.216	$\int \sqrt{dx} (a + b \sin^{-1}(cx))^3 dx$	707
3.217	$\int \frac{(a+b \sin^{-1}(cx))^3}{\sqrt{dx}} dx$	709
3.218	$\int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{3/2}} dx$	711
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3.220	$\int \frac{(dx)^{3/2}}{a+b \sin^{-1}(cx)} dx$	715
3.221	$\int \frac{\sqrt{dx}}{a+b \sin^{-1}(cx)} dx$	717
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3.223	$\int \frac{1}{(dx)^{3/2} (a+b \sin^{-1}(cx))} dx$	721
3.224	$\int \frac{(dx)^{3/2}}{(a+b \sin^{-1}(cx))^2} dx$	723
3.225	$\int \frac{\sqrt{dx}}{(a+b \sin^{-1}(cx))^2} dx$	725

3.226	$\int \frac{1}{\sqrt{dx}(a+b \sin^{-1}(cx))^2} dx$	727
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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 227 ]. This is test number [ 142 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$  functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 227 )	% 0.00 ( 0 )
Mathematica	% 99.56 ( 226 )	% 0.44 ( 1 )
Maple	% 95.15 ( 216 )	% 4.85 ( 11 )
Maxima	% 29.07 ( 66 )	% 70.93 ( 161 )
Fricas	% 34.80 ( 79 )	% 65.20 ( 148 )
Sympy	% 44.05 ( 100 )	% 55.95 ( 127 )
Giac	% 71.81 ( 163 )	% 28.19 ( 64 )
Mupad	% 33.04 ( 75 )	% 66.96 ( 152 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

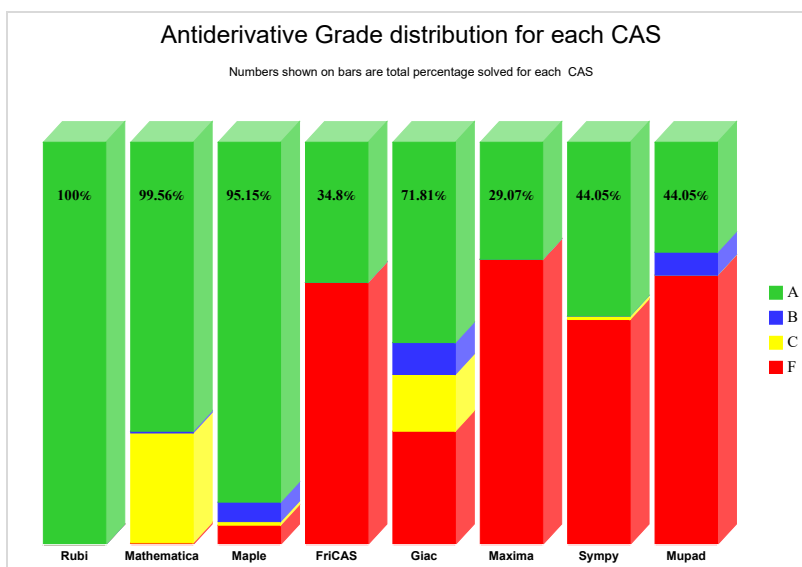
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

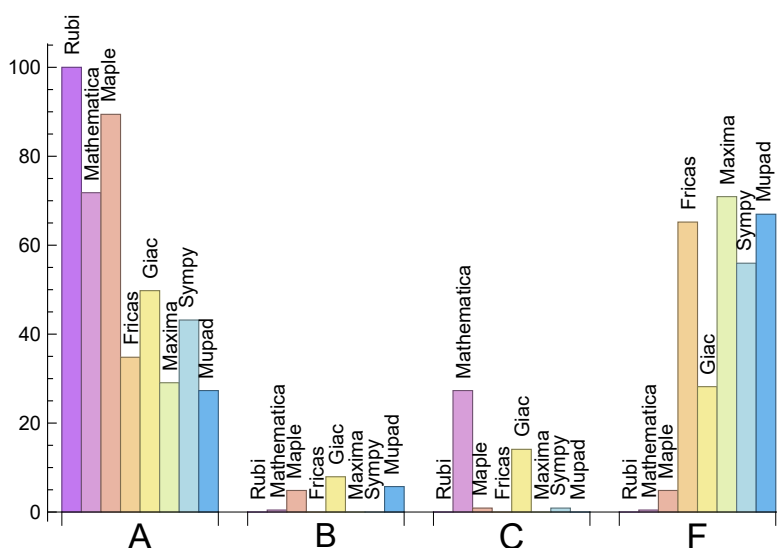
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	71.81	0.44	27.31	0.44
Maple	89.43	4.85	0.88	4.85
Maxima	29.07	0.00	0.00	70.93
Fricas	34.80	0.00	0.00	65.20
Sympy	43.17	0.00	0.88	55.95
Giac	49.78	7.93	14.10	28.19
Mupad	27.31	5.73	0.00	66.96

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	0.00 %	100.00 %	0.00 %
Maple	11	81.82 %	0.00 %	18.18 %
Maxima	161	50.31 %	12.42 %	37.27 %
Fricas	148	46.62 %	0.00 %	53.38 %
Sympy	127	90.55 %	2.36 %	7.09 %
Giac	64	79.69 %	0.00 %	20.31 %
Mupad	152	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

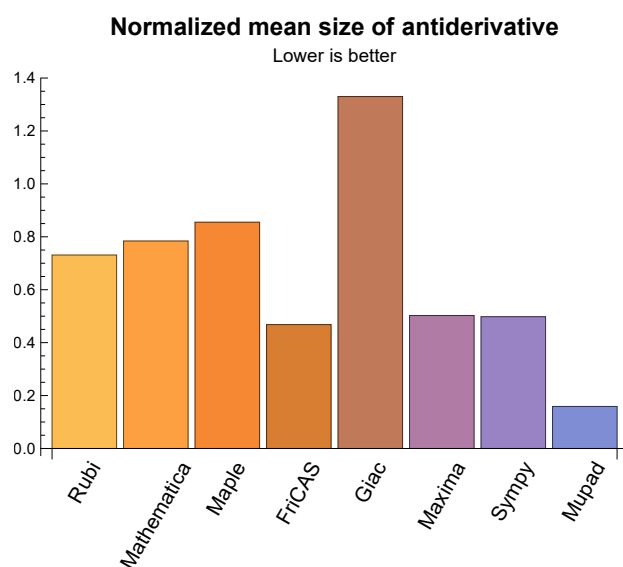
## 1.3 Performance

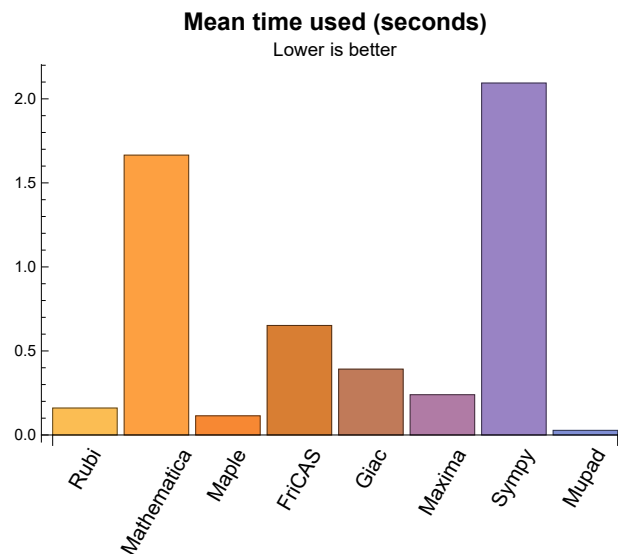
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	81.81	0.73	75.00	1.00
Mathematica	1.67	86.96	0.78	69.50	0.83
Maple	0.11	98.88	0.86	69.50	0.90
Maxima	0.24	39.24	0.50	0.00	0.00
Fricas	0.65	38.10	0.47	25.00	0.54
Sympy	2.09	45.44	0.50	0.00	0.00
Giac	0.39	167.51	1.33	68.00	1.11
Mupad	0.03	9.67	0.16	-1.00	-0.05

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 161, 162, 166, 167, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227}

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

**Mathematica** {17, 18, 20, 27, 28, 30, 38, 39, 40, 41, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 144, 151, 152, 156, 157, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```



```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

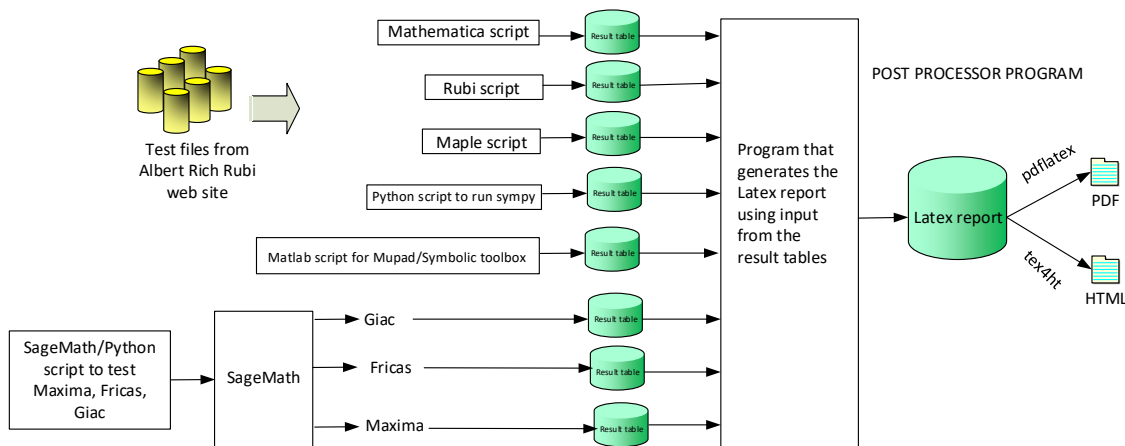
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"  
*The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.  
*The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 157 }

C grade: { 11, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208 }

F grade: { 216 }

#### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135,

136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 181, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 201, 202, 203, 204, 205, 206, 207, 208, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 151, 156, 157, 178, 180, 183, 184, 185, 198, 199, 200 }

C grade: { 132, 133 }

F grade: { 121, 122, 130, 131, 209, 210, 211, 212, 213, 214, 217 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 19, 21, 22, 24, 26, 33, 35, 37, 49, 50, 58, 65, 66, 72, 73, 123, 140, 141, 142, 143, 145, 146, 147, 148, 150, 153, 155, 161, 162, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 215, 216, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { }

C grade: { }

F grade: { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 149, 151, 152, 154, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 217, 218, 219 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 119, 120, 123, 124, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 153, 154, 155, 161, 162, 166, 167, 171, 172, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { }

C grade: { }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 125, 126, 127, 128, 130, 131, 132, 133, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 153, 154, 155, 161, 162, 166, 167, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 203, 204, 205, 215, 216, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { }

C grade: { 7, 8 }

F grade: { 6, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 136, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169,

170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 206, 207, 208, 209, 210, 211, 212, 213, 214, 217, 218, 219 }

## 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 150, 155, 158, 159, 160, 162, 166, 167, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 197, 202, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 8, 10, 19, 21, 51, 145, 146, 147, 148, 149, 153, 154, 163, 164, 165, 168, 169, 170 }

C grade: { 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190 }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 144, 151, 152, 156, 157, 161, 193, 194, 195, 196, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216 }

## 2.1.8 Mupad

A grade: { 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 161, 162, 166, 167, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade: { 4, 5, 6, 7, 16, 26, 37, 142, 143, 144, 145, 150, 155 }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 140, 141, 146, 147, 148, 149, 151, 152, 153, 154, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	72	71	49	70	113	-1
normalized size	1	1.00	0.68	0.96	0.95	0.65	0.93	1.51	-0.01
time (sec)	N/A	0.049	0.075	0.069	0.827	1.541	1.725	0.370	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	50	60	61	47	61	84	-1
normalized size	1	1.00	0.72	0.87	0.88	0.68	0.88	1.22	-0.01
time (sec)	N/A	0.029	0.030	0.007	0.466	0.570	0.862	0.170	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	52	50	40	48	64	-1
normalized size	1	1.00	0.76	0.96	0.93	0.74	0.89	1.19	-0.02
time (sec)	N/A	0.035	0.025	0.005	0.479	1.053	0.440	0.169	0.000
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	40	40	40	36	37	46	38
normalized size	1	1.00	0.89	0.89	0.89	0.80	0.82	1.02	0.84
time (sec)	N/A	0.016	0.015	0.005	0.421	0.595	0.190	0.124	0.083
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	24	24	20	24	23
normalized size	1	1.00	1.00	1.00	0.96	0.96	0.80	0.96	0.92
time (sec)	N/A	0.008	0.009	0.003	0.417	0.650	0.121	0.131	0.108

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	111	0	0	0	0	41
normalized size	1	1.00	0.90	2.18	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.058	0.088	0.395	0.000	0.652	0.000	0.000	0.079
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	31	39	49	32	48	26
normalized size	1	1.00	1.00	1.11	1.39	1.75	1.14	1.71	0.93
time (sec)	N/A	0.022	0.002	0.005	0.401	0.844	1.443	0.151	0.023
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	38	28	25	51	68	-1
normalized size	1	1.00	0.85	1.12	0.82	0.74	1.50	2.00	-0.03
time (sec)	N/A	0.014	0.008	0.005	0.405	0.774	1.141	0.205	0.000
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	53	60	73	109	77	-1
normalized size	1	1.00	0.95	0.95	1.07	1.30	1.95	1.38	-0.02
time (sec)	N/A	0.033	0.018	0.004	0.416	0.719	2.328	0.148	0.000
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	58	50	37	100	130	-1
normalized size	1	1.00	0.71	1.00	0.86	0.64	1.72	2.24	-0.02
time (sec)	N/A	0.022	0.020	0.005	0.435	0.846	1.728	0.181	0.000
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	51	73	82	85	182	101	-1
normalized size	1	1.00	0.64	0.91	1.02	1.06	2.28	1.26	-0.01
time (sec)	N/A	0.045	0.014	0.005	0.417	0.578	4.235	0.142	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	82	76	102	76	114	169	-1
normalized size	1	1.00	0.68	0.63	0.85	0.63	0.95	1.41	-0.01
time (sec)	N/A	0.194	0.033	0.191	0.569	0.744	3.140	0.189	0.000
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	74	93	0	70	90	133	-1
normalized size	1	1.00	0.76	0.95	0.00	0.71	0.92	1.36	-0.01
time (sec)	N/A	0.163	0.028	0.141	0.000	0.753	1.847	0.158	0.000
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	64	59	72	59	76	97	-1
normalized size	1	1.00	0.78	0.72	0.88	0.72	0.93	1.18	-0.01
time (sec)	N/A	0.121	0.026	0.122	0.794	0.562	0.888	0.142	0.000
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	65	0	51	51	73	-1
normalized size	1	1.00	0.92	1.08	0.00	0.85	0.85	1.22	-0.02
time (sec)	N/A	0.093	0.016	0.041	0.000	0.697	0.446	0.152	0.000
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	33	36	32	33	32
normalized size	1	1.00	1.00	1.06	0.94	1.03	0.91	0.94	0.91
time (sec)	N/A	0.045	0.011	0.034	0.706	0.504	0.182	0.123	0.137
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	169	0	0	0	0	-1
normalized size	1	1.00	1.00	2.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.039	0.061	0.000	0.736	0.000	0.000	0.000



Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	87	119	0	0	0	0	-1
normalized size	1	1.00	1.32	1.80	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.166	0.041	0.000	0.631	0.000	0.000	0.000
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	40	44	0	82	-1
normalized size	1	1.00	1.00	0.98	0.91	1.00	0.00	1.86	-0.02
time (sec)	N/A	0.080	0.023	0.043	0.532	1.309	0.000	0.179	0.000
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	139	157	0	0	0	0	-1
normalized size	1	1.00	1.20	1.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.600	0.322	0.000	0.830	0.000	0.000	0.000
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	69	76	74	62	0	185	-1
normalized size	1	1.00	0.79	0.87	0.85	0.71	0.00	2.13	-0.01
time (sec)	N/A	0.140	0.035	0.053	0.493	0.756	0.000	0.775	0.000
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	122	159	171	105	196	249	-1
normalized size	1	1.00	0.61	0.79	0.85	0.52	0.98	1.24	-0.00
time (sec)	N/A	0.385	0.069	0.081	0.518	0.695	5.683	0.184	0.000
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	112	154	0	96	160	185	-1
normalized size	1	1.00	0.67	0.92	0.00	0.57	0.96	1.11	-0.01
time (sec)	N/A	0.297	0.045	0.088	0.000	0.528	3.341	0.196	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	106	120	79	128	142	-1
normalized size	1	1.00	0.70	0.78	0.88	0.58	0.94	1.04	-0.01
time (sec)	N/A	0.225	0.041	0.062	0.520	1.844	1.867	0.225	0.000
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	96	0	69	92	101	-1
normalized size	1	1.00	0.83	0.97	0.00	0.70	0.93	1.02	-0.01
time (sec)	N/A	0.156	0.029	0.059	0.000	0.742	0.902	0.141	0.000
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	57	57	44	54	56	40
normalized size	1	1.00	1.00	0.95	0.95	0.73	0.90	0.93	0.67
time (sec)	N/A	0.080	0.011	0.040	0.540	0.649	0.447	0.143	0.144
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	229	0	0	0	0	-1
normalized size	1	1.00	1.00	2.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.055	0.065	0.000	0.484	0.000	0.000	0.000
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	133	179	0	0	0	0	-1
normalized size	1	1.00	1.23	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.120	0.113	0.000	1.282	0.000	0.000	0.000
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	92	163	0	0	0	0	-1
normalized size	1	1.00	0.90	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.260	0.143	0.000	1.401	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	284	250	0	0	0	0	-1
normalized size	1	1.00	1.59	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	2.734	0.224	0.000	0.494	0.000	0.000	0.000
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	116	225	0	0	0	0	-1
normalized size	1	1.00	0.69	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.633	0.215	0.000	0.921	0.000	0.000	0.000
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	167	320	0	153	269	362	-1
normalized size	1	1.00	0.59	1.13	0.00	0.54	0.95	1.28	-0.00
time (sec)	N/A	0.869	0.082	0.182	0.000	0.474	15.746	0.152	0.000
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	150	197	207	134	241	305	-1
normalized size	1	1.00	0.60	0.79	0.83	0.54	0.96	1.22	-0.00
time (sec)	N/A	0.663	0.072	0.081	0.457	0.415	9.506	0.148	0.000
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	135	209	0	121	190	234	-1
normalized size	1	1.00	0.68	1.06	0.00	0.61	0.96	1.18	-0.01
time (sec)	N/A	0.516	0.058	0.092	0.000	1.292	6.098	0.164	0.000
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	114	130	147	99	158	176	-1
normalized size	1	1.00	0.69	0.78	0.89	0.60	0.95	1.06	-0.01
time (sec)	N/A	0.353	0.048	0.067	0.509	0.630	3.313	0.176	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	96	117	0	82	104	127	-1
normalized size	1	1.00	0.86	1.05	0.00	0.74	0.94	1.14	-0.01
time (sec)	N/A	0.238	0.026	0.059	0.000	0.483	1.895	0.152	0.000
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	67	75	55	65	65	48
normalized size	1	1.00	1.00	0.97	1.09	0.80	0.94	0.94	0.70
time (sec)	N/A	0.118	0.021	0.041	0.540	0.409	0.859	0.124	0.139
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	287	0	0	0	0	-1
normalized size	1	1.00	1.00	2.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.046	0.062	0.000	0.483	0.000	0.000	0.000
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	198	241	0	0	0	0	-1
normalized size	1	1.00	1.27	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.260	0.114	0.000	0.560	0.000	0.000	0.000
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	124	227	0	0	0	0	-1
normalized size	1	1.00	1.04	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.265	0.161	0.000	0.642	0.000	0.000	0.000
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	399	409	0	0	0	0	-1
normalized size	1	1.00	1.45	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	3.974	0.225	0.000	1.057	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	47	-1
normalized size	1	1.00	0.73	0.73	0.00	0.00	0.00	0.85	-0.02
time (sec)	N/A	0.097	0.015	0.061	0.000	0.428	0.000	0.151	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	37	-1
normalized size	1	1.00	0.77	0.77	0.00	0.00	0.00	0.86	-0.02
time (sec)	N/A	0.080	0.120	0.056	0.000	0.515	0.000	0.139	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	35	-1
normalized size	1	1.00	0.76	0.76	0.00	0.00	0.00	0.85	-0.02
time (sec)	N/A	0.081	0.008	0.030	0.000	0.532	0.000	0.130	0.000
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	25	-1
normalized size	1	1.00	0.83	0.83	0.00	0.00	0.00	0.86	-0.03
time (sec)	N/A	0.062	0.076	0.030	0.000	0.540	0.000	0.146	0.000
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	0	0	0	23	-1
normalized size	1	1.00	0.81	0.81	0.00	0.00	0.00	0.85	-0.04
time (sec)	N/A	0.063	0.007	0.030	0.000	0.540	0.000	0.159	0.000
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	12	-1
normalized size	1	1.00	1.00	0.93	0.00	0.00	0.00	0.86	-0.07
time (sec)	N/A	0.035	0.020	0.043	0.000	0.457	0.000	0.148	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	9	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	0.00	1.00	-0.11
time (sec)	N/A	0.017	0.009	0.023	0.000	0.548	0.000	0.181	0.000
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	0.196	0.145	0.000	0.595	0.000	0.000	0.000
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	1.025	0.181	0.000	0.543	0.000	0.000	0.000
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	105	0	0	0	161	-1
normalized size	1	1.00	1.04	1.27	0.00	0.00	0.00	1.94	-0.01
time (sec)	N/A	0.074	0.248	0.066	0.000	0.490	0.000	0.150	0.000
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	78	78	0	0	0	120	-1
normalized size	1	1.00	1.10	1.10	0.00	0.00	0.00	1.69	-0.01
time (sec)	N/A	0.063	0.043	0.056	0.000	0.634	0.000	0.142	0.000
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	81	0	0	0	115	-1
normalized size	1	1.00	0.88	1.17	0.00	0.00	0.00	1.67	-0.01
time (sec)	N/A	0.058	0.184	0.034	0.000	0.474	0.000	0.155	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	54	0	0	0	72	-1
normalized size	1	1.00	0.98	0.95	0.00	0.00	0.00	1.26	-0.02
time (sec)	N/A	0.049	0.016	0.032	0.000	0.480	0.000	0.206	0.000
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	57	0	0	0	68	-1
normalized size	1	1.00	0.91	1.04	0.00	0.00	0.00	1.24	-0.02
time (sec)	N/A	0.044	0.159	0.031	0.000	0.492	0.000	0.206	0.000
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	28	0	0	0	36	-1
normalized size	1	1.00	0.84	0.74	0.00	0.00	0.00	0.95	-0.03
time (sec)	N/A	0.025	0.003	0.038	0.000	0.507	0.000	0.259	0.000
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	33	0	0	0	34	-1
normalized size	1	1.00	0.89	0.92	0.00	0.00	0.00	0.94	-0.03
time (sec)	N/A	0.078	0.066	0.025	0.000	0.457	0.000	0.226	0.000
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	0.861	0.100	0.000	0.494	0.000	0.000	0.000
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	9.009	0.162	0.000	0.770	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	103	121	0	0	0	170	-1
normalized size	1	1.00	1.05	1.23	0.00	0.00	0.00	1.73	-0.01
time (sec)	N/A	0.343	0.162	0.059	0.000	0.753	0.000	0.278	0.000
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	82	0	0	0	125	-1
normalized size	1	1.00	0.88	0.99	0.00	0.00	0.00	1.51	-0.01
time (sec)	N/A	0.300	0.186	0.050	0.000	0.553	0.000	0.269	0.000
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	82	0	0	0	102	-1
normalized size	1	1.00	0.83	1.00	0.00	0.00	0.00	1.24	-0.01
time (sec)	N/A	0.249	0.112	0.033	0.000	0.564	0.000	0.208	0.000
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	45	0	0	0	67	-1
normalized size	1	1.00	0.95	0.70	0.00	0.00	0.00	1.05	-0.02
time (sec)	N/A	0.168	0.068	0.041	0.000	0.519	0.000	0.210	0.000
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	43	0	0	0	43	-1
normalized size	1	1.00	0.94	0.84	0.00	0.00	0.00	0.84	-0.02
time (sec)	N/A	0.085	0.022	0.026	0.000	0.512	0.000	0.205	0.000
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	0.502	0.105	0.000	0.431	0.000	0.000	0.000



Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	6.370	0.170	0.000	0.446	0.000	0.000	0.000
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	159	171	0	0	0	250	-1
normalized size	1	1.00	1.01	1.08	0.00	0.00	0.00	1.58	-0.01
time (sec)	N/A	0.314	0.293	0.064	0.000	0.416	0.000	0.277	0.000
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	107	114	0	0	0	174	-1
normalized size	1	1.00	0.74	0.79	0.00	0.00	0.00	1.21	-0.01
time (sec)	N/A	0.282	0.355	0.055	0.000	0.485	0.000	0.232	0.000
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	102	117	0	0	0	148	-1
normalized size	1	1.00	0.72	0.83	0.00	0.00	0.00	1.05	-0.01
time (sec)	N/A	0.303	0.227	0.036	0.000	0.441	0.000	0.212	0.000
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	60	0	0	0	92	-1
normalized size	1	1.00	0.89	0.62	0.00	0.00	0.00	0.95	-0.01
time (sec)	N/A	0.164	0.118	0.043	0.000	0.433	0.000	0.380	0.000
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	63	0	0	0	66	-1
normalized size	1	1.00	0.90	0.81	0.00	0.00	0.00	0.85	-0.01
time (sec)	N/A	0.153	0.055	0.032	0.000	0.521	0.000	0.249	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	2.330	0.105	0.000	0.623	0.000	0.000	0.000
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	17.403	0.169	0.000	0.452	0.000	0.000	0.000
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	204	143	0	0	0	247	-1
normalized size	1	1.00	1.69	1.18	0.00	0.00	0.00	2.04	-0.01
time (sec)	N/A	0.242	0.100	0.129	0.000	0.000	0.000	0.297	0.000
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	138	90	0	0	0	153	-1
normalized size	1	1.00	1.45	0.95	0.00	0.00	0.00	1.61	-0.01
time (sec)	N/A	0.189	0.059	0.083	0.000	0.000	0.000	0.531	0.000
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	126	96	0	0	0	165	-1
normalized size	1	1.00	1.47	1.12	0.00	0.00	0.00	1.92	-0.01
time (sec)	N/A	0.181	0.045	0.076	0.000	0.000	0.000	0.268	0.000
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	42	0	0	0	71	-1
normalized size	1	1.00	1.37	0.71	0.00	0.00	0.00	1.20	-0.02
time (sec)	N/A	0.151	0.019	0.056	0.000	0.000	0.000	0.571	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	66	49	0	0	0	83	-1
normalized size	1	1.00	1.50	1.11	0.00	0.00	0.00	1.89	-0.02
time (sec)	N/A	0.090	0.026	0.050	0.000	0.000	0.000	0.224	0.000
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.462	0.124	0.000	0.000	0.000	0.000	0.000
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	282	202	193	0	0	0	355	-1
normalized size	1	1.32	0.94	0.90	0.00	0.00	0.00	1.66	-0.00
time (sec)	N/A	0.533	0.065	0.122	0.000	0.000	0.000	0.371	0.000
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	130	121	0	0	0	225	-1
normalized size	1	1.00	0.83	0.77	0.00	0.00	0.00	1.43	-0.01
time (sec)	N/A	0.379	0.033	0.089	0.000	0.000	0.000	0.310	0.000
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	136	131	0	0	0	237	-1
normalized size	1	1.00	0.93	0.89	0.00	0.00	0.00	1.61	-0.01
time (sec)	N/A	0.303	0.050	0.080	0.000	0.000	0.000	0.471	0.000
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	64	0	0	0	107	-1
normalized size	1	1.00	0.80	0.72	0.00	0.00	0.00	1.20	-0.01
time (sec)	N/A	0.183	0.014	0.076	0.000	0.000	0.000	0.273	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	76	72	0	0	0	119	-1
normalized size	1	1.00	1.01	0.96	0.00	0.00	0.00	1.59	-0.01
time (sec)	N/A	0.099	0.038	0.053	0.000	0.000	0.000	0.333	0.000
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.373	0.118	0.000	0.000	0.000	0.000	0.000
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	298	204	233	0	0	0	463	-1
normalized size	1	1.13	0.78	0.89	0.00	0.00	0.00	1.76	-0.00
time (sec)	N/A	0.803	0.067	0.140	0.000	0.000	0.000	0.321	0.000
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	140	154	0	0	0	297	-1
normalized size	1	1.00	0.68	0.75	0.00	0.00	0.00	1.45	-0.00
time (sec)	N/A	0.600	0.039	0.095	0.000	0.000	0.000	0.311	0.000
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	125	156	0	0	0	309	-1
normalized size	1	1.00	0.70	0.88	0.00	0.00	0.00	1.74	-0.01
time (sec)	N/A	0.469	0.058	0.091	0.000	0.000	0.000	0.536	0.000
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	81	79	0	0	0	143	-1
normalized size	1	1.00	0.68	0.66	0.00	0.00	0.00	1.20	-0.01
time (sec)	N/A	0.313	0.015	0.067	0.000	0.000	0.000	0.283	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	88	0	0	0	155	-1
normalized size	1	1.00	0.77	1.00	0.00	0.00	0.00	1.76	-0.01
time (sec)	N/A	0.165	0.036	0.061	0.000	0.000	0.000	0.347	0.000
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.381	0.115	0.000	0.000	0.000	0.000	0.000
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	193	72	0	0	0	139	-1
normalized size	1	1.00	1.82	0.68	0.00	0.00	0.00	1.31	-0.01
time (sec)	N/A	0.112	0.051	0.101	0.000	0.000	0.000	0.559	0.000
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	128	44	0	0	0	81	-1
normalized size	1	1.00	1.97	0.68	0.00	0.00	0.00	1.25	-0.02
time (sec)	N/A	0.083	0.036	0.066	0.000	0.000	0.000	0.312	0.000
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	128	51	0	0	0	93	-1
normalized size	1	1.00	1.80	0.72	0.00	0.00	0.00	1.31	-0.01
time (sec)	N/A	0.089	0.047	0.068	0.000	0.000	0.000	0.370	0.000
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	71	21	0	0	0	35	-1
normalized size	1	1.00	2.54	0.75	0.00	0.00	0.00	1.25	-0.04
time (sec)	N/A	0.045	0.017	0.052	0.000	0.000	0.000	0.310	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	69	25	0	0	0	47	-1
normalized size	1	1.00	2.30	0.83	0.00	0.00	0.00	1.57	-0.03
time (sec)	N/A	0.024	0.025	0.047	0.000	0.000	0.000	0.590	0.000
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	0.354	0.108	0.000	0.000	0.000	0.000	0.000
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	2.973	0.213	0.000	0.000	0.000	0.000	0.000
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	427	184	0	0	0	0	-1
normalized size	1	1.00	2.50	1.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.219	0.142	0.000	0.000	0.000	0.000	0.000
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	231	121	0	0	0	0	-1
normalized size	1	1.00	1.82	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.142	0.112	0.000	0.000	0.000	0.000	0.000
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	319	141	0	0	0	0	-1
normalized size	1	1.00	2.35	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.115	0.079	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	154	83	0	0	0	0	-1
normalized size	1	1.00	1.71	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.043	0.063	0.000	0.000	0.000	0.000	0.000
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	211	95	0	0	0	0	-1
normalized size	1	1.00	2.20	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.067	0.065	0.000	0.000	0.000	0.000	0.000
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	91	43	0	0	0	0	-1
normalized size	1	1.00	1.65	0.78	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.029	0.052	0.000	0.000	0.000	0.000	0.000
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	87	65	0	0	0	0	-1
normalized size	1	1.00	1.47	1.10	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.092	0.055	0.000	0.000	0.000	0.000	0.000
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	0.449	0.109	0.000	0.000	0.000	0.000	0.000
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	235	418	173	0	0	0	0	-1
normalized size	1	1.37	2.44	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.434	0.295	0.117	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	200	109	0	0	0	0	-1
normalized size	1	1.00	1.59	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.338	0.380	0.081	0.000	0.000	0.000	0.000	0.000
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	277	117	0	0	0	0	-1
normalized size	1	1.00	2.22	0.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.301	0.150	0.069	0.000	0.000	0.000	0.000	0.000
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	112	56	0	0	0	0	-1
normalized size	1	1.00	1.26	0.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.212	0.057	0.000	0.000	0.000	0.000	0.000
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	138	83	0	0	0	0	-1
normalized size	1	1.00	1.82	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.174	0.059	0.000	0.000	0.000	0.000	0.000
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.520	0.099	0.000	0.000	0.000	0.000	0.000
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	417	225	0	0	0	0	-1
normalized size	1	1.00	1.58	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.396	0.741	0.126	0.000	0.000	0.000	0.000	0.000



Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	272	139	0	0	0	0	-1
normalized size	1	1.00	1.43	0.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	1.107	0.090	0.000	0.000	0.000	0.000	0.000
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	280	154	0	0	0	0	-1
normalized size	1	1.00	1.47	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.348	0.344	0.082	0.000	0.000	0.000	0.000	0.000
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	146	73	0	0	0	0	-1
normalized size	1	1.00	1.23	0.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.368	0.059	0.000	0.000	0.000	0.000	0.000
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	143	110	0	0	0	0	-1
normalized size	1	1.00	1.36	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.242	0.062	0.000	0.000	0.000	0.000	0.000
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.469	0.100	0.000	0.000	0.000	0.000	0.000
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.122	1.063	1.172	0.000	0.475	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.118	0.977	0.810	0.000	0.548	0.000	0.000	0.000
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	122	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.039	0.768	0.000	0.529	0.000	0.000	0.000
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.019	0.797	0.000	0.529	0.000	0.000	0.000
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.017	0.558	0.653	0.000	0.433	0.000	0.000	0.000
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	0.585	0.700	0.000	0.532	0.000	0.000	0.000
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.016	2.693	0.106	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	2.975	0.091	0.000	0.000	0.000	0.000	0.000
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	2.252	0.090	0.000	0.000	0.000	0.000	0.000
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	2.279	0.088	0.000	0.000	0.000	0.000	0.000
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.016	0.850	0.984	0.000	0.550	0.000	0.000	0.000
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	132	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.100	0.256	0.000	0.449	0.000	0.000	0.000
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	137	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.074	0.229	0.000	0.527	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	138	0	0	0	0	-1
normalized size	1	1.00	0.88	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.020	0.166	0.000	0.538	0.000	0.000	0.000
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	240	0	0	0	0	-1
normalized size	1	1.00	0.92	3.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.036	0.114	0.000	0.539	0.000	0.000	0.000
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.015	0.308	0.145	0.000	0.476	0.000	0.000	0.000
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.016	0.673	0.120	0.000	0.521	0.000	0.000	0.000
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.021	3.580	0.127	0.000	0.534	0.000	0.000	0.000
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.017	4.282	0.107	0.000	0.458	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.018	1.599	0.104	0.000	0.460	0.000	0.000	0.000
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.021	1.927	0.097	0.000	0.454	0.000	0.000	0.000
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	81	72	70	61	80	95	-1
normalized size	1	1.00	1.07	0.95	0.92	0.80	1.05	1.25	-0.01
time (sec)	N/A	0.035	0.030	0.008	0.449	0.427	1.162	0.298	0.000
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	64	59	53	65	74	-1
normalized size	1	1.00	0.82	1.07	0.98	0.88	1.08	1.23	-0.02
time (sec)	N/A	0.039	0.041	0.005	0.449	0.472	0.493	0.354	0.000
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	56	52	49	49	54	64	45
normalized size	1	1.00	1.10	1.02	0.96	0.96	1.06	1.25	0.88
time (sec)	N/A	0.019	0.020	0.007	0.434	0.452	0.256	0.518	0.147
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	30	29	31	26	29	28
normalized size	1	1.00	1.00	1.00	0.97	1.03	0.87	0.97	0.93
time (sec)	N/A	0.014	0.014	0.003	0.434	0.448	0.136	0.463	0.167

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	122	0	0	0	0	48
normalized size	1	1.00	0.83	1.94	0.00	0.00	0.00	0.00	0.76
time (sec)	N/A	0.070	0.032	0.049	0.000	0.610	0.000	0.000	0.150
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	36	43	47	55	39	325	34
normalized size	1	1.00	1.09	1.30	1.42	1.67	1.18	9.85	1.03
time (sec)	N/A	0.027	0.003	0.004	0.423	0.560	1.711	0.370	0.129
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	50	36	35	61	163	-1
normalized size	1	1.00	1.13	1.28	0.92	0.90	1.56	4.18	-0.03
time (sec)	N/A	0.019	0.012	0.006	0.471	0.544	1.448	0.285	0.000
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	65	69	80	119	284	-1
normalized size	1	1.00	1.08	1.05	1.11	1.29	1.92	4.58	-0.02
time (sec)	N/A	0.038	0.018	0.005	0.440	0.573	2.606	0.918	0.000
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	95	126	142	111	170	194	-1
normalized size	1	1.00	0.93	1.24	1.39	1.09	1.67	1.90	-0.01
time (sec)	N/A	0.154	0.178	0.034	0.458	0.446	1.063	0.387	0.000
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	120	0	99	126	155	-1
normalized size	1	1.00	0.96	1.58	0.00	1.30	1.66	2.04	-0.01
time (sec)	N/A	0.119	0.075	0.035	0.000	1.093	0.554	0.264	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	72	72	65	82	75	142
normalized size	1	1.00	1.00	1.53	1.53	1.38	1.74	1.60	3.02
time (sec)	N/A	0.060	0.044	0.038	0.466	0.400	0.267	0.400	0.347
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	143	319	0	0	0	0	-1
normalized size	1	1.00	1.59	3.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.152	0.046	0.000	0.480	0.000	0.000	0.000
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	171	0	0	0	0	-1
normalized size	1	1.00	1.56	2.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.225	0.028	0.000	0.537	0.000	0.000	0.000
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	163	235	273	194	328	368	-1
normalized size	1	1.00	0.92	1.32	1.53	1.09	1.84	2.07	-0.01
time (sec)	N/A	0.297	0.381	0.034	0.485	1.774	2.489	0.595	0.000
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	114	219	0	169	264	285	-1
normalized size	1	1.00	0.91	1.75	0.00	1.35	2.11	2.28	-0.01
time (sec)	N/A	0.205	0.140	0.039	0.000	0.545	1.296	0.420	0.000
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	132	141	108	160	150	242
normalized size	1	1.00	0.94	1.61	1.72	1.32	1.95	1.83	2.95
time (sec)	N/A	0.109	0.082	0.040	0.441	2.030	0.596	0.625	0.407

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	244	592	0	0	0	0	-1
normalized size	1	1.00	1.98	4.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.234	0.032	0.000	0.433	0.000	0.000	0.000
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	283	378	0	0	0	0	-1
normalized size	1	1.00	2.07	2.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.316	0.102	0.000	0.746	0.000	0.000	0.000
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	117	91	102	0	0	0	173	-1
normalized size	1	0.97	0.75	0.84	0.00	0.00	0.00	1.43	-0.01
time (sec)	N/A	0.254	0.201	0.033	0.000	0.523	0.000	0.448	0.000
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	86	-1
normalized size	1	1.00	0.89	0.92	0.00	0.00	0.00	1.37	-0.02
time (sec)	N/A	0.132	0.079	0.034	0.000	0.480	0.000	0.316	0.000
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	48	0	0	0	49	-1
normalized size	1	1.00	0.83	0.91	0.00	0.00	0.00	0.92	-0.02
time (sec)	N/A	0.067	0.073	0.037	0.000	0.647	0.000	0.644	0.000
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.024	0.251	0.214	0.000	0.974	0.000	0.000	0.000



Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	2.179	0.345	0.000	1.377	0.000	0.000	0.000
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	152	125	149	0	0	0	646	-1
normalized size	1	0.97	0.80	0.96	0.00	0.00	0.00	4.14	-0.01
time (sec)	N/A	0.182	0.561	0.037	0.000	0.499	0.000	0.613	0.000
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	77	0	0	0	326	-1
normalized size	1	1.00	0.88	0.86	0.00	0.00	0.00	3.62	-0.01
time (sec)	N/A	0.100	0.294	0.030	0.000	0.566	0.000	0.658	0.000
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	82	72	76	0	0	0	192	-1
normalized size	1	0.95	0.84	0.88	0.00	0.00	0.00	2.23	-0.01
time (sec)	N/A	0.169	0.151	0.042	0.000	0.674	0.000	0.493	0.000
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.023	4.117	0.228	0.000	0.584	0.000	0.000	0.000
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.023	34.250	0.447	0.000	0.657	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	245	168	290	0	0	0	1539	-1
normalized size	1	1.24	0.85	1.47	0.00	0.00	0.00	7.81	-0.01
time (sec)	N/A	0.542	0.874	0.045	0.000	1.182	0.000	0.822	0.000
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	108	157	0	0	0	864	-1
normalized size	1	1.00	0.83	1.21	0.00	0.00	0.00	6.65	-0.01
time (sec)	N/A	0.323	0.386	0.033	0.000	0.542	0.000	1.515	0.000
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	93	138	0	0	0	482	-1
normalized size	1	1.00	0.84	1.24	0.00	0.00	0.00	4.34	-0.01
time (sec)	N/A	0.172	0.464	0.043	0.000	0.484	0.000	1.073	0.000
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.023	2.405	0.352	0.000	0.513	0.000	0.000	0.000
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.023	19.117	0.613	0.000	0.522	0.000	0.000	0.000
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	246	356	0	0	0	1057	-1
normalized size	1	1.00	1.02	1.47	0.00	0.00	0.00	4.37	-0.00
time (sec)	N/A	0.756	0.320	0.147	0.000	0.000	0.000	2.133	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	141	173	0	0	0	448	-1
normalized size	1	1.00	1.03	1.26	0.00	0.00	0.00	3.27	-0.01
time (sec)	N/A	0.453	0.076	0.080	0.000	0.000	0.000	1.460	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	178	0	0	0	531	-1
normalized size	1	1.00	0.99	1.48	0.00	0.00	0.00	4.42	-0.01
time (sec)	N/A	0.331	0.099	0.069	0.000	0.000	0.000	2.051	0.000
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.046	3.487	0.140	0.000	0.000	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	11.904	0.269	0.000	0.000	0.000	0.000	0.000
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	245	542	0	0	0	1967	-1
normalized size	1	1.00	0.78	1.73	0.00	0.00	0.00	6.28	-0.00
time (sec)	N/A	1.046	0.306	0.170	0.000	0.000	0.000	4.168	0.000
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	126	267	0	0	0	845	-1
normalized size	1	1.00	0.73	1.55	0.00	0.00	0.00	4.91	-0.01
time (sec)	N/A	0.525	0.066	0.095	0.000	0.000	0.000	2.090	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	291	270	0	0	0	993	-1
normalized size	1	1.00	1.83	1.70	0.00	0.00	0.00	6.25	-0.01
time (sec)	N/A	0.281	2.976	0.089	0.000	0.000	0.000	2.792	0.000
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.045	3.123	0.159	0.000	0.000	0.000	0.000	0.000
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.042	10.575	0.282	0.000	0.000	0.000	0.000	0.000
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	228	792	0	0	0	2667	-1
normalized size	1	1.00	0.64	2.21	0.00	0.00	0.00	7.45	-0.00
time (sec)	N/A	1.409	0.284	0.191	0.000	0.000	0.000	6.680	0.000
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	141	394	0	0	0	1307	-1
normalized size	1	1.00	0.65	1.82	0.00	0.00	0.00	6.05	-0.00
time (sec)	N/A	0.743	0.100	0.111	0.000	0.000	0.000	2.729	0.000
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	379	393	0	0	0	1179	-1
normalized size	1	1.00	2.12	2.20	0.00	0.00	0.00	6.59	-0.01
time (sec)	N/A	0.502	3.190	0.105	0.000	0.000	0.000	4.075	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.044	3.248	0.146	0.000	0.000	0.000	0.000	0.000
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.043	10.601	0.280	0.000	0.000	0.000	0.000	0.000
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	228	168	0	0	0	317	-1
normalized size	1	1.00	1.02	0.75	0.00	0.00	0.00	1.42	-0.00
time (sec)	N/A	0.421	0.289	0.112	0.000	0.000	0.000	1.862	0.000
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	123	80	0	0	0	132	-1
normalized size	1	1.00	1.24	0.81	0.00	0.00	0.00	1.33	-0.01
time (sec)	N/A	0.176	0.077	0.056	0.000	0.000	0.000	2.057	0.000
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	121	83	0	0	0	159	-1
normalized size	1	1.00	1.20	0.82	0.00	0.00	0.00	1.57	-0.01
time (sec)	N/A	0.091	0.088	0.042	0.000	0.000	0.000	1.456	0.000
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	3.119	0.132	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	11.747	0.244	0.000	0.000	0.000	0.000	0.000
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	343	295	0	0	0	0	-1
normalized size	1	1.00	1.37	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.472	0.118	0.000	0.000	0.000	0.000	0.000
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	155	143	0	0	0	0	-1
normalized size	1	1.00	1.19	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.150	0.073	0.000	0.000	0.000	0.000	0.000
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	167	149	0	0	0	0	-1
normalized size	1	1.00	1.22	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.329	0.072	0.000	0.000	0.000	0.000	0.000
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.044	3.812	0.118	0.000	0.000	0.000	0.000	0.000
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.043	11.525	0.270	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	370	660	0	0	0	0	-1
normalized size	1	1.00	1.27	2.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.007	1.877	0.153	0.000	0.000	0.000	0.000	0.000
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	173	311	0	0	0	0	-1
normalized size	1	1.00	0.96	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.506	1.359	0.090	0.000	0.000	0.000	0.000	0.000
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	214	325	0	0	0	0	-1
normalized size	1	1.00	1.31	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.978	0.090	0.000	0.000	0.000	0.000	0.000
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.045	3.950	0.118	0.000	0.000	0.000	0.000	0.000
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.045	11.565	0.271	0.000	0.000	0.000	0.000	0.000
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	100	144	0	0	82	0	-1
normalized size	1	1.00	0.83	1.20	0.00	0.00	0.68	0.00	-0.01
time (sec)	N/A	0.076	0.039	0.045	0.000	1.305	104.624	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	66	138	0	0	82	0	-1
normalized size	1	1.00	0.53	1.11	0.00	0.00	0.66	0.00	-0.01
time (sec)	N/A	0.098	0.032	0.011	0.000	2.005	17.699	0.000	0.000
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	119	0	0	76	0	-1
normalized size	1	1.00	0.75	1.35	0.00	0.00	0.86	0.00	-0.01
time (sec)	N/A	0.045	0.017	0.011	0.000	0.566	2.638	0.000	0.000
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	45	98	0	0	0	0	-1
normalized size	1	1.00	0.51	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.012	0.010	0.000	0.469	0.000	0.000	0.000
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	85	0	0	0	0	-1
normalized size	1	1.00	0.73	1.55	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.014	0.011	0.000	0.611	0.000	0.000	0.000
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	42	129	0	0	0	0	-1
normalized size	1	1.00	0.34	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.016	0.014	0.000	0.539	0.000	0.000	0.000
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	90	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.062	0.180	0.000	0.446	0.000	0.000	0.000



Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	90	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.075	0.171	0.000	1.933	0.000	0.000	0.000
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	90	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.052	0.174	0.000	1.565	0.000	0.000	0.000
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F(-1)	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.051	180.000	0.000	0.486	0.000	0.000	0.000
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	87	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.051	0.184	0.000	0.445	0.000	0.000	0.000
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	87	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.060	0.175	0.000	0.503	0.000	0.000	0.000
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	39.368	0.180	0.000	0.417	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	180.003	0.172	0.000	0.484	0.000	0.000	0.000
Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	F(-1)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.160	9.011	180.000	0.000	0.470	0.000	0.000	0.000
Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.159	8.443	0.178	0.000	0.449	0.000	0.000	0.000
Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	14.411	0.178	0.000	0.456	0.000	0.000	0.000
Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.030	3.069	0.116	0.000	0.388	0.000	0.000	0.000
Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	2.897	0.122	0.000	0.386	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	0.858	0.126	0.000	0.388	0.000	0.000	0.000
Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.032	4.211	0.122	0.000	0.424	0.000	0.000	0.000
Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	6.205	0.122	0.000	0.481	0.000	0.000	0.000
Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	5.287	0.122	0.000	0.503	0.000	0.000	0.000
Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	9.361	0.132	0.000	0.398	0.000	0.000	0.000
Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	14.813	0.125	0.000	0.395	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [30] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	3	3	1.00	6	0.500
5	A	2	2	1.00	4	0.500
6	A	5	5	1.00	8	0.625
7	A	4	4	1.00	8	0.500
8	A	2	2	1.00	8	0.250
9	A	5	5	1.00	8	0.625
10	A	3	3	1.00	8	0.375
11	A	6	5	1.00	8	0.625
12	A	7	5	1.00	10	0.500
13	A	6	4	1.00	10	0.400
14	A	5	5	1.00	10	0.500
15	A	4	4	1.00	8	0.500
16	A	3	3	1.00	6	0.500
17	A	6	6	1.00	10	0.600
18	A	7	5	1.00	10	0.500
19	A	3	3	1.00	10	0.300
20	A	9	7	1.00	10	0.700
21	A	5	5	1.00	10	0.500
22	A	14	7	1.00	10	0.700
23	A	11	5	1.00	10	0.500
24	A	9	7	1.00	10	0.700
25	A	6	5	1.00	8	0.625
26	A	4	3	1.00	6	0.500
27	A	7	7	1.00	10	0.700

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	9	6	1.00	10	0.600
29	A	7	7	1.00	10	0.700
30	A	14	10	1.00	10	1.000
31	A	10	9	1.00	10	0.900
32	A	23	4	1.00	10	0.400
33	A	19	6	1.00	10	0.600
34	A	14	4	1.00	10	0.400
35	A	11	6	1.00	10	0.600
36	A	7	4	1.00	8	0.500
37	A	5	3	1.00	6	0.500
38	A	8	7	1.00	10	0.700
39	A	11	7	1.00	10	0.700
40	A	8	8	1.00	10	0.800
41	A	19	10	1.00	10	1.000
42	A	7	3	1.00	10	0.300
43	A	6	3	1.00	10	0.300
44	A	6	3	1.00	10	0.300
45	A	5	3	1.00	10	0.300
46	A	5	3	1.00	10	0.300
47	A	4	4	1.00	8	0.500
48	A	2	2	1.00	6	0.333
49	A	0	0	0.00	0	0.000
50	A	0	0	0.00	0	0.000
51	A	6	2	1.00	10	0.200
52	A	5	2	1.00	10	0.200
53	A	5	2	1.00	10	0.200
54	A	4	2	1.00	10	0.200
55	A	4	2	1.00	10	0.200
56	A	2	2	1.00	8	0.250
57	A	3	3	1.00	6	0.500
58	A	0	0	0.00	0	0.000
59	A	0	0	0.00	0	0.000
60	A	14	5	1.00	10	0.500
61	A	12	6	1.00	10	0.600
62	A	10	6	1.00	10	0.600
63	A	7	7	1.00	8	0.875

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	4	1.00	6	0.667
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	12	4	1.00	10	0.400
68	A	9	4	1.00	10	0.400
69	A	10	6	1.00	10	0.600
70	A	5	5	1.00	8	0.625
71	A	5	4	1.00	6	0.667
72	A	0	0	0.00	0	0.000
73	A	0	0	0.00	0	0.000
74	A	10	5	1.00	12	0.417
75	A	8	5	1.00	12	0.417
76	A	8	5	1.00	12	0.417
77	A	6	5	1.00	10	0.500
78	A	4	4	1.00	8	0.500
79	A	0	0	0.00	0	0.000
80	A	23	8	1.32	12	0.667
81	A	16	8	1.00	12	0.667
82	A	13	8	1.00	12	0.667
83	A	8	8	1.00	10	0.800
84	A	5	5	1.00	8	0.625
85	A	0	0	0.00	0	0.000
86	A	26	8	1.13	12	0.667
87	A	18	7	1.00	12	0.583
88	A	15	8	1.00	12	0.667
89	A	9	7	1.00	10	0.700
90	A	6	5	1.00	8	0.625
91	A	0	0	0.00	0	0.000
92	A	9	4	1.00	12	0.333
93	A	7	4	1.00	12	0.333
94	A	7	4	1.00	12	0.333
95	A	5	5	1.00	10	0.500
96	A	3	3	1.00	8	0.375
97	A	0	0	0.00	0	0.000
98	A	0	0	0.00	0	0.000
99	A	10	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	8	3	1.00	12	0.250
101	A	8	3	1.00	12	0.250
102	A	6	3	1.00	12	0.250
103	A	6	3	1.00	12	0.250
104	A	3	3	1.00	10	0.300
105	A	4	4	1.00	8	0.500
106	A	0	0	0.00	0	0.000
107	A	19	6	1.37	12	0.500
108	A	15	7	1.00	12	0.583
109	A	13	7	1.00	12	0.583
110	A	8	8	1.00	10	0.800
111	A	5	5	1.00	8	0.625
112	A	0	0	0.00	0	0.000
113	A	17	5	1.00	12	0.417
114	A	12	5	1.00	12	0.417
115	A	13	7	1.00	12	0.583
116	A	6	6	1.00	10	0.600
117	A	6	5	1.00	8	0.625
118	A	0	0	0.00	0	0.000
119	A	0	0	0.00	0	0.000
120	A	0	0	0.00	0	0.000
121	A	2	2	1.00	12	0.167
122	A	2	2	1.00	10	0.200
123	A	0	0	0.00	0	0.000
124	A	0	0	0.00	0	0.000
125	A	0	0	0.00	0	0.000
126	A	0	0	0.00	0	0.000
127	A	0	0	0.00	0	0.000
128	A	0	0	0.00	0	0.000
129	A	0	0	0.00	0	0.000
130	A	9	4	1.00	10	0.400
131	A	9	4	1.00	10	0.400
132	A	6	5	1.00	8	0.625
133	A	4	3	1.00	6	0.500
134	A	0	0	0.00	0	0.000
135	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	0	0	0.00	0	0.000
137	A	0	0	0.00	0	0.000
138	A	0	0	0.00	0	0.000
139	A	0	0	0.00	0	0.000
140	A	4	3	1.00	12	0.250
141	A	4	3	1.00	12	0.250
142	A	3	3	1.00	10	0.300
143	A	3	2	1.00	8	0.250
144	A	5	5	1.00	12	0.417
145	A	4	4	1.00	12	0.333
146	A	2	2	1.00	12	0.167
147	A	5	5	1.00	12	0.417
148	A	5	5	1.00	14	0.357
149	A	4	4	1.00	12	0.333
150	A	3	3	1.00	10	0.300
151	A	6	6	1.00	14	0.429
152	A	7	5	1.00	14	0.357
153	A	10	7	1.00	14	0.500
154	A	6	5	1.00	12	0.417
155	A	5	3	1.00	10	0.300
156	A	7	7	1.00	14	0.500
157	A	9	6	1.00	14	0.429
158	A	9	5	0.97	14	0.357
159	A	6	6	1.00	12	0.500
160	A	4	4	1.00	10	0.400
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	8	4	0.97	14	0.286
164	A	4	4	1.00	12	0.333
165	A	5	5	0.95	10	0.500
166	A	0	0	0.00	0	0.000
167	A	0	0	0.00	0	0.000
168	A	16	8	1.24	14	0.571
169	A	9	9	1.00	12	0.750
170	A	6	6	1.00	10	0.600
171	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	0	0	0.00	0	0.000
173	A	14	8	1.00	16	0.500
174	A	9	8	1.00	14	0.571
175	A	7	7	1.00	12	0.583
176	A	0	0	0.00	0	0.000
177	A	0	0	0.00	0	0.000
178	A	22	11	1.00	16	0.688
179	A	11	11	1.00	14	0.786
180	A	8	8	1.00	12	0.667
181	A	0	0	0.00	0	0.000
182	A	0	0	0.00	0	0.000
183	A	24	11	1.00	16	0.688
184	A	12	10	1.00	14	0.714
185	A	9	8	1.00	12	0.667
186	A	0	0	0.00	0	0.000
187	A	0	0	0.00	0	0.000
188	A	13	7	1.00	16	0.438
189	A	8	8	1.00	14	0.571
190	A	6	6	1.00	12	0.500
191	A	0	0	0.00	0	0.000
192	A	0	0	0.00	0	0.000
193	A	12	6	1.00	16	0.375
194	A	6	6	1.00	14	0.429
195	A	7	7	1.00	12	0.583
196	A	0	0	0.00	0	0.000
197	A	0	0	0.00	0	0.000
198	A	22	10	1.00	16	0.625
199	A	11	11	1.00	14	0.786
200	A	8	8	1.00	12	0.667
201	A	0	0	0.00	0	0.000
202	A	0	0	0.00	0	0.000
203	A	5	4	1.00	16	0.250
204	A	7	7	1.00	16	0.438
205	A	4	4	1.00	16	0.250
206	A	6	6	1.00	16	0.375
207	A	3	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	7	7	1.00	16	0.438
209	A	2	2	1.00	18	0.111
210	A	2	2	1.00	18	0.111
211	A	2	2	1.00	18	0.111
212	A	2	2	1.00	18	0.111
213	A	2	2	1.00	18	0.111
214	A	2	2	1.00	18	0.111
215	A	0	0	0.00	0	0.000
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	0	0	0.00	0	0.000
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	0	0	0.00	0	0.000
224	A	0	0	0.00	0	0.000
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000

# Chapter 3

## Listing of integrals

### 3.1 $\int x^4 \sin^{-1}(ax) dx$

Optimal. Leaf size=75

$$\frac{(1 - a^2x^2)^{5/2}}{25a^5} - \frac{2(1 - a^2x^2)^{3/2}}{15a^5} + \frac{\sqrt{1 - a^2x^2}}{5a^5} + \frac{1}{5}x^5 \sin^{-1}(ax)$$

[Out]  $-2/15*(-a^2*x^2+1)^{(3/2)}/a^5+1/25*(-a^2*x^2+1)^{(5/2)}/a^5+1/5*x^5*\arcsin(a*x)+1/5*(-a^2*x^2+1)^{(1/2)}/a^5$

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4627, 266, 43}

$$\frac{(1 - a^2x^2)^{5/2}}{25a^5} - \frac{2(1 - a^2x^2)^{3/2}}{15a^5} + \frac{\sqrt{1 - a^2x^2}}{5a^5} + \frac{1}{5}x^5 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^4\*ArcSin[a\*x],x]

[Out]  $\text{Sqrt}[1 - a^2*x^2]/(5*a^5) - (2*(1 - a^2*x^2)^{(3/2)})/(15*a^5) + (1 - a^2*x^2)^{(5/2)}/(25*a^5) + (x^5*ArcSin[a*x])/5$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int x^4 \sin^{-1}(ax) dx &= \frac{1}{5}x^5 \sin^{-1}(ax) - \frac{1}{5}a \int \frac{x^5}{\sqrt{1-a^2x^2}} dx \\
&= \frac{1}{5}x^5 \sin^{-1}(ax) - \frac{1}{10}a \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1-a^2x}} dx, x, x^2 \right) \\
&= \frac{1}{5}x^5 \sin^{-1}(ax) - \frac{1}{10}a \operatorname{Subst} \left( \int \left( \frac{1}{a^4\sqrt{1-a^2x}} - \frac{2\sqrt{1-a^2x}}{a^4} + \frac{(1-a^2x)^{3/2}}{a^4} \right) dx, x, x^2 \right) \\
&= \frac{\sqrt{1-a^2x^2}}{5a^5} - \frac{2(1-a^2x^2)^{3/2}}{15a^5} + \frac{(1-a^2x^2)^{5/2}}{25a^5} + \frac{1}{5}x^5 \sin^{-1}(ax)
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 51, normalized size = 0.68

$$\frac{\sqrt{1-a^2x^2} (3a^4x^4 + 4a^2x^2 + 8)}{75a^5} + \frac{1}{5}x^5 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcSin[a\*x],x]

[Out] (Sqrt[1 - a^2\*x^2]\*(8 + 4\*a^2\*x^2 + 3\*a^4\*x^4))/(75\*a^5) + (x^5\*ArcSin[a\*x])/5

**fricas** [A] time = 1.54, size = 49, normalized size = 0.65

$$\frac{15a^5x^5 \arcsin(ax) + (3a^4x^4 + 4a^2x^2 + 8)\sqrt{-a^2x^2 + 1}}{75a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x),x, algorithm="fricas")

[Out] 1/75\*(15\*a^5\*x^5\*arcsin(a\*x) + (3\*a^4\*x^4 + 4\*a^2\*x^2 + 8)\*sqrt(-a^2\*x^2 + 1))/a^5

**giac** [A] time = 0.37, size = 113, normalized size = 1.51

$$\frac{(a^2x^2 - 1)^2 x \arcsin(ax)}{5a^4} + \frac{2(a^2x^2 - 1)x \arcsin(ax)}{5a^4} + \frac{x \arcsin(ax)}{5a^4} + \frac{(a^2x^2 - 1)^2 \sqrt{-a^2x^2 + 1}}{25a^5} - \frac{2(-a^2x^2 + 1)^{3/2}}{15a^5} + \sqrt{-a^2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x),x, algorithm="giac")

[Out] 1/5\*(a^2\*x^2 - 1)^2\*x\*arcsin(a\*x)/a^4 + 2/5\*(a^2\*x^2 - 1)\*x\*arcsin(a\*x)/a^4 + 1/5\*x\*arcsin(a\*x)/a^4 + 1/25\*(a^2\*x^2 - 1)^2\*sqrt(-a^2\*x^2 + 1)/a^5 - 2/15\*(-a^2\*x^2 + 1)^(3/2)/a^5 + 1/5\*sqrt(-a^2\*x^2 + 1)/a^5

**maple** [A] time = 0.07, size = 72, normalized size = 0.96

$$\frac{\frac{a^5x^5 \arcsin(ax)}{5} + \frac{a^4x^4\sqrt{-a^2x^2+1}}{25} + \frac{4a^2x^2\sqrt{-a^2x^2+1}}{75} + \frac{8\sqrt{-a^2x^2+1}}{75}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arcsin(a\*x),x)

[Out]  $1/a^5*(1/5*a^5*x^5*\arcsin(a*x)+1/25*a^4*x^4*(-a^2*x^2+1)^{(1/2)}+4/75*a^2*x^2*(-a^2*x^2+1)^{(1/2)}+8/75*(-a^2*x^2+1)^{(1/2)})$

**maxima** [A] time = 0.83, size = 71, normalized size = 0.95

$$\frac{1}{5} x^5 \arcsin(ax) + \frac{1}{75} \left( \frac{3 \sqrt{-a^2 x^2 + 1} x^4}{a^2} + \frac{4 \sqrt{-a^2 x^2 + 1} x^2}{a^4} + \frac{8 \sqrt{-a^2 x^2 + 1}}{a^6} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arcsin(a*x),x, algorithm="maxima")`

[Out]  $1/5*x^5*\arcsin(a*x) + 1/75*(3*\sqrt{-a^2*x^2 + 1})*x^4/a^2 + 4*\sqrt{-a^2*x^2 + 1})*x^2/a^4 + 8*\sqrt{-a^2*x^2 + 1}/a^6)*a$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{asin}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*asin(a*x),x)`

[Out] `int(x^4*asin(a*x), x)`

**sympy** [A] time = 1.72, size = 70, normalized size = 0.93

$$\begin{cases} \frac{x^5 \operatorname{asin}(ax)}{5} + \frac{x^4 \sqrt{-a^2 x^2 + 1}}{25a} + \frac{4x^2 \sqrt{-a^2 x^2 + 1}}{75a^3} + \frac{8\sqrt{-a^2 x^2 + 1}}{75a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*asin(a*x),x)`

[Out] `Piecewise((x**5*asin(a*x)/5 + x**4*sqrt(-a**2*x**2 + 1)/(25*a) + 4*x**2*sqrt(-a**2*x**2 + 1)/(75*a**3) + 8*sqrt(-a**2*x**2 + 1)/(75*a**5), Ne(a, 0)), (0, True))`

## 3.2 $\int x^3 \sin^{-1}(ax) dx$

Optimal. Leaf size=69

$$-\frac{3 \sin^{-1}(ax)}{32a^4} + \frac{x^3 \sqrt{1-a^2x^2}}{16a} + \frac{3x \sqrt{1-a^2x^2}}{32a^3} + \frac{1}{4} x^4 \sin^{-1}(ax)$$

[Out]  $-3/32*\arcsin(a*x)/a^4+1/4*x^4*\arcsin(a*x)+3/32*x*(-a^2*x^2+1)^{(1/2)}/a^3+1/16*x^3*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4627, 321, 216}

$$\frac{x^3 \sqrt{1-a^2x^2}}{16a} + \frac{3x \sqrt{1-a^2x^2}}{32a^3} - \frac{3 \sin^{-1}(ax)}{32a^4} + \frac{1}{4} x^4 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcSin[a\*x],x]

[Out]  $(3*x*\text{Sqrt}[1 - a^2*x^2])/(32*a^3) + (x^3*\text{Sqrt}[1 - a^2*x^2])/(16*a) - (3*\text{ArcSin}[a*x])/(32*a^4) + (x^4*\text{ArcSin}[a*x])/4$

### Rule 216

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^(n\*(m-n+1)))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c^n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int x^3 \sin^{-1}(ax) dx &= \frac{1}{4} x^4 \sin^{-1}(ax) - \frac{1}{4} a \int \frac{x^4}{\sqrt{1-a^2x^2}} dx \\ &= \frac{x^3 \sqrt{1-a^2x^2}}{16a} + \frac{1}{4} x^4 \sin^{-1}(ax) - \frac{3 \int \frac{x^2}{\sqrt{1-a^2x^2}} dx}{16a} \\ &= \frac{3x \sqrt{1-a^2x^2}}{32a^3} + \frac{x^3 \sqrt{1-a^2x^2}}{16a} + \frac{1}{4} x^4 \sin^{-1}(ax) - \frac{3 \int \frac{1}{\sqrt{1-a^2x^2}} dx}{32a^3} \\ &= \frac{3x \sqrt{1-a^2x^2}}{32a^3} + \frac{x^3 \sqrt{1-a^2x^2}}{16a} - \frac{3 \sin^{-1}(ax)}{32a^4} + \frac{1}{4} x^4 \sin^{-1}(ax) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 50, normalized size = 0.72

$$\frac{(8a^4x^4 - 3) \sin^{-1}(ax) + ax\sqrt{1 - a^2x^2} (2a^2x^2 + 3)}{32a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcSin[a\*x], x]

[Out] (a\*x\*Sqrt[1 - a^2\*x^2]\*(3 + 2\*a^2\*x^2) + (-3 + 8\*a^4\*x^4)\*ArcSin[a\*x])/(32\*a^4)

**fricas [A]** time = 0.57, size = 47, normalized size = 0.68

$$\frac{(8a^4x^4 - 3) \arcsin(ax) + (2a^3x^3 + 3ax)\sqrt{-a^2x^2 + 1}}{32a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x), x, algorithm="fricas")

[Out] 1/32\*((8\*a^4\*x^4 - 3)\*arcsin(a\*x) + (2\*a^3\*x^3 + 3\*a\*x)\*sqrt(-a^2\*x^2 + 1))/a^4

**giac [A]** time = 0.17, size = 84, normalized size = 1.22

$$-\frac{(-a^2x^2 + 1)^{\frac{3}{2}}x}{16a^3} + \frac{(a^2x^2 - 1)^2 \arcsin(ax)}{4a^4} + \frac{5\sqrt{-a^2x^2 + 1}x}{32a^3} + \frac{(a^2x^2 - 1) \arcsin(ax)}{2a^4} + \frac{5 \arcsin(ax)}{32a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x), x, algorithm="giac")

[Out] -1/16\*(-a^2\*x^2 + 1)^(3/2)\*x/a^3 + 1/4\*(a^2\*x^2 - 1)^2\*arcsin(a\*x)/a^4 + 5/32\*sqrt(-a^2\*x^2 + 1)\*x/a^3 + 1/2\*(a^2\*x^2 - 1)\*arcsin(a\*x)/a^4 + 5/32\*arcsin(a\*x)/a^4

**maple [A]** time = 0.01, size = 60, normalized size = 0.87

$$\frac{\frac{a^4x^4 \arcsin(ax)}{4} + \frac{a^3x^3\sqrt{-a^2x^2+1}}{16} + \frac{3ax\sqrt{-a^2x^2+1}}{32} - \frac{3\arcsin(ax)}{32}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arcsin(a\*x), x)

[Out] 1/a^4\*(1/4\*a^4\*x^4\*arcsin(a\*x)+1/16\*a^3\*x^3\*(-a^2\*x^2+1)^(1/2)+3/32\*a\*x\*(-a^2\*x^2+1)^(1/2)-3/32\*arcsin(a\*x))

**maxima [A]** time = 0.47, size = 61, normalized size = 0.88

$$\frac{1}{4}x^4 \arcsin(ax) + \frac{1}{32} \left( \frac{2\sqrt{-a^2x^2+1}x^3}{a^2} + \frac{3\sqrt{-a^2x^2+1}x}{a^4} - \frac{3 \arcsin(ax)}{a^5} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x), x, algorithm="maxima")

[Out] 1/4\*x^4\*arcsin(a\*x) + 1/32\*(2\*sqrt(-a^2\*x^2 + 1)\*x^3/a^2 + 3\*sqrt(-a^2\*x^2 + 1)\*x/a^4 - 3\*arcsin(a\*x)/a^5)\*a

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asin}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*asin(a*x),x)`

[Out] `int(x^3*asin(a*x), x)`

sympy [A] time = 0.86, size = 61, normalized size = 0.88

$$\begin{cases} \frac{x^4 \operatorname{asin}(ax)}{4} + \frac{x^3 \sqrt{-a^2 x^2 + 1}}{16a} + \frac{3x \sqrt{-a^2 x^2 + 1}}{32a^3} - \frac{3 \operatorname{asin}(ax)}{32a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*asin(a*x),x)`

[Out] `Piecewise((x**4*asin(a*x)/4 + x**3*sqrt(-a**2*x**2 + 1)/(16*a) + 3*x*sqrt(-a**2*x**2 + 1)/(32*a**3) - 3*asin(a*x)/(32*a**4), Ne(a, 0)), (0, True))`



### 3.3 $\int x^2 \sin^{-1}(ax) dx$

Optimal. Leaf size=54

$$-\frac{(1-a^2x^2)^{3/2}}{9a^3} + \frac{\sqrt{1-a^2x^2}}{3a^3} + \frac{1}{3}x^3 \sin^{-1}(ax)$$

[Out]  $-1/9*(-a^2*x^2+1)^{(3/2)}/a^3+1/3*x^3*\arcsin(a*x)+1/3*(-a^2*x^2+1)^{(1/2)}/a^3$

**Rubi [A]** time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4627, 266, 43}

$$-\frac{(1-a^2x^2)^{3/2}}{9a^3} + \frac{\sqrt{1-a^2x^2}}{3a^3} + \frac{1}{3}x^3 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcSin[a\*x],x]

[Out] Sqrt[1 - a^2\*x^2]/(3\*a^3) - (1 - a^2\*x^2)^(3/2)/(9\*a^3) + (x^3\*ArcSin[a\*x])/3

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 \sin^{-1}(ax) dx &= \frac{1}{3}x^3 \sin^{-1}(ax) - \frac{1}{3}a \int \frac{x^3}{\sqrt{1-a^2x^2}} dx \\ &= \frac{1}{3}x^3 \sin^{-1}(ax) - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{x}{\sqrt{1-a^2x}} dx, x, x^2\right) \\ &= \frac{1}{3}x^3 \sin^{-1}(ax) - \frac{1}{6}a \operatorname{Subst}\left(\int \left(\frac{1}{a^2\sqrt{1-a^2x}} - \frac{\sqrt{1-a^2x}}{a^2}\right) dx, x, x^2\right) \\ &= \frac{\sqrt{1-a^2x^2}}{3a^3} - \frac{(1-a^2x^2)^{3/2}}{9a^3} + \frac{1}{3}x^3 \sin^{-1}(ax) \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 41, normalized size = 0.76

$$\frac{1}{9} \left( \frac{\sqrt{1-a^2x^2} (a^2x^2+2)}{a^3} + 3x^3 \sin^{-1}(ax) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcSin[a\*x],x]

[Out] ((Sqrt[1 - a^2\*x^2]\*(2 + a^2\*x^2))/a^3 + 3\*x^3\*ArcSin[a\*x])/9

**fricas** [A] time = 1.05, size = 40, normalized size = 0.74

$$\frac{3a^3x^3 \arcsin(ax) + (a^2x^2 + 2)\sqrt{-a^2x^2 + 1}}{9a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x),x, algorithm="fricas")

[Out] 1/9\*(3\*a^3\*x^3\*arcsin(a\*x) + (a^2\*x^2 + 2)\*sqrt(-a^2\*x^2 + 1))/a^3

**giac** [A] time = 0.17, size = 64, normalized size = 1.19

$$\frac{(a^2x^2 - 1)x \arcsin(ax)}{3a^2} + \frac{x \arcsin(ax)}{3a^2} - \frac{(-a^2x^2 + 1)^{\frac{3}{2}}}{9a^3} + \frac{\sqrt{-a^2x^2 + 1}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x),x, algorithm="giac")

[Out] 1/3\*(a^2\*x^2 - 1)\*x\*arcsin(a\*x)/a^2 + 1/3\*x\*arcsin(a\*x)/a^2 - 1/9\*(-a^2\*x^2 + 1)^(3/2)/a^3 + 1/3\*sqrt(-a^2\*x^2 + 1)/a^3

**maple** [A] time = 0.00, size = 52, normalized size = 0.96

$$\frac{\frac{a^3x^3 \arcsin(ax)}{3} + \frac{a^2x^2\sqrt{-a^2x^2+1}}{9} + \frac{2\sqrt{-a^2x^2+1}}{9}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arcsin(a\*x),x)

[Out] 1/a^3\*(1/3\*a^3\*x^3\*arcsin(a\*x)+1/9\*a^2\*x^2\*(-a^2\*x^2+1)^(1/2)+2/9\*(-a^2\*x^2+1)^(1/2))

**maxima** [A] time = 0.48, size = 50, normalized size = 0.93

$$\frac{1}{3} x^3 \arcsin(ax) + \frac{1}{9} a \left( \frac{\sqrt{-a^2x^2 + 1} x^2}{a^2} + \frac{2\sqrt{-a^2x^2 + 1}}{a^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x),x, algorithm="maxima")

[Out] 1/3\*x^3\*arcsin(a\*x) + 1/9\*a\*(sqrt(-a^2\*x^2 + 1)\*x^2/a^2 + 2\*sqrt(-a^2\*x^2 + 1)/a^4)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\sqrt{\frac{1}{a^2}-x^2} \left(\frac{2}{a^2}+x^2\right)}{9} + \frac{x^3 \operatorname{asin}(ax)}{3} & \text{if } 0 < a \\ \int x^2 \operatorname{asin}(ax) dx & \text{if } -0 < a \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*asin(a*x),x)`

[Out] `piecewise(0 < a, ((1/a^2 - x^2)^(1/2)*(2/a^2 + x^2))/9 + (x^3*asin(a*x))/3, ~0 < a, int(x^2*asin(a*x), x))`

sympy [A] time = 0.44, size = 48, normalized size = 0.89

$$\begin{cases} \frac{x^3 \operatorname{asin}(ax)}{3} + \frac{x^2 \sqrt{-a^2 x^2 + 1}}{9a} + \frac{2\sqrt{-a^2 x^2 + 1}}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*asin(a*x),x)`

[Out] `Piecewise((x**3*asin(a*x)/3 + x**2*sqrt(-a**2*x**2 + 1)/(9*a) + 2*sqrt(-a**2*x**2 + 1)/(9*a**3), Ne(a, 0)), (0, True))`

### 3.4 $\int x \sin^{-1}(ax) dx$

Optimal. Leaf size=45

$$\frac{x\sqrt{1-a^2x^2}}{4a} - \frac{\sin^{-1}(ax)}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)$$

[Out]  $-1/4*\arcsin(a*x)/a^2+1/2*x^2*\arcsin(a*x)+1/4*x*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 321, 216}

$$\frac{x\sqrt{1-a^2x^2}}{4a} - \frac{\sin^{-1}(ax)}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x\*ArcSin[a\*x], x]

[Out]  $(x*\text{Sqrt}[1 - a^2*x^2])/(4*a) - \text{ArcSin}[a*x]/(4*a^2) + (x^2*\text{ArcSin}[a*x])/2$

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^(n\*(m-n+1)))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x \sin^{-1}(ax) dx &= \frac{1}{2}x^2 \sin^{-1}(ax) - \frac{1}{2}a \int \frac{x^2}{\sqrt{1-a^2x^2}} dx \\ &= \frac{x\sqrt{1-a^2x^2}}{4a} + \frac{1}{2}x^2 \sin^{-1}(ax) - \frac{\int \frac{1}{\sqrt{1-a^2x^2}} dx}{4a} \\ &= \frac{x\sqrt{1-a^2x^2}}{4a} - \frac{\sin^{-1}(ax)}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.89

$$\frac{ax\sqrt{1-a^2x^2} + (2a^2x^2 - 1)\sin^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcSin[a\*x],x]

[Out] (a\*x\*Sqrt[1 - a^2\*x^2] + (-1 + 2\*a^2\*x^2)\*ArcSin[a\*x])/(4\*a^2)

**fricas** [A] time = 0.60, size = 36, normalized size = 0.80

$$\frac{\sqrt{-a^2x^2 + 1}ax + (2a^2x^2 - 1)\arcsin(ax)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x),x, algorithm="fricas")

[Out] 1/4\*(sqrt(-a^2\*x^2 + 1)\*a\*x + (2\*a^2\*x^2 - 1)\*arcsin(a\*x))/a^2

**giac** [A] time = 0.12, size = 46, normalized size = 1.02

$$\frac{\sqrt{-a^2x^2 + 1}x}{4a} + \frac{(a^2x^2 - 1)\arcsin(ax)}{2a^2} + \frac{\arcsin(ax)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x),x, algorithm="giac")

[Out] 1/4\*sqrt(-a^2\*x^2 + 1)\*x/a + 1/2\*(a^2\*x^2 - 1)\*arcsin(a\*x)/a^2 + 1/4\*arcsin(a\*x)/a^2

**maple** [A] time = 0.00, size = 40, normalized size = 0.89

$$\frac{\frac{a^2x^2\arcsin(ax)}{2} + \frac{ax\sqrt{-a^2x^2+1}}{4} - \frac{\arcsin(ax)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arcsin(a\*x),x)

[Out] 1/a^2\*(1/2\*a^2\*x^2\*arcsin(a\*x)+1/4\*a\*x\*(-a^2\*x^2+1)^(1/2)-1/4\*arcsin(a\*x))

**maxima** [A] time = 0.42, size = 40, normalized size = 0.89

$$\frac{1}{2}x^2\arcsin(ax) + \frac{1}{4}a\left(\frac{\sqrt{-a^2x^2 + 1}x}{a^2} - \frac{\arcsin(ax)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x),x, algorithm="maxima")

[Out] 1/2\*x^2\*arcsin(a\*x) + 1/4\*a\*(sqrt(-a^2\*x^2 + 1)\*x/a^2 - arcsin(a\*x)/a^3)

**mupad** [B] time = 0.08, size = 38, normalized size = 0.84

$$\frac{\operatorname{asin}(ax)(2a^2x^2 - 1)}{4a^2} + \frac{x\sqrt{1 - a^2x^2}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*asin(a\*x),x)

[Out] (asin(a\*x)\*(2\*a^2\*x^2 - 1))/(4\*a^2) + (x\*(1 - a^2\*x^2)^(1/2))/(4\*a)

**sympy** [A] time = 0.19, size = 37, normalized size = 0.82

$$\begin{cases} \frac{x^2\operatorname{asin}(ax)}{2} + \frac{x\sqrt{-a^2x^2+1}}{4a} - \frac{\operatorname{asin}(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(a*x),x)
```

```
[Out] Piecewise((x**2*asin(a*x)/2 + x*sqrt(-a**2*x**2 + 1)/(4*a) - asin(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

### 3.5 $\int \sin^{-1}(ax) dx$

**Optimal.** Leaf size=25

$$\frac{\sqrt{1-a^2x^2}}{a} + x \sin^{-1}(ax)$$

[Out]  $x*\arcsin(a*x)+(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4619, 261}

$$\frac{\sqrt{1-a^2x^2}}{a} + x \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x],x]

[Out] Sqrt[1 - a^2\*x^2]/a + x\*ArcSin[a\*x]

**Rule 261**

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

**Rule 4619**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

**Rubi steps**

$$\begin{aligned} \int \sin^{-1}(ax) dx &= x \sin^{-1}(ax) - a \int \frac{x}{\sqrt{1-a^2x^2}} dx \\ &= \frac{\sqrt{1-a^2x^2}}{a} + x \sin^{-1}(ax) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}}{a} + x \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x],x]

[Out] Sqrt[1 - a^2\*x^2]/a + x\*ArcSin[a\*x]

**fricas [A]** time = 0.65, size = 24, normalized size = 0.96

$$\frac{ax \arcsin(ax) + \sqrt{-a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x),x, algorithm="fricas")

[Out] (a\*x\*arcsin(a\*x) + sqrt(-a^2\*x^2 + 1))/a

**giac** [A] time = 0.13, size = 24, normalized size = 0.96

$$\frac{ax \arcsin(ax) + \sqrt{-a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x),x, algorithm="giac")

[Out] (a\*x\*arcsin(a\*x) + sqrt(-a^2\*x^2 + 1))/a

**maple** [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax \arcsin(ax) + \sqrt{-a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x),x)

[Out] 1/a\*(a\*x\*arcsin(a\*x)+(-a^2\*x^2+1)^(1/2))

**maxima** [A] time = 0.42, size = 24, normalized size = 0.96

$$\frac{ax \arcsin(ax) + \sqrt{-a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x),x, algorithm="maxima")

[Out] (a\*x\*arcsin(a\*x) + sqrt(-a^2\*x^2 + 1))/a

**mupad** [B] time = 0.11, size = 23, normalized size = 0.92

$$x \operatorname{asin}(ax) + \frac{\sqrt{1 - a^2x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x),x)

[Out] x\*asin(a\*x) + (1 - a^2\*x^2)^(1/2)/a

**sympy** [A] time = 0.12, size = 20, normalized size = 0.80

$$\begin{cases} x \operatorname{asin}(ax) + \frac{\sqrt{-a^2x^2+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x),x)

[Out] Piecewise((x\*asin(a\*x) + sqrt(-a\*\*2\*x\*\*2 + 1)/a, Ne(a, 0)), (0, True))



### 3.6 $\int \frac{\sin^{-1}(ax)}{x} dx$

**Optimal.** Leaf size=51

$$-\frac{1}{2}i\text{Li}_2\left(e^{2i\sin^{-1}(ax)}\right) - \frac{1}{2}i\sin^{-1}(ax)^2 + \sin^{-1}(ax)\log\left(1 - e^{2i\sin^{-1}(ax)}\right)$$

[Out]  $-1/2*I*\arcsin(a*x)^2 + \arcsin(a*x)*\ln(1 - (I*a*x + (-a^2*x^2 + 1)^{(1/2)})^2) - 1/2*I*\text{polylog}(2, (I*a*x + (-a^2*x^2 + 1)^{(1/2)})^2)$

**Rubi [A]** time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4625, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2, e^{2i\sin^{-1}(ax)}\right) - \frac{1}{2}i\sin^{-1}(ax)^2 + \sin^{-1}(ax)\log\left(1 - e^{2i\sin^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]/x, x]

[Out]  $(-I/2)*\text{ArcSin}[a*x]^2 + \text{ArcSin}[a*x]*\text{Log}[1 - E^{((2*I)*\text{ArcSin}[a*x])}] - (I/2)*\text{PolyLog}[2, E^{((2*I)*\text{ArcSin}[a*x])}]$

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 3717

Int[(((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m+1))/(d\*(m+1)), x] - Dist[2\*I, Int[((c + d\*x)^m \* E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 4625

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)}{x} dx &= \text{Subst} \left( \int x \cot(x) dx, x, \sin^{-1}(ax) \right) \\
&= -\frac{1}{2}i \sin^{-1}(ax)^2 - 2i \text{Subst} \left( \int \frac{e^{2ix}x}{1-e^{2ix}} dx, x, \sin^{-1}(ax) \right) \\
&= -\frac{1}{2}i \sin^{-1}(ax)^2 + \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) - \text{Subst} \left( \int \log(1 - e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\
&= -\frac{1}{2}i \sin^{-1}(ax)^2 + \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) + \frac{1}{2}i \text{Subst} \left( \int \frac{\log(1-x)}{x} dx, x, e^{2i \sin^{-1}(ax)} \right) \\
&= -\frac{1}{2}i \sin^{-1}(ax)^2 + \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) - \frac{1}{2}i \text{Li}_2(e^{2i \sin^{-1}(ax)})
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 46, normalized size = 0.90

$$\sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) - \frac{1}{2}i (\sin^{-1}(ax)^2 + \text{Li}_2(e^{2i \sin^{-1}(ax)}))$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]/x,x]

[Out] ArcSin[a\*x]\*Log[1 - E^((2\*I)\*ArcSin[a\*x])] - (I/2)\*(ArcSin[a\*x]^2 + PolyLog[2, E^((2\*I)\*ArcSin[a\*x])])

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)/x, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)/x, x)

**maple [A]** time = 0.40, size = 111, normalized size = 2.18

$$-\frac{i \arcsin(ax)^2}{2} + \arcsin(ax) \ln(1 + iax + \sqrt{-a^2x^2 + 1}) + \arcsin(ax) \ln(1 - iax - \sqrt{-a^2x^2 + 1}) - i \text{polylog}(2, -ia$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)/x,x)

[Out] -1/2\*I\*arcsin(a\*x)^2+arcsin(a\*x)\*ln(1+I\*a\*x+(-a^2\*x^2+1)^(1/2))+arcsin(a\*x)\*ln(1-I\*a\*x-(-a^2\*x^2+1)^(1/2))-I\*polylog(2,-I\*a\*x-(-a^2\*x^2+1)^(1/2))-I\*polylog(2,I\*a\*x+(-a^2\*x^2+1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x,x, algorithm="maxima")

[Out] integrate(arcsin(a\*x)/x, x)

**mupad** [B] time = 0.08, size = 41, normalized size = 0.80

$$\ln(1 - e^{\operatorname{asin}(ax)2i}) \operatorname{asin}(ax) - \frac{\operatorname{polylog}(2, e^{\operatorname{asin}(ax)2i}) 1i}{2} - \frac{\operatorname{asin}(ax)^2 1i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)/x,x)

[Out] log(1 - exp(asin(a\*x)\*2i))\*asin(a\*x) - (polylog(2, exp(asin(a\*x)\*2i))\*1i)/2 - (asin(a\*x)^2\*1i)/2

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)/x,x)

[Out] Integral(asin(a\*x)/x, x)

### 3.7 $\int \frac{\sin^{-1}(ax)}{x^2} dx$

Optimal. Leaf size=28

$$-a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{\sin^{-1}(ax)}{x}$$

[Out] `-arcsin(a*x)/x-a*arctanh((-a^2*x^2+1)^(1/2))`

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 266, 63, 208}

$$-a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{\sin^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] `Int[ArcSin[a*x]/x^2,x]`

[Out] `-(ArcSin[a*x]/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]`

#### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

#### Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

#### Rule 266

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

#### Rule 4627

`Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

#### Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)}{x^2} dx &= -\frac{\sin^{-1}(ax)}{x} + a \int \frac{1}{x\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sin^{-1}(ax)}{x} + \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{\sin^{-1}(ax)}{x} - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\sin^{-1}(ax)}{x} - a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 28, normalized size = 1.00

$$-a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{\sin^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]/x^2,x]

[Out] -(ArcSin[a\*x]/x) - a\*ArcTanh[Sqrt[1 - a^2\*x^2]]

**fricas [A]** time = 0.84, size = 49, normalized size = 1.75

$$\frac{ax \log\left(\sqrt{-a^2x^2+1}+1\right) - ax \log\left(\sqrt{-a^2x^2+1}-1\right) + 2 \arcsin(ax)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^2,x, algorithm="fricas")

[Out] -1/2\*(a\*x\*log(sqrt(-a^2\*x^2 + 1) + 1) - a\*x\*log(sqrt(-a^2\*x^2 + 1) - 1) + 2\*arcsin(a\*x))/x

**giac [A]** time = 0.15, size = 48, normalized size = 1.71

$$-\frac{1}{2}a\left(\log\left(\sqrt{-a^2x^2+1}+1\right) - \log\left(-\sqrt{-a^2x^2+1}+1\right)\right) - \frac{\arcsin(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^2,x, algorithm="giac")

[Out] -1/2\*a\*(log(sqrt(-a^2\*x^2 + 1) + 1) - log(-sqrt(-a^2\*x^2 + 1) + 1)) - arcsin(a\*x)/x

**maple [A]** time = 0.00, size = 31, normalized size = 1.11

$$a\left(-\frac{\arcsin(ax)}{ax} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)/x^2,x)

[Out] a\*(-arcsin(a\*x)/a/x-arctanh(1/(-a^2\*x^2+1)^(1/2)))

**maxima** [A] time = 0.40, size = 39, normalized size = 1.39

$$-a \log \left( \frac{2 \sqrt{-a^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) - \frac{\arcsin(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^2,x, algorithm="maxima")

[Out] -a\*log(2\*sqrt(-a^2\*x^2 + 1)/abs(x) + 2/abs(x)) - arcsin(a\*x)/x

**mupad** [B] time = 0.02, size = 26, normalized size = 0.93

$$-\frac{\operatorname{asin}(ax)}{x} - a \operatorname{atanh} \left( \frac{1}{\sqrt{1 - a^2 x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)/x^2,x)

[Out] - asin(a\*x)/x - a\*atanh(1/(1 - a^2\*x^2)^(1/2))

**sympy** [C] time = 1.44, size = 32, normalized size = 1.14

$$a \left( \begin{cases} -\operatorname{acosh} \left( \frac{1}{ax} \right) & \text{for } \frac{1}{|a^2 x^2|} > 1 \\ i \operatorname{asin} \left( \frac{1}{ax} \right) & \text{otherwise} \end{cases} \right) - \frac{\operatorname{asin}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)/x\*\*2,x)

[Out] a\*Piecewise((-acosh(1/(a\*x)), 1/Abs(a\*\*2\*x\*\*2) > 1), (I\*asin(1/(a\*x)), True)) - asin(a\*x)/x

$$3.8 \quad \int \frac{\sin^{-1}(ax)}{x^3} dx$$

**Optimal.** Leaf size=34

$$-\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\sin^{-1}(ax)}{2x^2}$$

[Out]  $-1/2*\arcsin(a*x)/x^2-1/2*a*(-a^2*x^2+1)^{(1/2)}/x$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4627, 264}

$$-\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\sin^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]/x^3,x]

[Out]  $-(a*\text{Sqrt}[1 - a^2*x^2])/(2*x) - \text{ArcSin}[a*x]/(2*x^2)$

**Rule 264**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

**Rule 4627**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{\sin^{-1}(ax)}{x^3} dx &= -\frac{\sin^{-1}(ax)}{2x^2} + \frac{1}{2}a \int \frac{1}{x^2\sqrt{1-a^2x^2}} dx \\ &= -\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\sin^{-1}(ax)}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.85

$$-\frac{ax\sqrt{1-a^2x^2} + \sin^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]/x^3,x]

[Out]  $-1/2*(a*x*\text{Sqrt}[1 - a^2*x^2] + \text{ArcSin}[a*x])/x^2$

**fricas [A]** time = 0.77, size = 25, normalized size = 0.74

$$-\frac{\sqrt{-a^2x^2 + 1} ax + \arcsin(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^3,x, algorithm="fricas")

[Out] -1/2\*(sqrt(-a^2\*x^2 + 1)\*a\*x + arcsin(a\*x))/x^2

**giac** [B] time = 0.21, size = 68, normalized size = 2.00

$$\frac{1}{4} \left( \frac{a^4 x}{\left( \sqrt{-a^2 x^2 + 1} |a| + a \right) |a|} - \frac{\sqrt{-a^2 x^2 + 1} |a| + a}{x |a|} \right) a - \frac{\arcsin(ax)}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^3,x, algorithm="giac")

[Out] 1/4\*(a^4\*x/((sqrt(-a^2\*x^2 + 1)\*abs(a) + a)\*abs(a)) - (sqrt(-a^2\*x^2 + 1)\*abs(a) + a)/(x\*abs(a)))\*a - 1/2\*arcsin(a\*x)/x^2

**maple** [A] time = 0.00, size = 38, normalized size = 1.12

$$a^2 \left( -\frac{\arcsin(ax)}{2a^2x^2} - \frac{\sqrt{-a^2x^2 + 1}}{2ax} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)/x^3,x)

[Out] a^2\*(-1/2\*arcsin(a\*x)/a^2/x^2-1/2/a/x\*(-a^2\*x^2+1)^(1/2))

**maxima** [A] time = 0.40, size = 28, normalized size = 0.82

$$-\frac{\sqrt{-a^2x^2 + 1} a}{2x} - \frac{\arcsin(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^3,x, algorithm="maxima")

[Out] -1/2\*sqrt(-a^2\*x^2 + 1)\*a/x - 1/2\*arcsin(a\*x)/x^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asin}(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)/x^3,x)

[Out] int(asin(a\*x)/x^3, x)

**sympy** [C] time = 1.14, size = 51, normalized size = 1.50

$$\frac{a \left( \begin{cases} -\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } |a^2x^2| > 1 \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise} \end{cases} \right)}{2} - \frac{\operatorname{asin}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)/x\*\*3,x)

[Out] a\*Piecewise((-I\*sqrt(a\*\*2\*x\*\*2 - 1)/x, Abs(a\*\*2\*x\*\*2) > 1), (-sqrt(-a\*\*2\*x\*\*2 + 1)/x, True))/2 - asin(a\*x)/(2\*x\*\*2)



### 3.9 $\int \frac{\sin^{-1}(ax)}{x^4} dx$

**Optimal.** Leaf size=56

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{\sin^{-1}(ax)}{3x^3}$$

[Out]  $-1/3*\arcsin(a*x)/x^3-1/6*a^3*\operatorname{arctanh}((-a^2*x^2+1)^{(1/2)})-1/6*a*(-a^2*x^2+1)^{(1/2)}/x^2$

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4627, 266, 51, 63, 208}

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{\sin^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]/x^4,x]

[Out]  $-(a*\operatorname{Sqrt}[1-a^2*x^2])/(6*x^2) - \operatorname{ArcSin}[a*x]/(3*x^3) - (a^3*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-a^2*x^2]])/6$

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*((d\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)}{x^4} dx &= -\frac{\sin^{-1}(ax)}{3x^3} + \frac{1}{3}a \int \frac{1}{x^3\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sin^{-1}(ax)}{3x^3} + \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\sin^{-1}(ax)}{3x^3} + \frac{1}{12}a^3 \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\sin^{-1}(ax)}{3x^3} - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2} - \frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\sin^{-1}(ax)}{3x^3} - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 53, normalized size = 0.95

$$\frac{ax\sqrt{1-a^2x^2} + a^3x^3 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + 2\sin^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]/x^4,x]

[Out] -1/6\*(a\*x\*Sqrt[1 - a^2\*x^2] + 2\*ArcSin[a\*x] + a^3\*x^3\*ArcTanh[Sqrt[1 - a^2\*x^2]])/x^3

**fricas [A]** time = 0.72, size = 73, normalized size = 1.30

$$\frac{a^3x^3 \log\left(\sqrt{-a^2x^2+1} + 1\right) - a^3x^3 \log\left(\sqrt{-a^2x^2+1} - 1\right) + 2\sqrt{-a^2x^2+1}ax + 4 \arcsin(ax)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^4,x, algorithm="fricas")

[Out] -1/12\*(a^3\*x^3\*log(sqrt(-a^2\*x^2 + 1) + 1) - a^3\*x^3\*log(sqrt(-a^2\*x^2 + 1) - 1) + 2\*sqrt(-a^2\*x^2 + 1)\*a\*x + 4\*arcsin(a\*x))/x^3

**giac [A]** time = 0.15, size = 77, normalized size = 1.38

$$\frac{a^4 \log\left(\sqrt{-a^2x^2+1} + 1\right) - a^4 \log\left(-\sqrt{-a^2x^2+1} + 1\right) + \frac{2\sqrt{-a^2x^2+1}a^2}{x^2}}{12a} - \frac{\arcsin(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^4,x, algorithm="giac")

[Out] -1/12\*(a^4\*log(sqrt(-a^2\*x^2 + 1) + 1) - a^4\*log(-sqrt(-a^2\*x^2 + 1) + 1) + 2\*sqrt(-a^2\*x^2 + 1)\*a^2/x^2)/a - 1/3\*arcsin(a\*x)/x^3

**maple [A]** time = 0.00, size = 53, normalized size = 0.95

$$a^3 \left( \frac{\arcsin(ax)}{3a^3x^3} - \frac{\sqrt{-a^2x^2+1}}{6a^2x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)/x^4,x)

[Out]  $a^3*(-1/3*\arcsin(a*x)/a^3/x^3-1/6/a^2/x^2*(-a^2*x^2+1)^{(1/2)}-1/6*\operatorname{arctanh}(1/(-a^2*x^2+1)^{(1/2)}))$

**maxima** [A] time = 0.42, size = 60, normalized size = 1.07

$$-\frac{1}{6} \left( a^2 \log \left( \frac{2 \sqrt{-a^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-a^2 x^2 + 1}}{x^2} \right) a - \frac{\arcsin(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^4,x, algorithm="maxima")

[Out]  $-1/6*(a^2*\log(2*\sqrt{-a^2*x^2 + 1}/\operatorname{abs}(x) + 2/\operatorname{abs}(x)) + \sqrt{-a^2*x^2 + 1}/x^2)*a - 1/3*\arcsin(a*x)/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)/x^4,x)

[Out] int(asin(a\*x)/x^4, x)

**sympy** [A] time = 2.33, size = 109, normalized size = 1.95

$$\frac{a \left( \begin{array}{l} \left( \frac{a^2 \operatorname{acosh}\left(\frac{1}{ax}\right)}{2} - \frac{a \sqrt{-1 + \frac{1}{a^2 x^2}}}{2x} \right) \quad \text{for } \frac{1}{|a^2 x^2|} > 1 \\ \left( \frac{ia^2 \operatorname{asin}\left(\frac{1}{ax}\right)}{2} - \frac{ia}{2x \sqrt{1 - \frac{1}{a^2 x^2}}} + \frac{i}{2ax^3 \sqrt{1 - \frac{1}{a^2 x^2}}} \right) \quad \text{otherwise} \end{array} \right)}{3} - \frac{\operatorname{asin}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)/x\*\*4,x)

[Out]  $a*\operatorname{Piecewise}((-a**2*\operatorname{acosh}(1/(a*x)))/2 - a*\sqrt{-1 + 1/(a**2*x**2)})/(2*x), 1/\operatorname{abs}(a**2*x**2) > 1), (I*a**2*\operatorname{asin}(1/(a*x)))/2 - I*a/(2*x*\sqrt{1 - 1/(a**2*x**2)}) + I/(2*a*x**3*\sqrt{1 - 1/(a**2*x**2)}), \operatorname{True}))/3 - \operatorname{asin}(a*x)/(3*x**3)$

### 3.10 $\int \frac{\sin^{-1}(ax)}{x^5} dx$

**Optimal.** Leaf size=58

$$-\frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{a^3\sqrt{1-a^2x^2}}{6x} - \frac{\sin^{-1}(ax)}{4x^4}$$

[Out]  $-1/4*\arcsin(a*x)/x^4-1/12*a*(-a^2*x^2+1)^{(1/2)}/x^3-1/6*a^3*(-a^2*x^2+1)^{(1/2)}/x$

**Rubi [A]** time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4627, 271, 264}

$$-\frac{a^3\sqrt{1-a^2x^2}}{6x} - \frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{\sin^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]/x^5,x]

[Out]  $-(a*\text{Sqrt}[1 - a^2*x^2])/(12*x^3) - (a^3*\text{Sqrt}[1 - a^2*x^2])/(6*x) - \text{ArcSin}[a*x]/(4*x^4)$

#### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 271

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(ax)}{x^5} dx &= -\frac{\sin^{-1}(ax)}{4x^4} + \frac{1}{4}a \int \frac{1}{x^4\sqrt{1-a^2x^2}} dx \\ &= -\frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{\sin^{-1}(ax)}{4x^4} + \frac{1}{6}a^3 \int \frac{1}{x^2\sqrt{1-a^2x^2}} dx \\ &= -\frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{a^3\sqrt{1-a^2x^2}}{6x} - \frac{\sin^{-1}(ax)}{4x^4} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 41, normalized size = 0.71

$$\frac{ax\sqrt{1-a^2x^2}(2a^2x^2+1)+3\sin^{-1}(ax)}{12x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]/x^5,x]

[Out] -1/12\*(a\*x\*Sqrt[1 - a^2\*x^2]\*(1 + 2\*a^2\*x^2) + 3\*ArcSin[a\*x])/x^4

**fricas** [A] time = 0.85, size = 37, normalized size = 0.64

$$-\frac{(2a^3x^3 + ax)\sqrt{-a^2x^2 + 1} + 3 \arcsin(ax)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^5,x, algorithm="fricas")

[Out] -1/12\*((2\*a^3\*x^3 + a\*x)\*sqrt(-a^2\*x^2 + 1) + 3\*arcsin(a\*x))/x^4

**giac** [B] time = 0.18, size = 130, normalized size = 2.24

$$\frac{1}{96} \left( \frac{\left( a^4 + \frac{9(\sqrt{-a^2x^2+1}|a|+a)^2}{x^2} \right) a^6 x^3}{(\sqrt{-a^2x^2+1}|a|+a)^3 |a|} - \frac{\frac{9(\sqrt{-a^2x^2+1}|a|+a)a^4}{x} + \frac{(\sqrt{-a^2x^2+1}|a|+a)^3}{x^3}}{a^2|a|} \right) a - \frac{\arcsin(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^5,x, algorithm="giac")

[Out] 1/96\*((a^4 + 9\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^2/x^2)\*a^6\*x^3/((sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^3\*abs(a)) - (9\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)\*a^4/x + (sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^3/x^3)/(a^2\*abs(a)))\*a - 1/4\*arcsin(a\*x)/x^4

**maple** [A] time = 0.00, size = 58, normalized size = 1.00

$$a^4 \left( -\frac{\arcsin(ax)}{4a^4x^4} - \frac{\sqrt{-a^2x^2+1}}{12a^3x^3} - \frac{\sqrt{-a^2x^2+1}}{6ax} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)/x^5,x)

[Out] a^4\*(-1/4\*arcsin(a\*x)/a^4/x^4-1/12/a^3/x^3\*(-a^2\*x^2+1)^(1/2)-1/6/a/x\*(-a^2\*x^2+1)^(1/2))

**maxima** [A] time = 0.43, size = 50, normalized size = 0.86

$$-\frac{1}{12} \left( \frac{2\sqrt{-a^2x^2+1}a^2}{x} + \frac{\sqrt{-a^2x^2+1}}{x^3} \right) a - \frac{\arcsin(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^5,x, algorithm="maxima")

[Out] -1/12\*(2\*sqrt(-a^2\*x^2 + 1)\*a^2/x + sqrt(-a^2\*x^2 + 1)/x^3)\*a - 1/4\*arcsin(a\*x)/x^4

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)/x^5,x)`

[Out] `int(asin(a*x)/x^5, x)`

sympy [A] time = 1.73, size = 100, normalized size = 1.72

$$\frac{a \left( \begin{cases} -\frac{2ia^2\sqrt{a^2x^2-1}}{3x} - \frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2| > 1 \\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x} - \frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise} \end{cases} \right)}{4} - \frac{\operatorname{asin}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)/x**5,x)`

[Out] `a*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))/4 - asin(a*x)/(4*x**4)`

### 3.11 $\int \frac{\sin^{-1}(ax)}{x^6} dx$

**Optimal.** Leaf size=80

$$-\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3}{40}a^5 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\sin^{-1}(ax)}{5x^5}$$

[Out]  $-1/5*\arcsin(a*x)/x^5-3/40*a^5*\operatorname{arctanh}((-a^2*x^2+1)^{(1/2)})-1/20*a*(-a^2*x^2+1)^{(1/2)}/x^4-3/40*a^3*(-a^2*x^2+1)^{(1/2)}/x^2$

**Rubi [A]** time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4627, 266, 51, 63, 208}

$$-\frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3}{40}a^5 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{\sin^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]/x^6,x]

[Out]  $-(a*\operatorname{Sqrt}[1-a^2*x^2])/(20*x^4) - (3*a^3*\operatorname{Sqrt}[1-a^2*x^2])/(40*x^2) - \operatorname{ArcSin}[a*x]/(5*x^5) - (3*a^5*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-a^2*x^2]])/40$

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] ] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)}{x^6} dx &= -\frac{\sin^{-1}(ax)}{5x^5} + \frac{1}{5}a \int \frac{1}{x^5\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sin^{-1}(ax)}{5x^5} + \frac{1}{10}a \operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{\sin^{-1}(ax)}{5x^5} + \frac{1}{40}(3a^3) \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\sin^{-1}(ax)}{5x^5} + \frac{1}{80}(3a^5) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\sin^{-1}(ax)}{5x^5} - \frac{1}{40}(3a^3) \operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2} - \frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\sin^{-1}(ax)}{5x^5} - \frac{3}{40}a^5 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

**Mathematica** [C] time = 0.01, size = 51, normalized size = 0.64

$$-\frac{1}{5}a^5\sqrt{1-a^2x^2} {}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; 1-a^2x^2\right) - \frac{\sin^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]/x^6,x]

[Out] -1/5\*ArcSin[a\*x]/x^5 - (a^5\*Sqrt[1 - a^2\*x^2]\*Hypergeometric2F1[1/2, 3, 3/2, 1 - a^2\*x^2])/5

**fricas** [A] time = 0.58, size = 85, normalized size = 1.06

$$\frac{3a^5x^5 \log\left(\sqrt{-a^2x^2+1}+1\right) - 3a^5x^5 \log\left(\sqrt{-a^2x^2+1}-1\right) + 2\left(3a^3x^3+2ax\right)\sqrt{-a^2x^2+1} + 16 \arcsin(ax)}{80x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^6,x, algorithm="fricas")

[Out] -1/80\*(3\*a^5\*x^5\*log(sqrt(-a^2\*x^2+1)+1) - 3\*a^5\*x^5\*log(sqrt(-a^2\*x^2+1)-1) + 2\*(3\*a^3\*x^3+2\*a\*x)\*sqrt(-a^2\*x^2+1) + 16\*arcsin(a\*x))/x^5

**giac** [A] time = 0.14, size = 101, normalized size = 1.26

$$\frac{3a^6 \log\left(\sqrt{-a^2x^2+1}+1\right) - 3a^6 \log\left(-\sqrt{-a^2x^2+1}+1\right) - \frac{2\left(3(-a^2x^2+1)^{\frac{3}{2}}a^6-5\sqrt{-a^2x^2+1}a^6\right)}{a^4x^4}}{80a} - \frac{\arcsin(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)/x^6,x, algorithm="giac")

[Out] -1/80\*(3\*a^6\*log(sqrt(-a^2\*x^2+1)+1) - 3\*a^6\*log(-sqrt(-a^2\*x^2+1)+1) + 1) - 2\*(3\*(-a^2\*x^2+1)^(3/2)\*a^6 - 5\*sqrt(-a^2\*x^2+1)\*a^6)/(a^4\*x^4)/a - 1/5\*arcsin(a\*x)/x^5

**maple** [A] time = 0.00, size = 73, normalized size = 0.91

$$a^5 \left( -\frac{\arcsin(ax)}{5a^5x^5} - \frac{\sqrt{-a^2x^2+1}}{20a^4x^4} - \frac{3\sqrt{-a^2x^2+1}}{40a^2x^2} - \frac{3 \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{40} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(a*x)/x^6,x)`

[Out]  $a^5*(-1/5*\arcsin(a*x)/a^5/x^5-1/20/a^4/x^4*(-a^2*x^2+1)^{(1/2)}-3/40/a^2/x^2*(-a^2*x^2+1)^{(1/2)}-3/40*\operatorname{arctanh}(1/(-a^2*x^2+1)^{(1/2)}))$

**maxima** [A] time = 0.42, size = 82, normalized size = 1.02

$$-\frac{1}{40} \left( 3a^4 \log \left( \frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{3\sqrt{-a^2x^2+1}a^2}{x^2} + \frac{2\sqrt{-a^2x^2+1}}{x^4} \right) a - \frac{\arcsin(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(a*x)/x^6,x, algorithm="maxima")`

[Out]  $-1/40*(3*a^4*\log(2*\sqrt{-a^2*x^2+1}/\operatorname{abs}(x)+2/\operatorname{abs}(x))+3*\sqrt{-a^2*x^2+1}*a^2/x^2+2*\sqrt{-a^2*x^2+1}/x^4)*a-1/5*\arcsin(a*x)/x^5$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)/x^6,x)`

[Out] `int(asin(a*x)/x^6, x)`

**sympy** [A] time = 4.23, size = 182, normalized size = 2.28

$$a \left( \begin{array}{l} \left( \frac{3a^4 \operatorname{acosh}\left(\frac{1}{ax}\right)}{8} + \frac{3a^3}{8x\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{a}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{1}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}} \right) \text{ for } \frac{1}{|a^2x^2|} > 1 \\ \left( \frac{3ia^4 \operatorname{asin}\left(\frac{1}{ax}\right)}{8} - \frac{3ia^3}{8x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{ia}{8x^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{i}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} \right) \text{ otherwise} \end{array} \right) - \frac{\operatorname{asin}(ax)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)/x**6,x)`

[Out]  $a*\operatorname{Piecewise}((-3*a**4*\operatorname{acosh}(1/(a*x))/8+3*a**3/(8*x*\sqrt{-1+1/(a**2*x**2)})-a/(8*x**3*\sqrt{-1+1/(a**2*x**2)})-1/(4*a*x**5*\sqrt{-1+1/(a**2*x**2)})),1/\operatorname{Abs}(a**2*x**2)>1),(3*I*a**4*\operatorname{asin}(1/(a*x))/8-3*I*a**3/(8*x*\sqrt{1-1/(a**2*x**2)})+I*a/(8*x**3*\sqrt{1-1/(a**2*x**2)})+I/(4*a*x**5*\sqrt{1-1/(a**2*x**2)})),\operatorname{True}))/5-\operatorname{asin}(a*x)/(5*x**5)$

### 3.12 $\int x^4 \sin^{-1}(ax)^2 dx$

**Optimal.** Leaf size=120

$$-\frac{16x}{75a^4} - \frac{8x^3}{225a^2} + \frac{2x^4\sqrt{1-a^2x^2}\sin^{-1}(ax)}{25a} + \frac{16\sqrt{1-a^2x^2}\sin^{-1}(ax)}{75a^5} + \frac{8x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)}{75a^3} + \frac{1}{5}x^5\sin^{-1}(ax)^2 - \frac{2x^5}{125}$$

[Out]  $-16/75*x/a^4-8/225*x^3/a^2-2/125*x^5+1/5*x^5*\arcsin(a*x)^2+16/75*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a^5+8/75*x^2*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a^3+2/25*x^4*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.19, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 4707, 4677, 8, 30}

$$-\frac{8x^3}{225a^2} + \frac{2x^4\sqrt{1-a^2x^2}\sin^{-1}(ax)}{25a} + \frac{8x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)}{75a^3} + \frac{16\sqrt{1-a^2x^2}\sin^{-1}(ax)}{75a^5} - \frac{16x}{75a^4} + \frac{1}{5}x^5\sin^{-1}(ax)^2 - \frac{2x^5}{125}$$

Antiderivative was successfully verified.

[In] `Int[x^4*ArcSin[a*x]^2,x]`

[Out]  $(-16*x)/(75*a^4) - (8*x^3)/(225*a^2) - (2*x^5)/125 + (16*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(75*a^5) + (8*x^2*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(75*a^3) + (2*x^4*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(25*a) + (x^5*\text{ArcSin}[a*x]^2)/5$

#### Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 4627

`Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((d_)*(x_)^(m_)), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

#### Rule 4677

`Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p + 1)), x] + Dist[(b*n*d*IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

#### Rule 4707

`Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_)^(m_))/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[(f*x)^(m - 2)*(a + b*ArcSin[c*x])^n]/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]`

Rubi steps

$$\begin{aligned}
\int x^4 \sin^{-1}(ax)^2 dx &= \frac{1}{5}x^5 \sin^{-1}(ax)^2 - \frac{1}{5}(2a) \int \frac{x^5 \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx \\
&= \frac{2x^4\sqrt{1-a^2x^2} \sin^{-1}(ax)}{25a} + \frac{1}{5}x^5 \sin^{-1}(ax)^2 - \frac{2 \int x^4 dx}{25} - \frac{8 \int \frac{x^3 \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx}{25a} \\
&= -\frac{2x^5}{125} + \frac{8x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{75a^3} + \frac{2x^4\sqrt{1-a^2x^2} \sin^{-1}(ax)}{25a} + \frac{1}{5}x^5 \sin^{-1}(ax)^2 - \frac{16 \int \frac{x \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx}{75a^3} \\
&= -\frac{8x^3}{225a^2} - \frac{2x^5}{125} + \frac{16\sqrt{1-a^2x^2} \sin^{-1}(ax)}{75a^5} + \frac{8x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{75a^3} + \frac{2x^4\sqrt{1-a^2x^2} \sin^{-1}(ax)}{25a} \\
&= -\frac{16x}{75a^4} - \frac{8x^3}{225a^2} - \frac{2x^5}{125} + \frac{16\sqrt{1-a^2x^2} \sin^{-1}(ax)}{75a^5} + \frac{8x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{75a^3} + \frac{2x^4\sqrt{1-a^2x^2} \sin^{-1}(ax)}{25a}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 82, normalized size = 0.68

$$\frac{225a^5x^5 \sin^{-1}(ax)^2 - 2ax(9a^4x^4 + 20a^2x^2 + 120) + 30\sqrt{1-a^2x^2}(3a^4x^4 + 4a^2x^2 + 8) \sin^{-1}(ax)}{1125a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcSin[a\*x]^2,x]

[Out] (-2\*a\*x\*(120 + 20\*a^2\*x^2 + 9\*a^4\*x^4) + 30\*Sqrt[1 - a^2\*x^2]\*(8 + 4\*a^2\*x^2 + 3\*a^4\*x^4)\*ArcSin[a\*x] + 225\*a^5\*x^5\*ArcSin[a\*x]^2)/(1125\*a^5)

**fricas [A]** time = 0.74, size = 76, normalized size = 0.63

$$\frac{225a^5x^5 \arcsin(ax)^2 - 18a^5x^5 - 40a^3x^3 + 30(3a^4x^4 + 4a^2x^2 + 8)\sqrt{-a^2x^2 + 1} \arcsin(ax) - 240ax}{1125a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^2,x, algorithm="fricas")

[Out] 1/1125\*(225\*a^5\*x^5\*arcsin(a\*x)^2 - 18\*a^5\*x^5 - 40\*a^3\*x^3 + 30\*(3\*a^4\*x^4 + 4\*a^2\*x^2 + 8)\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x) - 240\*a\*x)/a^5

**giac [A]** time = 0.19, size = 169, normalized size = 1.41

$$\frac{(a^2x^2 - 1)^2 x \arcsin(ax)^2}{5a^4} + \frac{2(a^2x^2 - 1)x \arcsin(ax)^2}{5a^4} - \frac{2(a^2x^2 - 1)^2 x}{125a^4} + \frac{x \arcsin(ax)^2}{5a^4} + \frac{2(a^2x^2 - 1)^2 \sqrt{-a^2x^2 + 1}}{25a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^2,x, algorithm="giac")

[Out] 1/5\*(a^2\*x^2 - 1)^2\*x\*arcsin(a\*x)^2/a^4 + 2/5\*(a^2\*x^2 - 1)\*x\*arcsin(a\*x)^2/a^4 - 2/125\*(a^2\*x^2 - 1)^2\*x/a^4 + 1/5\*x\*arcsin(a\*x)^2/a^4 + 2/25\*(a^2\*x^2 - 1)^2\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)/a^5 - 76/1125\*(a^2\*x^2 - 1)\*x/a^4 - 4/15\*(-a^2\*x^2 + 1)^(3/2)\*arcsin(a\*x)/a^5 - 298/1125\*x/a^4 + 2/5\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)/a^5

**maple [A]** time = 0.19, size = 76, normalized size = 0.63

$$\frac{a^5x^5 \arcsin(ax)^2}{5} + \frac{2 \arcsin(ax)(3a^4x^4 + 4a^2x^2 + 8)\sqrt{-a^2x^2 + 1}}{75} - \frac{2a^5x^5}{125} - \frac{8a^3x^3}{225} - \frac{16ax}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arcsin(a*x)^2,x)`

[Out]  $1/a^5*(1/5*a^5*x^5*arcsin(a*x)^2+2/75*arcsin(a*x)*(3*a^4*x^4+4*a^2*x^2+8)*(-a^2*x^2+1)^{(1/2)}-2/125*a^5*x^5-8/225*a^3*x^3-16/75*a*x)$

**maxima** [A] time = 0.57, size = 102, normalized size = 0.85

$$\frac{1}{5}x^5 \arcsin(ax)^2 + \frac{2}{75} \left( \frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6} \right) a \arcsin(ax) - \frac{2(9a^4x^5 + 20a^2x^3 + 120x)}{1125a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arcsin(a*x)^2,x, algorithm="maxima")`

[Out]  $1/5*x^5*arcsin(a*x)^2 + 2/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arcsin(a*x) - 2/1125*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)/a^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{asin}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*asin(a*x)^2,x)`

[Out] `int(x^4*asin(a*x)^2, x)`

**sympy** [A] time = 3.14, size = 114, normalized size = 0.95

$$\begin{cases} \frac{x^5 \operatorname{asin}^2(ax)}{5} - \frac{2x^5}{125} + \frac{2x^4 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{25a} - \frac{8x^3}{225a^2} + \frac{8x^2 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{75a^3} - \frac{16x}{75a^4} + \frac{16 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{75a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*asin(a*x)**2,x)`

[Out] `Piecewise((x**5*asin(a*x)**2/5 - 2*x**5/125 + 2*x**4*sqrt(-a**2*x**2 + 1)*asin(a*x)/(25*a) - 8*x**3/(225*a**2) + 8*x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)/(75*a**3) - 16*x/(75*a**4) + 16*sqrt(-a**2*x**2 + 1)*asin(a*x)/(75*a**5), Ne(a, 0)), (0, True))`

### 3.13 $\int x^3 \sin^{-1}(ax)^2 dx$

**Optimal.** Leaf size=98

$$-\frac{3 \sin^{-1}(ax)^2}{32a^4} - \frac{3x^2}{32a^2} + \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{8a} + \frac{3x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{16a^3} + \frac{1}{4} x^4 \sin^{-1}(ax)^2 - \frac{x^4}{32}$$

[Out]  $-3/32*x^2/a^2-1/32*x^4-3/32*\arcsin(a*x)^2/a^4+1/4*x^4*\arcsin(a*x)^2+3/16*x*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a^3+1/8*x^3*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4627, 4707, 4641, 30}

$$-\frac{3x^2}{32a^2} + \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{8a} + \frac{3x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{16a^3} - \frac{3 \sin^{-1}(ax)^2}{32a^4} + \frac{1}{4} x^4 \sin^{-1}(ax)^2 - \frac{x^4}{32}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcSin[a\*x]^2,x]

[Out]  $(-3*x^2)/(32*a^2) - x^4/32 + (3*x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(16*a^3) + (x^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(8*a) - (3*\text{ArcSin}[a*x]^2)/(32*a^4) + (x^4*\text{ArcSin}[a*x]^2)/4$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])^(n+1)/(b\*c\*Sqrt[d]\*(n+1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int((((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((f\_)\*(x\_))^(m\_))/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m-1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m-1))/(c^2\*m), Int[((f\*x)^(m-2)\*(a + b\*ArcSin[c\*x])^n)/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1-c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m-1)\*(a + b\*ArcSin[c\*x])^(n-1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rubi steps

$$\begin{aligned}
\int x^3 \sin^{-1}(ax)^2 dx &= \frac{1}{4}x^4 \sin^{-1}(ax)^2 - \frac{1}{2}a \int \frac{x^4 \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx \\
&= \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{8a} + \frac{1}{4}x^4 \sin^{-1}(ax)^2 - \frac{\int x^3 dx}{8} - \frac{3 \int \frac{x^2 \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx}{8a} \\
&= -\frac{x^4}{32} + \frac{3x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{16a^3} + \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{8a} + \frac{1}{4}x^4 \sin^{-1}(ax)^2 - \frac{3 \int \frac{\sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx}{16a^3} \\
&= -\frac{3x^2}{32a^2} - \frac{x^4}{32} + \frac{3x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{16a^3} + \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{8a} - \frac{3 \sin^{-1}(ax)^2}{32a^4} + \frac{1}{4}x^4 \sin^{-1}(ax)^2
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 74, normalized size = 0.76

$$\frac{(8a^4x^4 - 3) \sin^{-1}(ax)^2 - a^2x^2 (a^2x^2 + 3) + 2ax \sqrt{1-a^2x^2} (2a^2x^2 + 3) \sin^{-1}(ax)}{32a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcSin[a\*x]^2,x]

[Out]  $(-(a^2x^2(3 + a^2x^2)) + 2ax\sqrt{1-a^2x^2}(3 + 2a^2x^2)\text{ArcSin}[ax] + (-3 + 8a^4x^4)\text{ArcSin}[ax]^2)/(32a^4)$

**fricas** [A] time = 0.75, size = 70, normalized size = 0.71

$$\frac{a^4x^4 + 3a^2x^2 - (8a^4x^4 - 3) \arcsin(ax)^2 - 2(2a^3x^3 + 3ax)\sqrt{-a^2x^2 + 1} \arcsin(ax)}{32a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^2,x, algorithm="fricas")

[Out]  $-1/32*(a^4x^4 + 3a^2x^2 - (8a^4x^4 - 3)*\arcsin(a*x)^2 - 2*(2a^3x^3 + 3a*x)*\sqrt{-a^2x^2 + 1}*\arcsin(a*x))/a^4$

**giac** [A] time = 0.16, size = 133, normalized size = 1.36

$$-\frac{(-a^2x^2 + 1)^{\frac{3}{2}}x \arcsin(ax)}{8a^3} + \frac{(a^2x^2 - 1)^2 \arcsin(ax)^2}{4a^4} + \frac{5\sqrt{-a^2x^2 + 1}x \arcsin(ax)}{16a^3} + \frac{(a^2x^2 - 1) \arcsin(ax)^2}{2a^4} - \frac{(a^2x^2 - 1)^2 \arcsin(ax)^2}{32a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^2,x, algorithm="giac")

[Out]  $-1/8*(-a^2x^2 + 1)^{(3/2)}x*\arcsin(a*x)/a^3 + 1/4*(a^2x^2 - 1)^2*\arcsin(a*x)^2/a^4 + 5/16*\sqrt{-a^2x^2 + 1}*x*\arcsin(a*x)/a^3 + 1/2*(a^2x^2 - 1)*\arcsin(a*x)^2/a^4 - 1/32*(a^2x^2 - 1)^2/a^4 + 5/32*\arcsin(a*x)^2/a^4 - 5/32*(a^2x^2 - 1)/a^4 - 17/256/a^4$

**maple** [A] time = 0.14, size = 93, normalized size = 0.95

$$\frac{\frac{a^4x^4 \arcsin(ax)^2}{4} - \frac{\arcsin(ax)(-2a^3x^3\sqrt{-a^2x^2+1} - 3ax\sqrt{-a^2x^2+1} + 3\arcsin(ax))}{16}}{a^4} + \frac{3\arcsin(ax)^2}{32} - \frac{a^4x^4}{32} - \frac{3a^2x^2}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arcsin(a\*x)^2,x)

[Out]  $1/a^4*(1/4*a^4*x^4*\arcsin(a*x)^2-1/16*\arcsin(a*x)*(-2*a^3*x^3*(-a^2*x^2+1)^{(1/2)}-3*a*x*(-a^2*x^2+1)^{(1/2)}+3*\arcsin(a*x))+3/32*\arcsin(a*x)^2-1/32*a^4*x^4-3/32*a^2*x^2)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^4 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^2 + a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^4 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)}{2(a^2x^2-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^2,x, algorithm="maxima")

[Out]  $1/4*x^4*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^2 + a*\integrate(1/2*\sqrt{a*x+1}*\sqrt{-a*x+1}*x^4*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})/(a^2*x^2-1), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asin}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*asin(a\*x)^2,x)

[Out] int(x^3\*asin(a\*x)^2, x)

**sympy** [A] time = 1.85, size = 90, normalized size = 0.92

$$\begin{cases} \frac{x^4 \operatorname{asin}^2(ax)}{4} - \frac{x^4}{32} + \frac{x^3 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{8a} - \frac{3x^2}{32a^2} + \frac{3x \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{16a^3} - \frac{3 \operatorname{asin}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*asin(a\*x)\*\*2,x)

[Out] Piecewise((x\*\*4\*asin(a\*x)\*\*2/4 - x\*\*4/32 + x\*\*3\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)/(8\*a) - 3\*x\*\*2/(32\*a\*\*2) + 3\*x\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)/(16\*a\*\*3) - 3\*asin(a\*x)\*\*2/(32\*a\*\*4), Ne(a, 0)), (0, True))

### 3.14 $\int x^2 \sin^{-1}(ax)^2 dx$

Optimal. Leaf size=82

$$\frac{2x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)}{9a} - \frac{4x}{9a^2} + \frac{4\sqrt{1-a^2x^2}\sin^{-1}(ax)}{9a^3} + \frac{1}{3}x^3\sin^{-1}(ax)^2 - \frac{2x^3}{27}$$

[Out]  $-4/9*x/a^2-2/27*x^3+1/3*x^3*\arcsin(a*x)^2+4/9*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a^3+2/9*x^2*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 4707, 4677, 8, 30}

$$\frac{2x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)}{9a} + \frac{4\sqrt{1-a^2x^2}\sin^{-1}(ax)}{9a^3} - \frac{4x}{9a^2} + \frac{1}{3}x^3\sin^{-1}(ax)^2 - \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcSin[a\*x]^2,x]

[Out]  $(-4*x)/(9*a^2) - (2*x^3)/27 + (4*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(9*a^3) + (2*x^2*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(9*a) + (x^3*\text{ArcSin}[a*x]^2)/3$

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c^n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4677

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4707

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((f\_)\*(x\_))^(m\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rubi steps



$$\begin{aligned}
\int x^2 \sin^{-1}(ax)^2 dx &= \frac{1}{3}x^3 \sin^{-1}(ax)^2 - \frac{1}{3}(2a) \int \frac{x^3 \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx \\
&= \frac{2x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{9a} + \frac{1}{3}x^3 \sin^{-1}(ax)^2 - \frac{2 \int x^2 dx}{9} - \frac{4 \int \frac{x \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx}{9a} \\
&= -\frac{2x^3}{27} + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)}{9a^3} + \frac{2x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{9a} + \frac{1}{3}x^3 \sin^{-1}(ax)^2 - \frac{4 \int 1 dx}{9a^2} \\
&= -\frac{4x}{9a^2} - \frac{2x^3}{27} + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)}{9a^3} + \frac{2x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{9a} + \frac{1}{3}x^3 \sin^{-1}(ax)^2
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 64, normalized size = 0.78

$$\frac{9a^3x^3 \sin^{-1}(ax)^2 - 2ax(a^2x^2 + 6) + 6\sqrt{1-a^2x^2}(a^2x^2 + 2) \sin^{-1}(ax)}{27a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcSin[a\*x]^2,x]

[Out] (-2\*a\*x\*(6 + a^2\*x^2) + 6\*Sqrt[1 - a^2\*x^2]\*(2 + a^2\*x^2)\*ArcSin[a\*x] + 9\*a^3\*x^3\*ArcSin[a\*x]^2)/(27\*a^3)

**fricas [A]** time = 0.56, size = 59, normalized size = 0.72

$$\frac{9a^3x^3 \arcsin(ax)^2 - 2a^3x^3 + 6(a^2x^2 + 2)\sqrt{-a^2x^2 + 1} \arcsin(ax) - 12ax}{27a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^2,x, algorithm="fricas")

[Out] 1/27\*(9\*a^3\*x^3\*arcsin(a\*x)^2 - 2\*a^3\*x^3 + 6\*(a^2\*x^2 + 2)\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x) - 12\*a\*x)/a^3

**giac [A]** time = 0.14, size = 97, normalized size = 1.18

$$\frac{(a^2x^2 - 1)x \arcsin(ax)^2}{3a^2} + \frac{x \arcsin(ax)^2}{3a^2} - \frac{2(a^2x^2 - 1)x}{27a^2} - \frac{2(-a^2x^2 + 1)^{\frac{3}{2}} \arcsin(ax)}{9a^3} - \frac{14x}{27a^2} + \frac{2\sqrt{-a^2x^2 + 1} \arcsin(ax)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^2,x, algorithm="giac")

[Out] 1/3\*(a^2\*x^2 - 1)\*x\*arcsin(a\*x)^2/a^2 + 1/3\*x\*arcsin(a\*x)^2/a^2 - 2/27\*(a^2\*x^2 - 1)\*x/a^2 - 2/9\*(-a^2\*x^2 + 1)^(3/2)\*arcsin(a\*x)/a^3 - 14/27\*x/a^2 + 2/3\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)/a^3

**maple [A]** time = 0.12, size = 59, normalized size = 0.72

$$\frac{\frac{a^3x^3 \arcsin(ax)^2}{3} + \frac{2 \arcsin(ax)(a^2x^2+2)\sqrt{-a^2x^2+1}}{9} - \frac{2a^3x^3}{27} - \frac{4ax}{9}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arcsin(a\*x)^2,x)

[Out]  $1/a^3*(1/3*a^3*x^3*\arcsin(ax)^2+2/9*\arcsin(ax)*(a^2*x^2+2)*(-a^2*x^2+1)^{(1/2)}-2/27*a^3*x^3-4/9*ax)$

**maxima** [A] time = 0.79, size = 72, normalized size = 0.88

$$\frac{1}{3}x^3 \arcsin(ax)^2 + \frac{2}{9}a \left( \frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4} \right) \arcsin(ax) - \frac{2(a^2x^3+6x)}{27a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsin(a*x)^2,x, algorithm="maxima")`

[Out]  $1/3*x^3*\arcsin(a*x)^2 + 2/9*a*(\text{sqrt}(-a^2*x^2 + 1)*x^2/a^2 + 2*\text{sqrt}(-a^2*x^2 + 1)/a^4)*\arcsin(a*x) - 2/27*(a^2*x^3 + 6*x)/a^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \arcsin(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*asin(a*x)^2,x)`

[Out] `int(x^2*asin(a*x)^2, x)`

**sympy** [A] time = 0.89, size = 76, normalized size = 0.93

$$\begin{cases} \frac{x^3 \arcsin^2(ax)}{3} - \frac{2x^3}{27} + \frac{2x^2 \sqrt{-a^2x^2+1} \arcsin(ax)}{9a} - \frac{4x}{9a^2} + \frac{4\sqrt{-a^2x^2+1} \arcsin(ax)}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*asin(a*x)**2,x)`

[Out] `Piecewise((x**3*asin(a*x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)/(9*a) - 4*x/(9*a**2) + 4*sqrt(-a**2*x**2 + 1)*asin(a*x)/(9*a**3), Ne(a, 0)), (0, True))`

### 3.15 $\int x \sin^{-1}(ax)^2 dx$

Optimal. Leaf size=60

$$\frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} - \frac{\sin^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^2 - \frac{x^2}{4}$$

[Out]  $-1/4*x^2-1/4*\arcsin(a*x)^2/a^2+1/2*x^2*\arcsin(a*x)^2+1/2*x*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 4707, 4641, 30}

$$\frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} - \frac{\sin^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^2 - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcSin[a\*x]^2,x]

[Out]  $-x^2/4 + (x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(2*a) - \text{ArcSin}[a*x]^2/(4*a^2) + (x^2*\text{ArcSin}[a*x]^2)/2$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[((f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n)/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int x \sin^{-1}(ax)^2 dx &= \frac{1}{2}x^2 \sin^{-1}(ax)^2 - a \int \frac{x^2 \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx \\ &= \frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} + \frac{1}{2}x^2 \sin^{-1}(ax)^2 - \frac{\int x dx}{2} - \frac{\int \frac{\sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx}{2a} \\ &= -\frac{x^2}{4} + \frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} - \frac{\sin^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^2 \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 55, normalized size = 0.92

$$\frac{-a^2x^2 + 2ax\sqrt{1-a^2x^2} \sin^{-1}(ax) + (2a^2x^2 - 1) \sin^{-1}(ax)^2}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcSin[a\*x]^2,x]

[Out]  $(-(a^2x^2) + 2a*x*sqrt[1 - a^2*x^2]*ArcSin[a*x] + (-1 + 2*a^2*x^2)*ArcSin[a*x]^2)/(4*a^2)$

**fricas** [A] time = 0.70, size = 51, normalized size = 0.85

$$\frac{a^2x^2 - 2\sqrt{-a^2x^2 + 1} ax \arcsin(ax) - (2a^2x^2 - 1) \arcsin(ax)^2}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^2,x, algorithm="fricas")

[Out]  $-1/4*(a^2*x^2 - 2*sqrt(-a^2*x^2 + 1)*a*x*arcsin(a*x) - (2*a^2*x^2 - 1)*arcsin(a*x)^2)/a^2$

**giac** [A] time = 0.15, size = 73, normalized size = 1.22

$$\frac{\sqrt{-a^2x^2 + 1} x \arcsin(ax)}{2a} + \frac{(a^2x^2 - 1) \arcsin(ax)^2}{2a^2} + \frac{\arcsin(ax)^2}{4a^2} - \frac{a^2x^2 - 1}{4a^2} - \frac{1}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^2,x, algorithm="giac")

[Out]  $1/2*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)/a + 1/2*(a^2*x^2 - 1)*arcsin(a*x)^2/a^2 + 1/4*arcsin(a*x)^2/a^2 - 1/4*(a^2*x^2 - 1)/a^2 - 1/8/a^2$

**maple** [A] time = 0.04, size = 65, normalized size = 1.08

$$\frac{\frac{(a^2x^2-1) \arcsin(ax)^2}{2} + \frac{\arcsin(ax)(ax\sqrt{-a^2x^2+1} + \arcsin(ax))}{2}}{a^2} - \frac{\arcsin(ax)^2}{4} - \frac{a^2x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arcsin(a\*x)^2,x)

[Out]  $1/a^2*(1/2*(a^2*x^2-1)*arcsin(a*x)^2+1/2*arcsin(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+arcsin(a*x))-1/4*arcsin(a*x)^2-1/4*a^2*x^2)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^2 + a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^2,x, algorithm="maxima")

[Out] 1/2\*x^2\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2 + a\*integrate(sqrt(a\*x + 1)\*sqrt(-a\*x + 1)\*x^2\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))/(a^2\*x^2 - 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{asin}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*asin(a\*x)^2,x)

[Out] int(x\*asin(a\*x)^2, x)

**sympy** [A] time = 0.45, size = 51, normalized size = 0.85

$$\begin{cases} \frac{x^2 \operatorname{asin}^2(ax)}{2} - \frac{x^2}{4} + \frac{x \sqrt{-a^2 x^2 + 1} \operatorname{asin}(ax)}{2a} - \frac{\operatorname{asin}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*asin(a\*x)\*\*2,x)

[Out] Piecewise((x\*\*2\*asin(a\*x)\*\*2/2 - x\*\*2/4 + x\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)/(2\*a) - asin(a\*x)\*\*2/(4\*a\*\*2), Ne(a, 0)), (0, True))

### 3.16 $\int \sin^{-1}(ax)^2 dx$

Optimal. Leaf size=35

$$\frac{2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} + x \sin^{-1}(ax)^2 - 2x$$

[Out]  $-2*x+x*\arcsin(a*x)^2+2*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4619, 4677, 8}

$$\frac{2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} + x \sin^{-1}(ax)^2 - 2x$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^2,x]

[Out]  $-2*x + (2*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/a + x*\text{ArcSin}[a*x]^2$

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p+1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p+1)), x] + Dist[(b\*n\*d\*IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p+1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p+1/2)\*(a + b\*ArcSin[c\*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \sin^{-1}(ax)^2 dx &= x \sin^{-1}(ax)^2 - (2a) \int \frac{x \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx \\ &= \frac{2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} + x \sin^{-1}(ax)^2 - 2 \int 1 dx \\ &= -2x + \frac{2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} + x \sin^{-1}(ax)^2 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 35, normalized size = 1.00

$$\frac{2\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} + x \sin^{-1}(ax)^2 - 2x$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^2,x]

[Out]  $-2*x + (2*\sqrt{1 - a^2*x^2}*\text{ArcSin}[a*x])/a + x*\text{ArcSin}[a*x]^2$

**fricas** [A] time = 0.50, size = 36, normalized size = 1.03

$$\frac{ax \arcsin(ax)^2 - 2ax + 2\sqrt{-a^2x^2 + 1} \arcsin(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2,x, algorithm="fricas")

[Out]  $(a*x*\arcsin(a*x)^2 - 2*a*x + 2*\sqrt{-a^2*x^2 + 1}*\arcsin(a*x))/a$

**giac** [A] time = 0.12, size = 33, normalized size = 0.94

$$x \arcsin(ax)^2 - 2x + \frac{2\sqrt{-a^2x^2 + 1} \arcsin(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2,x, algorithm="giac")

[Out]  $x*\arcsin(a*x)^2 - 2*x + 2*\sqrt{-a^2*x^2 + 1}*\arcsin(a*x)/a$

**maple** [A] time = 0.03, size = 37, normalized size = 1.06

$$\frac{ax \arcsin(ax)^2 - 2ax + 2 \arcsin(ax) \sqrt{-a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^2,x)

[Out]  $1/a*(a*x*\arcsin(a*x)^2-2*a*x+2*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)})$

**maxima** [A] time = 0.71, size = 33, normalized size = 0.94

$$x \arcsin(ax)^2 - 2x + \frac{2\sqrt{-a^2x^2 + 1} \arcsin(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2,x, algorithm="maxima")

[Out]  $x*\arcsin(a*x)^2 - 2*x + 2*\sqrt{-a^2*x^2 + 1}*\arcsin(a*x)/a$

**mupad** [B] time = 0.14, size = 32, normalized size = 0.91

$$x \left( \arcsin(ax)^2 - 2 \right) + \frac{2 \arcsin(ax) \sqrt{1 - a^2 x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^2,x)

[Out]  $x*(\arcsin(a*x)^2 - 2) + (2*\arcsin(a*x)*(1 - a^2*x^2)^{(1/2)})/a$

**sympy** [A] time = 0.18, size = 32, normalized size = 0.91

$$\begin{cases} x \arcsin^2(ax) - 2x + \frac{2\sqrt{-a^2x^2+1} \arcsin(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(a*x)**2,x)
```

```
[Out] Piecewise((x*asin(a*x)**2 - 2*x + 2*sqrt(-a**2*x**2 + 1)*asin(a*x)/a, Ne(a, 0)), (0, True))
```



$$3.17 \quad \int \frac{\sin^{-1}(ax)^2}{x} dx$$

**Optimal.** Leaf size=71

$$-i \sin^{-1}(ax) \operatorname{Li}_2(e^{2i \sin^{-1}(ax)}) + \frac{1}{2} \operatorname{Li}_3(e^{2i \sin^{-1}(ax)}) - \frac{1}{3} i \sin^{-1}(ax)^3 + \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)})$$

[Out]  $-1/3*I*\arcsin(a*x)^3 + \arcsin(a*x)^2*\ln(1 - (I*a*x + (-a^2*x^2+1)^{(1/2)})^2) - I*\arcsin(a*x)*\operatorname{polylog}(2, (I*a*x + (-a^2*x^2+1)^{(1/2)})^2) + 1/2*\operatorname{polylog}(3, (I*a*x + (-a^2*x^2+1)^{(1/2)})^2)$

**Rubi [A]** time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4625, 3717, 2190, 2531, 2282, 6589}

$$-i \sin^{-1}(ax) \operatorname{PolyLog}(2, e^{2i \sin^{-1}(ax)}) + \frac{1}{2} \operatorname{PolyLog}(3, e^{2i \sin^{-1}(ax)}) - \frac{1}{3} i \sin^{-1}(ax)^3 + \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)})$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^2/x, x]

[Out]  $(-I/3)*\operatorname{ArcSin}[a*x]^3 + \operatorname{ArcSin}[a*x]^2*\operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcSin}[a*x])}] - I*\operatorname{ArcSin}[a*x]*\operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcSin}[a*x])}] + \operatorname{PolyLog}[3, E^{((2*I)*\operatorname{ArcSin}[a*x])}]]/2$

**Rule 2190**

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^((n\_)\*((c\_) + (d\_)\*(x\_))^(m\_)))/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^((n\_))), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2282**

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)^v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

**Rule 2531**

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^((n\_)))]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

**Rule 3717**

Int[(((c\_) + (d\_)\*(x\_))^(m\_))\*tan[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

**Rule 4625**

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned}
 \int \frac{\sin^{-1}(ax)^2}{x} dx &= \text{Subst} \left( \int x^2 \cot(x) dx, x, \sin^{-1}(ax) \right) \\
 &= -\frac{1}{3}i \sin^{-1}(ax)^3 - 2i \text{Subst} \left( \int \frac{e^{2ix} x^2}{1 - e^{2ix}} dx, x, \sin^{-1}(ax) \right) \\
 &= -\frac{1}{3}i \sin^{-1}(ax)^3 + \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)}) - 2 \text{Subst} \left( \int x \log(1 - e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\
 &= -\frac{1}{3}i \sin^{-1}(ax)^3 + \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)}) - i \sin^{-1}(ax) \text{Li}_2(e^{2i \sin^{-1}(ax)}) + i \text{Subst} \left( \int \text{Li}_2(e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\
 &= -\frac{1}{3}i \sin^{-1}(ax)^3 + \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)}) - i \sin^{-1}(ax) \text{Li}_2(e^{2i \sin^{-1}(ax)}) + \frac{1}{2} \text{Subst} \left( \int \text{Li}_3(e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\
 &= -\frac{1}{3}i \sin^{-1}(ax)^3 + \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)}) - i \sin^{-1}(ax) \text{Li}_2(e^{2i \sin^{-1}(ax)}) + \frac{1}{2} \text{Li}_3(e^{2i \sin^{-1}(ax)})
 \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 71, normalized size = 1.00

$$i \sin^{-1}(ax) \text{Li}_2(e^{-2i \sin^{-1}(ax)}) + \frac{1}{2} \text{Li}_3(e^{-2i \sin^{-1}(ax)}) + \frac{1}{3} i \sin^{-1}(ax)^3 + \sin^{-1}(ax)^2 \log(1 - e^{-2i \sin^{-1}(ax)})$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^2/x, x]

[Out] (I/3)\*ArcSin[a\*x]^3 + ArcSin[a\*x]^2\*Log[1 - E^((-2\*I)\*ArcSin[a\*x])] + I\*ArcSin[a\*x]\*PolyLog[2, E^((-2\*I)\*ArcSin[a\*x])] + PolyLog[3, E^((-2\*I)\*ArcSin[a\*x])]/2

**fricas** [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x, x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^2/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x, x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^2/x, x)

**maple** [A] time = 0.06, size = 169, normalized size = 2.38

$$-\frac{i \arcsin(ax)^3}{3} + \arcsin(ax)^2 \ln(1 + iax + \sqrt{-a^2x^2 + 1}) - 2i \arcsin(ax) \text{polylog}(2, -iax - \sqrt{-a^2x^2 + 1}) + 2 \text{polylog}(3, -iax - \sqrt{-a^2x^2 + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(a*x)^2/x,x)`

[Out]  $-1/3*I*\arcsin(ax)^3+\arcsin(ax)^2*\ln(1+I*ax+(-a^2*x^2+1)^{1/2})-2*I*\arcsin(ax)*\operatorname{polylog}(2,-I*ax-(-a^2*x^2+1)^{1/2})+2*\operatorname{polylog}(3,-I*ax-(-a^2*x^2+1)^{1/2})+\arcsin(ax)^2*\ln(1-I*ax-(-a^2*x^2+1)^{1/2})-2*I*\arcsin(ax)*\operatorname{polylog}(2,I*ax+(-a^2*x^2+1)^{1/2})+2*\operatorname{polylog}(3,I*ax+(-a^2*x^2+1)^{1/2})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(a*x)^2/x,x, algorithm="maxima")`

[Out] `integrate(arcsin(a*x)^2/x, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^2/x,x)`

[Out] `int(asin(a*x)^2/x, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^2(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)**2/x,x)`

[Out] `Integral(asin(a*x)**2/x, x)`

$$3.18 \quad \int \frac{\sin^{-1}(ax)^2}{x^2} dx$$

**Optimal.** Leaf size=66

$$2ia\text{Li}_2\left(-e^{i\sin^{-1}(ax)}\right) - 2ia\text{Li}_2\left(e^{i\sin^{-1}(ax)}\right) - \frac{\sin^{-1}(ax)^2}{x} - 4a\sin^{-1}(ax)\tanh^{-1}\left(e^{i\sin^{-1}(ax)}\right)$$

[Out] -arcsin(a\*x)^2/x-4\*a\*arcsin(a\*x)\*arctanh(I\*a\*x+(-a^2\*x^2+1)^(1/2))+2\*I\*a\*polylog(2,-I\*a\*x-(-a^2\*x^2+1)^(1/2))-2\*I\*a\*polylog(2,I\*a\*x+(-a^2\*x^2+1)^(1/2))

**Rubi [A]** time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 4709, 4183, 2279, 2391}

$$2ia\text{PolyLog}\left(2, -e^{i\sin^{-1}(ax)}\right) - 2ia\text{PolyLog}\left(2, e^{i\sin^{-1}(ax)}\right) - \frac{\sin^{-1}(ax)^2}{x} - 4a\sin^{-1}(ax)\tanh^{-1}\left(e^{i\sin^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^2/x^2,x]

[Out] -(ArcSin[a\*x]^2/x) - 4\*a\*ArcSin[a\*x]\*ArcTanh[E^(I\*ArcSin[a\*x])] + (2\*I)\*a\*polyLog[2, -E^(I\*ArcSin[a\*x])] - (2\*I)\*a\*PolyLog[2, E^(I\*ArcSin[a\*x])]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4183

Int[csc[(e\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4709

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_)]/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] :> Dist[1/(c^(m + 1)\*Sqrt[d]), Subst[Int[(a + b\*x)^n\*Sin[x]^m, x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)^2}{x^2} dx &= -\frac{\sin^{-1}(ax)^2}{x} + (2a) \int \frac{\sin^{-1}(ax)}{x\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sin^{-1}(ax)^2}{x} + (2a) \text{Subst} \left( \int x \csc(x) dx, x, \sin^{-1}(ax) \right) \\
&= -\frac{\sin^{-1}(ax)^2}{x} - 4a \sin^{-1}(ax) \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) - (2a) \text{Subst} \left( \int \log(1 - e^{ix}) dx, x, \sin^{-1}(ax) \right) \\
&= -\frac{\sin^{-1}(ax)^2}{x} - 4a \sin^{-1}(ax) \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) + (2ia) \text{Subst} \left( \int \frac{\log(1-x)}{x} dx, x, e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{\sin^{-1}(ax)^2}{x} - 4a \sin^{-1}(ax) \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) + 2ia \text{Li}_2 \left( -e^{i \sin^{-1}(ax)} \right) - 2ia \text{Li}_2 \left( e^{i \sin^{-1}(ax)} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 87, normalized size = 1.32

$$a \left( 2i \text{Li}_2 \left( -e^{i \sin^{-1}(ax)} \right) - 2i \text{Li}_2 \left( e^{i \sin^{-1}(ax)} \right) - \sin^{-1}(ax) \left( \frac{\sin^{-1}(ax)}{ax} - 2 \log(1 - e^{i \sin^{-1}(ax)}) + 2 \log(1 + e^{i \sin^{-1}(ax)}) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^2/x^2,x]

[Out] a\*(-(ArcSin[a\*x]\*(ArcSin[a\*x]/(a\*x) - 2\*Log[1 - E^(I\*ArcSin[a\*x])]) + 2\*Log[1 + E^(I\*ArcSin[a\*x])])) + (2\*I)\*PolyLog[2, -E^(I\*ArcSin[a\*x])] - (2\*I)\*PolyLog[2, E^(I\*ArcSin[a\*x])]

**fricas [F]** time = 0.63, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^2}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^2,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^2/x^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^2,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^2/x^2, x)

**maple [A]** time = 0.04, size = 119, normalized size = 1.80

$$-\frac{\arcsin(ax)^2}{x} + 2a \arcsin(ax) \ln \left( 1 - iax - \sqrt{-a^2x^2 + 1} \right) - 2a \arcsin(ax) \ln \left( 1 + iax + \sqrt{-a^2x^2 + 1} \right) + 2ia \text{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^2/x^2,x)

[Out] -arcsin(a\*x)^2/x + 2\*a\*arcsin(a\*x)\*ln(1-I\*a\*x-(-a^2\*x^2+1)^(1/2))-2\*a\*arcsin(a\*x)\*ln(1+I\*a\*x+(-a^2\*x^2+1)^(1/2))+2\*I\*a\*dilog(1+I\*a\*x+(-a^2\*x^2+1)^(1/2))-2\*I\*a\*dilog(1-I\*a\*x-(-a^2\*x^2+1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2ax \int \frac{\sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}{\sqrt{ax+1}(ax-1)x} dx + \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^2,x, algorithm="maxima")

[Out] -(2\*a\*x\*integrate(sqrt(a\*x + 1)\*sqrt(-a\*x + 1)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))/(a^2\*x^3 - x), x) + arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2)/x

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}(ax)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^2/x^2,x)

[Out] int(asin(a\*x)^2/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^2(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*2/x\*\*2,x)

[Out] Integral(asin(a\*x)\*\*2/x\*\*2, x)

### 3.19 $\int \frac{\sin^{-1}(ax)^2}{x^3} dx$

**Optimal.** Leaf size=44

$$-\frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)}{x} + a^2\log(x) - \frac{\sin^{-1}(ax)^2}{2x^2}$$

[Out]  $-1/2*\arcsin(a*x)^2/x^2+a^2*\ln(x)-a*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/x$

**Rubi [A]** time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4627, 4681, 29}

$$-\frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)}{x} + a^2\log(x) - \frac{\sin^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^2/x^3,x]

[Out]  $-((a*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/x) - \text{ArcSin}[a*x]^2/(2*x^2) + a^2*\text{Log}[x]$

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^ (n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4681

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^ (n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(d+e\*x^2)^(p+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*f\*(m+1)), x] - Dist[(b\*c\*n\*d^IntPart[p]\*(d+e\*x^2)^FracPart[p])/(f\*(m+1)\*(1-c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m+1)\*(1-c^2\*x^2)^(p+1/2)\*(a+b\*ArcSin[c\*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2\*d+e, 0] && GtQ[n, 0] && EqQ[m+2\*p+3, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(ax)^2}{x^3} dx &= -\frac{\sin^{-1}(ax)^2}{2x^2} + a \int \frac{\sin^{-1}(ax)}{x^2\sqrt{1-a^2x^2}} dx \\ &= -\frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)}{x} - \frac{\sin^{-1}(ax)^2}{2x^2} + a^2 \int \frac{1}{x} dx \\ &= -\frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)}{x} - \frac{\sin^{-1}(ax)^2}{2x^2} + a^2 \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 44, normalized size = 1.00

$$-\frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)}{x} + a^2\log(x) - \frac{\sin^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^2/x^3,x]

[Out] -((a\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/x) - ArcSin[a\*x]^2/(2\*x^2) + a^2\*Log[x]

**fricas** [A] time = 1.31, size = 44, normalized size = 1.00

$$\frac{2 a^2 x^2 \log(x) - 2 \sqrt{-a^2 x^2 + 1} a x \arcsin(ax) - \arcsin(ax)^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^3,x, algorithm="fricas")

[Out] 1/2\*(2\*a^2\*x^2\*log(x) - 2\*sqrt(-a^2\*x^2 + 1)\*a\*x\*arcsin(a\*x) - arcsin(a\*x)^2)/x^2

**giac** [B] time = 0.18, size = 82, normalized size = 1.86

$$\frac{1}{2} \left( \left( \frac{a^4 x}{(\sqrt{-a^2 x^2 + 1} |a| + a) |a|} - \frac{\sqrt{-a^2 x^2 + 1} |a| + a}{x |a|} \right) \arcsin(ax) + 2 a \log(|x|) \right) a - \frac{\arcsin(ax)^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^3,x, algorithm="giac")

[Out] 1/2\*((a^4\*x/((sqrt(-a^2\*x^2 + 1)\*abs(a) + a)\*abs(a)) - (sqrt(-a^2\*x^2 + 1)\*abs(a) + a)/(x\*abs(a)))\*arcsin(a\*x) + 2\*a\*log(abs(x)))\*a - 1/2\*arcsin(a\*x)^2/x^2

**maple** [A] time = 0.04, size = 43, normalized size = 0.98

$$-\frac{\arcsin(ax)^2}{2x^2} - \frac{a \arcsin(ax) \sqrt{-a^2 x^2 + 1}}{x} + a^2 \ln(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^2/x^3,x)

[Out] -1/2\*arcsin(a\*x)^2/x^2 - a\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/x + a^2\*ln(a\*x)

**maxima** [A] time = 0.53, size = 40, normalized size = 0.91

$$a^2 \log(x) - \frac{\sqrt{-a^2 x^2 + 1} a \arcsin(ax)}{x} - \frac{\arcsin(ax)^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^3,x, algorithm="maxima")

[Out] a^2\*log(x) - sqrt(-a^2\*x^2 + 1)\*a\*arcsin(a\*x)/x - 1/2\*arcsin(a\*x)^2/x^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arcsin(ax)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^2/x^3,x)

[Out] int(asin(a\*x)^2/x^3, x)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^2(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(a*x)**2/x**3,x)
```

```
[Out] Integral(asin(a*x)**2/x**3, x)
```

## 3.20 $\int \frac{\sin^{-1}(ax)^2}{x^4} dx$

**Optimal.** Leaf size=116

$$\frac{1}{3}ia^3\text{Li}_2(-e^{i\sin^{-1}(ax)}) - \frac{1}{3}ia^3\text{Li}_2(e^{i\sin^{-1}(ax)}) - \frac{2}{3}a^3\sin^{-1}(ax)\tanh^{-1}(e^{i\sin^{-1}(ax)}) - \frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)}{3x^2} - \frac{a^2}{3x} - \frac{\sin^{-1}(ax)}{3x^3}$$

[Out]  $-1/3*a^2/x - 1/3*\arcsin(ax)^2/x^3 - 2/3*a^3*\arcsin(ax)*\arctanh(I*a*x + (-a^2*x^2+1)^{(1/2)}) + 1/3*I*a^3*\text{polylog}(2, -I*a*x - (-a^2*x^2+1)^{(1/2)}) - 1/3*I*a^3*\text{polylog}(2, I*a*x + (-a^2*x^2+1)^{(1/2)}) - 1/3*a*\arcsin(ax)*(-a^2*x^2+1)^{(1/2)}/x^2$

**Rubi [A]** time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4627, 4701, 4709, 4183, 2279, 2391, 30}

$$\frac{1}{3}ia^3\text{PolyLog}(2, -e^{i\sin^{-1}(ax)}) - \frac{1}{3}ia^3\text{PolyLog}(2, e^{i\sin^{-1}(ax)}) - \frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)}{3x^2} - \frac{a^2}{3x} - \frac{2}{3}a^3\sin^{-1}(ax)\tanh^{-1}(e^{i\sin^{-1}(ax)})$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^2/x^4, x]

[Out]  $-a^2/(3*x) - (a*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(3*x^2) - \text{ArcSin}[a*x]^2/(3*x^3) - (2*a^3*\text{ArcSin}[a*x]*\text{ArcTanh}[E^(I*\text{ArcSin}[a*x])])/3 + (I/3)*a^3*\text{PolyLog}[2, -E^(I*\text{ArcSin}[a*x])] - (I/3)*a^3*\text{PolyLog}[2, E^(I*\text{ArcSin}[a*x])]$

### Rule 30

Int[(x\_)^(m\_), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 4183

Int[csc[(e\_) + (f\_)\*(x\_)]\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

### Rule 4701

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)
)*(x_)^(p_), x_Symbol] := Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcSin[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && LtQ[m, -1] && IntegerQ[m]
```

### Rule 4709

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_.) + (e_.)*
(x_)^2], x_Symbol] := Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin
[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d +
e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(ax)^2}{x^4} dx &= -\frac{\sin^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a) \int \frac{\sin^{-1}(ax)}{x^3\sqrt{1-a^2x^2}} dx \\ &= -\frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x^2} - \frac{\sin^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^2 \int \frac{1}{x^2} dx + \frac{1}{3}a^3 \int \frac{\sin^{-1}(ax)}{x\sqrt{1-a^2x^2}} dx \\ &= -\frac{a^2}{3x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x^2} - \frac{\sin^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^3 \operatorname{Subst}\left(\int x \csc(x) dx, x, \sin^{-1}(ax)\right) \\ &= -\frac{a^2}{3x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x^2} - \frac{\sin^{-1}(ax)^2}{3x^3} - \frac{2}{3}a^3 \sin^{-1}(ax) \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) - \frac{1}{3}a^3 \operatorname{Subst}\left(\int \frac{1}{x} dx, x, \sin^{-1}(ax)\right) \\ &= -\frac{a^2}{3x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x^2} - \frac{\sin^{-1}(ax)^2}{3x^3} - \frac{2}{3}a^3 \sin^{-1}(ax) \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) + \frac{1}{3}(ia^3) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, \sin^{-1}(ax)\right) \\ &= -\frac{a^2}{3x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x^2} - \frac{\sin^{-1}(ax)^2}{3x^3} - \frac{2}{3}a^3 \sin^{-1}(ax) \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) + \frac{1}{3}ia^3 \operatorname{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) \end{aligned}$$

**Mathematica [A]** time = 0.60, size = 139, normalized size = 1.20

$$\frac{-ia^3x^3\operatorname{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) + ia^3x^3\operatorname{Li}_2\left(e^{i \sin^{-1}(ax)}\right) - a^3x^3 \sin^{-1}(ax) \log\left(1 - e^{i \sin^{-1}(ax)}\right) + a^3x^3 \sin^{-1}(ax) \log\left(1 + e^{i \sin^{-1}(ax)}\right)}{3x^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^2/x^4,x]

[Out]  $-\frac{1}{3}(a^2x^2 + a*x*\sqrt{1 - a^2*x^2})*\operatorname{ArcSin}[a*x] + \operatorname{ArcSin}[a*x]^2 - a^3*x^3*\operatorname{ArcSin}[a*x]*\operatorname{Log}[1 - E^{(I*\operatorname{ArcSin}[a*x])}] + a^3*x^3*\operatorname{ArcSin}[a*x]*\operatorname{Log}[1 + E^{(I*\operatorname{ArcSin}[a*x])}] - I*a^3*x^3*\operatorname{PolyLog}[2, -E^{(I*\operatorname{ArcSin}[a*x])}] + I*a^3*x^3*\operatorname{PolyLog}[2, E^{(I*\operatorname{ArcSin}[a*x])}])/x^3$

**fricas [F]** time = 0.83, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\arcsin(ax)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^4,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^2/x^4, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^4,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^2/x^4, x)

**maple** [A] time = 0.32, size = 157, normalized size = 1.35

$$\frac{a \arcsin(ax) \sqrt{-a^2x^2+1}}{3x^2} - \frac{a^2 \arcsin(ax)^2}{3x} - \frac{a^3 \arcsin(ax) \ln\left(1+iax+\sqrt{-a^2x^2+1}\right)}{3} + \frac{ia^3 \operatorname{polylog}\left(2,-iax-\sqrt{-a^2x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^2/x^4,x)

[Out] -1/3\*a\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/x^2-1/3\*a^2/x-1/3\*arcsin(a\*x)^2/x^3-1/3\*a^3\*arcsin(a\*x)\*ln(1+I\*a\*x+(-a^2\*x^2+1)^(1/2))+1/3\*I\*a^3\*polylog(2,-I\*a\*x-(-a^2\*x^2+1)^(1/2))+1/3\*a^3\*arcsin(a\*x)\*ln(1-I\*a\*x-(-a^2\*x^2+1)^(1/2))-1/3\*I\*a^3\*polylog(2,I\*a\*x+(-a^2\*x^2+1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2ax^3 \int \frac{\sqrt{ax+1} \sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}{a^2x^5-x^3} dx + \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^4,x, algorithm="maxima")

[Out] -1/3\*(6\*a\*x^3\*integrate(1/3\*sqrt(a\*x+1)\*sqrt(-a\*x+1)\*arctan2(a\*x, sqrt(a\*x+1)\*sqrt(-a\*x+1))/(a^2\*x^5-x^3), x) + arctan2(a\*x, sqrt(a\*x+1)\*sqrt(-a\*x+1))^2)/x^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^2/x^4,x)

[Out] int(asin(a\*x)^2/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^2(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*2/x\*\*4,x)

[Out] Integral(asin(a\*x)\*\*2/x\*\*4, x)

### 3.21 $\int \frac{\sin^{-1}(ax)^2}{x^5} dx$

**Optimal.** Leaf size=87

$$\frac{1}{3}a^4 \log(x) - \frac{a^2}{12x^2} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{6x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x} - \frac{\sin^{-1}(ax)^2}{4x^4}$$

[Out]  $-1/12*a^2/x^2-1/4*\arcsin(a*x)^2/x^4+1/3*a^4*\ln(x)-1/6*a*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/x^3-1/3*a^3*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/x$

**Rubi [A]** time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 4701, 4681, 29, 30}

$$-\frac{a^2}{12x^2} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{6x^3} + \frac{1}{3}a^4 \log(x) - \frac{\sin^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^2/x^5, x]

[Out]  $-a^2/(12*x^2) - (a*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(6*x^3) - (a^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(3*x) - \text{ArcSin}[a*x]^2/(4*x^4) + (a^4*\text{Log}[x])/3$

**Rule 29**

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

**Rule 30**

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rule 4627**

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

**Rule 4681**

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_) \* (x\_)^2)^(p\_), x\_Symbol] := Simp[((f\*x)^(m+1)\*(d + e\*x^2)^(p+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*f\*(m+1)), x] - Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(f\*(m+1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m+1)\*(1 - c^2\*x^2)^(p+1/2)\*(a + b\*ArcSin[c\*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && EqQ[m + 2\*p + 3, 0] && NeQ[m, -1]

**Rule 4701**

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_) \* (x\_)^2)^(p\_), x\_Symbol] := Simp[((f\*x)^(m+1)\*(d + e\*x^2)^(p+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*f\*(m+1)), x] + (Dist[(c^2\*(m+2\*p+3))/(f^2\*(m+1)), Int[(f\*x)^(m+2)\*(d + e\*x^2)^p\*(a + b\*ArcSin[c\*x])^n, x], x] - Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(f\*(m+1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m+1)\*(1 - c^2\*x^2)^(p+1/2)\*(a + b\*ArcSin[c\*x])^(n-1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n,

0] && LtQ[m, -1] && IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sin^{-1}(ax)^2}{x^5} dx &= -\frac{\sin^{-1}(ax)^2}{4x^4} + \frac{1}{2}a \int \frac{\sin^{-1}(ax)}{x^4\sqrt{1-a^2x^2}} dx \\
 &= -\frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{6x^3} - \frac{\sin^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2 \int \frac{1}{x^3} dx + \frac{1}{3}a^3 \int \frac{\sin^{-1}(ax)}{x^2\sqrt{1-a^2x^2}} dx \\
 &= -\frac{a^2}{12x^2} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{6x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x} - \frac{\sin^{-1}(ax)^2}{4x^4} + \frac{1}{3}a^4 \int \frac{1}{x} dx \\
 &= -\frac{a^2}{12x^2} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)}{6x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)}{3x} - \frac{\sin^{-1}(ax)^2}{4x^4} + \frac{1}{3}a^4 \log(x)
 \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 69, normalized size = 0.79

$$\frac{1}{3}a^4 \log(x) - \frac{a^2}{12x^2} - \frac{a\sqrt{1-a^2x^2} (2a^2x^2 + 1) \sin^{-1}(ax)}{6x^3} - \frac{\sin^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^2/x^5,x]

[Out] -1/12\*a^2/x^2 - (a\*Sqrt[1 - a^2\*x^2]\*(1 + 2\*a^2\*x^2)\*ArcSin[a\*x])/(6\*x^3) - ArcSin[a\*x]^2/(4\*x^4) + (a^4\*Log[x])/3

**fricas** [A] time = 0.76, size = 62, normalized size = 0.71

$$\frac{4a^4x^4 \log(x) - a^2x^2 - 2(2a^3x^3 + ax)\sqrt{-a^2x^2 + 1} \arcsin(ax) - 3 \arcsin(ax)^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^5,x, algorithm="fricas")

[Out] 1/12\*(4\*a^4\*x^4\*log(x) - a^2\*x^2 - 2\*(2\*a^3\*x^3 + a\*x)\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x) - 3\*arcsin(a\*x)^2)/x^4

**giac** [B] time = 0.78, size = 185, normalized size = 2.13

$$\frac{1}{48} \left( \left( \frac{\left( a^4 + \frac{9(\sqrt{-a^2x^2+1}|a|+a)^2}{x^2} \right) a^6 x^3}{\left( \sqrt{-a^2x^2+1}|a|+a \right)^3 |a|} - \frac{\frac{9(\sqrt{-a^2x^2+1}|a|+a)a^4}{x} + \frac{(\sqrt{-a^2x^2+1}|a|+a)^3}{x^3}}{a^2|a|} \right) \arcsin(ax) + \frac{4 \left( 2a^4 \log(a^2x^2) - \frac{2(a^2x^2)}{a} \right)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^5,x, algorithm="giac")

[Out] 1/48\*(((a^4 + 9\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^2/x^2)\*a^6\*x^3/((sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^3\*abs(a)) - (9\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)\*a^4/x + (sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^3/x^3)/(a^2\*abs(a)))\*arcsin(a\*x) + 4\*(2\*a^4\*log(a^2\*x^2) - (2\*(a^2\*x^2 - 1)\*a^4 + 3\*a^4)/(a^2\*x^2))/a)\*a - 1/4\*arcsin(a\*x)^2/x^4

**maple** [A] time = 0.05, size = 76, normalized size = 0.87

$$-\frac{\arcsin(ax)^2}{4x^4} - \frac{a \arcsin(ax) \sqrt{-a^2x^2+1}}{6x^3} - \frac{a^2}{12x^2} - \frac{a^3 \arcsin(ax) \sqrt{-a^2x^2+1}}{3x} + \frac{a^4 \ln(ax)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^2/x^5,x)

[Out] -1/4\*arcsin(a\*x)^2/x^4-1/6\*a\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/x^3-1/12\*a^2/x^2-1/3\*a^3\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/x+1/3\*a^4\*ln(a\*x)

**maxima** [A] time = 0.49, size = 74, normalized size = 0.85

$$\frac{1}{12} \left( 4a^2 \log(x) - \frac{1}{x^2} \right) a^2 - \frac{1}{6} \left( \frac{2\sqrt{-a^2x^2+1}a^2}{x} + \frac{\sqrt{-a^2x^2+1}}{x^3} \right) a \arcsin(ax) - \frac{\arcsin(ax)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^2/x^5,x, algorithm="maxima")

[Out] 1/12\*(4\*a^2\*log(x) - 1/x^2)\*a^2 - 1/6\*(2\*sqrt(-a^2\*x^2 + 1)\*a^2/x + sqrt(-a^2\*x^2 + 1)/x^3)\*a\*arcsin(a\*x) - 1/4\*arcsin(a\*x)^2/x^4

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\arcsin(ax)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^2/x^5,x)

[Out] int(asin(a\*x)^2/x^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin^2(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*2/x\*\*5,x)

[Out] Integral(asin(a\*x)\*\*2/x\*\*5, x)

### 3.22 $\int x^4 \sin^{-1}(ax)^3 dx$

**Optimal.** Leaf size=201

$$-\frac{16x \sin^{-1}(ax)}{25a^4} - \frac{8x^3 \sin^{-1}(ax)}{75a^2} + \frac{3x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{25a} - \frac{6(1-a^2x^2)^{5/2}}{625a^5} + \frac{76(1-a^2x^2)^{3/2}}{1125a^5} - \frac{298\sqrt{1-a^2x^2}}{375a^5} + \dots$$

[Out]  $\frac{76}{1125}(-a^2x^2+1)^{(3/2)}/a^5 - \frac{6}{625}(-a^2x^2+1)^{(5/2)}/a^5 - \frac{16}{25}x \arcsin(ax)/a^4 - \frac{8}{75}x^3 \arcsin(ax)/a^2 - \frac{6}{125}x^5 \arcsin(ax) + \frac{1}{5}x^5 \arcsin(ax)^3 - \frac{298}{375}(-a^2x^2+1)^{(1/2)}/a^5 + \frac{8}{25} \arcsin(ax)^2 (-a^2x^2+1)^{(1/2)}/a^5 + \frac{4}{25}x^2 \arcsin(ax)^2 (-a^2x^2+1)^{(1/2)}/a^3 + \frac{3}{25}x^4 \arcsin(ax)^2 (-a^2x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.38, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4627, 4707, 4677, 4619, 261, 266, 43}

$$-\frac{6(1-a^2x^2)^{5/2}}{625a^5} + \frac{76(1-a^2x^2)^{3/2}}{1125a^5} - \frac{298\sqrt{1-a^2x^2}}{375a^5} + \frac{3x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{25a} - \frac{8x^3 \sin^{-1}(ax)}{75a^2} + \frac{4x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{25a^3} - \dots$$

Antiderivative was successfully verified.

[In] Int[x^4\*ArcSin[a\*x]^3,x]

[Out]  $(-298\sqrt{1-a^2x^2})/(375a^5) + (76(1-a^2x^2)^{(3/2)})/(1125a^5) - (6(1-a^2x^2)^{(5/2)})/(625a^5) - (16x \arcsin(ax))/(25a^4) - (8x^3 \arcsin(ax))/(75a^2) - (6x^5 \arcsin(ax))/125 + (8\sqrt{1-a^2x^2} \arcsin(ax)^2)/(25a^5) + (4x^2 \sqrt{1-a^2x^2} \arcsin(ax)^2)/(25a^3) + (3x^4 \sqrt{1-a^2x^2} \arcsin(ax)^2)/(25a) + (x^5 \arcsin(ax)^3)/5$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, n}, x] && GtQ[m, 0]



\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4707

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rubi steps

$$\begin{aligned}
 \int x^4 \sin^{-1}(ax)^3 dx &= \frac{1}{5}x^5 \sin^{-1}(ax)^3 - \frac{1}{5}(3a) \int \frac{x^5 \sin^{-1}(ax)^2}{\sqrt{1 - a^2x^2}} dx \\
 &= \frac{3x^4 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{25a} + \frac{1}{5}x^5 \sin^{-1}(ax)^3 - \frac{6}{25} \int x^4 \sin^{-1}(ax) dx - \frac{12 \int \frac{x^3 \sin^{-1}(ax)^2}{\sqrt{1 - a^2x^2}} dx}{25a} \\
 &= -\frac{6}{125}x^5 \sin^{-1}(ax) + \frac{4x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{25a^3} + \frac{3x^4 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{25a} + \frac{1}{5}x^5 \sin^{-1}(ax) \\
 &= -\frac{8x^3 \sin^{-1}(ax)}{75a^2} - \frac{6}{125}x^5 \sin^{-1}(ax) + \frac{8\sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{25a^5} + \frac{4x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{25a^3} \\
 &= -\frac{16x \sin^{-1}(ax)}{25a^4} - \frac{8x^3 \sin^{-1}(ax)}{75a^2} - \frac{6}{125}x^5 \sin^{-1}(ax) + \frac{8\sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{25a^5} + \frac{4x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{25a^3} \\
 &= -\frac{86\sqrt{1 - a^2x^2}}{125a^5} + \frac{4(1 - a^2x^2)^{3/2}}{125a^5} - \frac{6(1 - a^2x^2)^{5/2}}{625a^5} - \frac{16x \sin^{-1}(ax)}{25a^4} - \frac{8x^3 \sin^{-1}(ax)}{75a^2} - \frac{6}{125}x^5 \sin^{-1}(ax) \\
 &= -\frac{298\sqrt{1 - a^2x^2}}{375a^5} + \frac{76(1 - a^2x^2)^{3/2}}{1125a^5} - \frac{6(1 - a^2x^2)^{5/2}}{625a^5} - \frac{16x \sin^{-1}(ax)}{25a^4} - \frac{8x^3 \sin^{-1}(ax)}{75a^2} - \frac{6}{125}x^5 \sin^{-1}(ax)
 \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 122, normalized size = 0.61

$$\frac{1125a^5x^5 \sin^{-1}(ax)^3 - 2\sqrt{1 - a^2x^2} (27a^4x^4 + 136a^2x^2 + 2072) - 30ax (9a^4x^4 + 20a^2x^2 + 120) \sin^{-1}(ax) + 225a^5 \sin^{-1}(ax)^2}{5625a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcSin[a\*x]^3,x]

[Out] (-2\*Sqrt[1 - a^2\*x^2]\*(2072 + 136\*a^2\*x^2 + 27\*a^4\*x^4) - 30\*a\*x\*(120 + 20\*a^2\*x^2 + 9\*a^4\*x^4)\*ArcSin[a\*x] + 225\*Sqrt[1 - a^2\*x^2]\*(8 + 4\*a^2\*x^2 + 3\*a^4\*x^4)\*ArcSin[a\*x]^2 + 1125\*a^5\*x^5\*ArcSin[a\*x]^3)/(5625\*a^5)

**fricas** [A] time = 0.70, size = 105, normalized size = 0.52

$$\frac{1125 a^5 x^5 \arcsin(ax)^3 - 30 (9 a^5 x^5 + 20 a^3 x^3 + 120 ax) \arcsin(ax) - (54 a^4 x^4 + 272 a^2 x^2 - 225 (3 a^4 x^4 + 4 a^2 x^2 - 5625 a^5)) \arcsin(ax)^2 + 4144 \sqrt{-a^2 x^2 + 1}}{5625 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^3,x, algorithm="fricas")

[Out] 1/5625\*(1125\*a^5\*x^5\*arcsin(a\*x)^3 - 30\*(9\*a^5\*x^5 + 20\*a^3\*x^3 + 120\*a\*x)\*arcsin(a\*x) - (54\*a^4\*x^4 + 272\*a^2\*x^2 - 225\*(3\*a^4\*x^4 + 4\*a^2\*x^2 + 8))\*arcsin(a\*x)^2 + 4144)\*sqrt(-a^2\*x^2 + 1))/a^5

**giac** [A] time = 0.18, size = 249, normalized size = 1.24

$$\frac{(a^2 x^2 - 1)^2 x \arcsin(ax)^3}{5 a^4} + \frac{2(a^2 x^2 - 1)x \arcsin(ax)^3}{5 a^4} - \frac{6(a^2 x^2 - 1)^2 x \arcsin(ax)}{125 a^4} + \frac{x \arcsin(ax)^3}{5 a^4} + \frac{3(a^2 x^2 - 1)^2 \sqrt{-a^2 x^2 + 1}}{625 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^3,x, algorithm="giac")

[Out] 1/5\*(a^2\*x^2 - 1)^2\*x\*arcsin(a\*x)^3/a^4 + 2/5\*(a^2\*x^2 - 1)\*x\*arcsin(a\*x)^3/a^4 - 6/125\*(a^2\*x^2 - 1)^2\*x\*arcsin(a\*x)/a^4 + 1/5\*x\*arcsin(a\*x)^3/a^4 + 3/25\*(a^2\*x^2 - 1)^2\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)^2/a^5 - 76/375\*(a^2\*x^2 - 1)\*x\*arcsin(a\*x)/a^4 - 2/5\*(-a^2\*x^2 + 1)^(3/2)\*arcsin(a\*x)^2/a^5 - 298/375\*x\*arcsin(a\*x)/a^4 - 6/625\*(a^2\*x^2 - 1)^2\*sqrt(-a^2\*x^2 + 1)/a^5 + 3/5\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)^2/a^5 + 76/1125\*(-a^2\*x^2 + 1)^(3/2)/a^5 - 298/375\*sqrt(-a^2\*x^2 + 1)/a^5

**maple** [A] time = 0.08, size = 159, normalized size = 0.79

$$\frac{a^5 x^5 \arcsin(ax)^3}{5} + \frac{\arcsin(ax)^2 (3a^4 x^4 + 4a^2 x^2 + 8) \sqrt{-a^2 x^2 + 1}}{25} - \frac{16 \sqrt{-a^2 x^2 + 1}}{25} - \frac{16 a x \arcsin(ax)}{25} - \frac{6 a^5 x^5 \arcsin(ax)}{125} - \frac{2(3a^4 x^4 + 4a^2 x^2 + 8) \sqrt{-a^2 x^2 + 1}}{625 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arcsin(a\*x)^3,x)

[Out] 1/a^5\*(1/5\*a^5\*x^5\*arcsin(a\*x)^3+1/25\*arcsin(a\*x)^2\*(3\*a^4\*x^4+4\*a^2\*x^2+8)\*(-a^2\*x^2+1)^(1/2)-16/25\*(-a^2\*x^2+1)^(1/2)-16/25\*a\*x\*arcsin(a\*x)-6/125\*a^5\*x^5\*arcsin(a\*x)-2/625\*(3\*a^4\*x^4+4\*a^2\*x^2+8)\*(-a^2\*x^2+1)^(1/2)-8/75\*a^3\*x^3\*arcsin(a\*x)-8/225\*(a^2\*x^2+2)\*(-a^2\*x^2+1)^(1/2))

**maxima** [A] time = 0.52, size = 171, normalized size = 0.85

$$\frac{1}{5} x^5 \arcsin(ax)^3 + \frac{1}{25} \left( \frac{3 \sqrt{-a^2 x^2 + 1} x^4}{a^2} + \frac{4 \sqrt{-a^2 x^2 + 1} x^2}{a^4} + \frac{8 \sqrt{-a^2 x^2 + 1}}{a^6} \right) a \arcsin(ax)^2 - \frac{2}{5625} a \left( \frac{27 \sqrt{-a^2 x^2 + 1}}{a^5} - \frac{16 \sqrt{-a^2 x^2 + 1}}{a^5} - \frac{16 a x \arcsin(ax)}{a^5} - \frac{6 a^5 x^5 \arcsin(ax)}{a^5} - \frac{2(3 a^4 x^4 + 4 a^2 x^2 + 8) \sqrt{-a^2 x^2 + 1}}{a^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^3,x, algorithm="maxima")

[Out] 1/5\*x^5\*arcsin(a\*x)^3 + 1/25\*(3\*sqrt(-a^2\*x^2 + 1)\*x^4/a^2 + 4\*sqrt(-a^2\*x^2 + 1)\*x^2/a^4 + 8\*sqrt(-a^2\*x^2 + 1)/a^6)\*a\*arcsin(a\*x)^2 - 2/5625\*a\*((27\*sqrt(-a^2\*x^2 + 1)\*a^2\*x^4 + 136\*sqrt(-a^2\*x^2 + 1)\*x^2 + 2072\*sqrt(-a^2\*x^2 + 1)/a^2)/a^4 + 15\*(9\*a^4\*x^5 + 20\*a^2\*x^3 + 120\*x)\*arcsin(a\*x)/a^5)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{asin}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*asin(a*x)^3,x)`

[Out] `int(x^4*asin(a*x)^3, x)`

**sympy** [A] time = 5.68, size = 196, normalized size = 0.98

$$\left\{ \begin{array}{l} \frac{x^5 \operatorname{asin}^3(ax)}{5} - \frac{6x^5 \operatorname{asin}(ax)}{125} + \frac{3x^4 \sqrt{-a^2x^2+1} \operatorname{asin}^2(ax)}{25a} - \frac{6x^4 \sqrt{-a^2x^2+1}}{625a} - \frac{8x^3 \operatorname{asin}(ax)}{75a^2} + \frac{4x^2 \sqrt{-a^2x^2+1} \operatorname{asin}^2(ax)}{25a^3} - \frac{272x^2 \sqrt{-a^2x^2+1}}{5625a^3} \\ 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*asin(a*x)**3,x)`

[Out] `Piecewise((x**5*asin(a*x)**3/5 - 6*x**5*asin(a*x)/125 + 3*x**4*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(25*a) - 6*x**4*sqrt(-a**2*x**2 + 1)/(625*a) - 8*x**3*asin(a*x)/(75*a**2) + 4*x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(25*a**3) - 272*x**2*sqrt(-a**2*x**2 + 1)/(5625*a**3) - 16*x*asin(a*x)/(25*a**4) + 8*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(25*a**5) - 4144*sqrt(-a**2*x**2 + 1)/(5625*a**5), Ne(a, 0)), (0, True))`

### 3.23 $\int x^3 \sin^{-1}(ax)^3 dx$

**Optimal.** Leaf size=167

$$-\frac{3 \sin^{-1}(ax)^3}{32a^4} + \frac{45 \sin^{-1}(ax)}{256a^4} - \frac{9x^2 \sin^{-1}(ax)}{32a^2} - \frac{3x^3 \sqrt{1-a^2x^2}}{128a} + \frac{3x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{16a} - \frac{45x \sqrt{1-a^2x^2}}{256a^3} + \frac{9x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{32a^2}$$

[Out] 45/256\*arcsin(a\*x)/a^4-9/32\*x^2\*arcsin(a\*x)/a^2-3/32\*x^4\*arcsin(a\*x)-3/32\*a  
rccsin(a\*x)^3/a^4+1/4\*x^4\*arcsin(a\*x)^3-45/256\*x\*(-a^2\*x^2+1)^(1/2)/a^3-3/12  
8\*x^3\*(-a^2\*x^2+1)^(1/2)/a+9/32\*x\*arcsin(a\*x)^2\*(-a^2\*x^2+1)^(1/2)/a^3+3/16  
\*x^3\*arcsin(a\*x)^2\*(-a^2\*x^2+1)^(1/2)/a

**Rubi [A]** time = 0.30, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 10, number of rules / integrand size = 0.500, Rules used = {4627, 4707, 4641, 321, 216}

$$-\frac{3x^3 \sqrt{1-a^2x^2}}{128a} - \frac{45x \sqrt{1-a^2x^2}}{256a^3} + \frac{3x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{16a} - \frac{9x^2 \sin^{-1}(ax)}{32a^2} + \frac{9x \sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{32a^3} - \frac{3 \sin^{-1}(ax)}{32a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcSin[a\*x]^3,x]

[Out] (-45\*x\*Sqrt[1 - a^2\*x^2])/(256\*a^3) - (3\*x^3\*Sqrt[1 - a^2\*x^2])/(128\*a) + (45\*ArcSin[a\*x])/(256\*a^4) - (9\*x^2\*ArcSin[a\*x])/(32\*a^2) - (3\*x^4\*ArcSin[a\*x])/32 + (9\*x\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2)/(32\*a^3) + (3\*x^3\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2)/(16\*a) - (3\*ArcSin[a\*x]^3)/(32\*a^4) + (x^4\*ArcSin[a\*x]^3)/4

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*

$x^2])/(c*\text{Sqrt}[d + e*x^2]), \text{Int}[(f*x)^{(m-1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{GtQ}[m, 1] \&\& \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned} \int x^3 \sin^{-1}(ax)^3 dx &= \frac{1}{4}x^4 \sin^{-1}(ax)^3 - \frac{1}{4}(3a) \int \frac{x^4 \sin^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx \\ &= \frac{3x^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{16a} + \frac{1}{4}x^4 \sin^{-1}(ax)^3 - \frac{3}{8} \int x^3 \sin^{-1}(ax) dx - \frac{9 \int \frac{x^2 \sin^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx}{16a} \\ &= -\frac{3}{32}x^4 \sin^{-1}(ax) + \frac{9x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{32a^3} + \frac{3x^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{16a} + \frac{1}{4}x^4 \sin^{-1}(ax) \\ &= -\frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2 \sin^{-1}(ax)}{32a^2} - \frac{3}{32}x^4 \sin^{-1}(ax) + \frac{9x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{32a^3} + \frac{3x^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{16a} \\ &= -\frac{45x\sqrt{1-a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2 \sin^{-1}(ax)}{32a^2} - \frac{3}{32}x^4 \sin^{-1}(ax) + \frac{9x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{32a^3} \\ &= -\frac{45x\sqrt{1-a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1-a^2x^2}}{128a} + \frac{45 \sin^{-1}(ax)}{256a^4} - \frac{9x^2 \sin^{-1}(ax)}{32a^2} - \frac{3}{32}x^4 \sin^{-1}(ax) + \frac{9x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{32a^3} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 112, normalized size = 0.67

$$\frac{8(8a^4x^4 - 3) \sin^{-1}(ax)^3 - 3ax\sqrt{1-a^2x^2} (2a^2x^2 + 15) + 24ax\sqrt{1-a^2x^2} (2a^2x^2 + 3) \sin^{-1}(ax)^2 - 3(8a^4x^4 + 24a^2x^2 - 15) \sin^{-1}(ax)}{256a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcSin[a\*x]^3,x]

[Out] (-3\*a\*x\*Sqrt[1 - a^2\*x^2]\*(15 + 2\*a^2\*x^2) - 3\*(-15 + 24\*a^2\*x^2 + 8\*a^4\*x^4)\*ArcSin[a\*x] + 24\*a\*x\*Sqrt[1 - a^2\*x^2]\*(3 + 2\*a^2\*x^2)\*ArcSin[a\*x]^2 + 8\*(-3 + 8\*a^4\*x^4)\*ArcSin[a\*x]^3)/(256\*a^4)

**fricas [A]** time = 0.53, size = 96, normalized size = 0.57

$$\frac{8(8a^4x^4 - 3) \arcsin(ax)^3 - 3(8a^4x^4 + 24a^2x^2 - 15) \arcsin(ax) - 3(2a^3x^3 - 8(2a^3x^3 + 3ax) \arcsin(ax)^2 - 15 \arcsin(ax)) \sqrt{1-a^2x^2}}{256a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^3,x, algorithm="fricas")

[Out] 1/256\*(8\*(8\*a^4\*x^4 - 3)\*arcsin(a\*x)^3 - 3\*(8\*a^4\*x^4 + 24\*a^2\*x^2 - 15)\*arcsin(a\*x) - 3\*(2\*a^3\*x^3 - 8\*(2\*a^3\*x^3 + 3\*a\*x)\*arcsin(a\*x)^2 + 15\*a\*x)\*sqrt(-a^2\*x^2 + 1))/a^4

**giac [A]** time = 0.20, size = 185, normalized size = 1.11

$$-\frac{3(-a^2x^2 + 1)^{\frac{3}{2}}x \arcsin(ax)^2}{16a^3} + \frac{(a^2x^2 - 1)^2 \arcsin(ax)^3}{4a^4} + \frac{15\sqrt{-a^2x^2 + 1}x \arcsin(ax)^2}{32a^3} + \frac{(a^2x^2 - 1) \arcsin(ax)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^3,x, algorithm="giac")

[Out]  $-3/16*(-a^2*x^2 + 1)^{(3/2)}*x*\arcsin(ax)^2/a^3 + 1/4*(a^2*x^2 - 1)^2*\arcsin(ax)^3/a^4 + 15/32*\sqrt{-a^2*x^2 + 1}*x*\arcsin(ax)^2/a^3 + 1/2*(a^2*x^2 - 1)*\arcsin(ax)^3/a^4 + 3/128*(-a^2*x^2 + 1)^{(3/2)}*x/a^3 - 3/32*(a^2*x^2 - 1)^2*\arcsin(ax)/a^4 + 5/32*\arcsin(ax)^3/a^4 - 51/256*\sqrt{-a^2*x^2 + 1}*x/a^3 - 15/32*(a^2*x^2 - 1)*\arcsin(ax)/a^4 - 51/256*\arcsin(ax)/a^4$

**maple** [A] time = 0.09, size = 154, normalized size = 0.92

$$\frac{a^4 x^4 \arcsin(ax)^3}{4} - \frac{3 \arcsin(ax)^2 (-2a^3 x^3 \sqrt{-a^2 x^2 + 1} - 3ax \sqrt{-a^2 x^2 + 1} + 3 \arcsin(ax))}{32} - \frac{3a^4 x^4 \arcsin(ax)}{32} - \frac{3ax(2a^2 x^2 + 3) \sqrt{-a^2 x^2 + 1}}{256} - \frac{27 \arcsin(ax)}{256} a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arcsin(a*x)^3,x)`

[Out]  $1/a^4*(1/4*a^4*x^4*\arcsin(ax)^3-3/32*\arcsin(ax)^2*(-2*a^3*x^3*(-a^2*x^2+1)^{(1/2)}-3*a*x*(-a^2*x^2+1)^{(1/2)}+3*\arcsin(ax))-3/32*a^4*x^4*\arcsin(ax)-3/256*a*x*(2*a^2*x^2+3)*(-a^2*x^2+1)^{(1/2)}-27/256*\arcsin(ax)-9/32*(a^2*x^2-1)*\arcsin(ax)-9/64*a*x*(-a^2*x^2+1)^{(1/2)}+3/16*\arcsin(ax)^3)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} x^4 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^3 + 3a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^4 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^2}{4(a^2 x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(a*x)^3,x, algorithm="maxima")`

[Out]  $1/4*x^4*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})^3 + 3*a*\integrate(1/4*\sqrt{ax+1}*\sqrt{-ax+1}*x^4*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})^2/(a^2*x^2-1), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asin}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*asin(a*x)^3,x)`

[Out] `int(x^3*asin(a*x)^3, x)`

**sympy** [A] time = 3.34, size = 160, normalized size = 0.96

$$\begin{cases} \frac{x^4 \operatorname{asin}^3(ax)}{4} - \frac{3x^4 \operatorname{asin}(ax)}{32} + \frac{3x^3 \sqrt{-a^2 x^2 + 1} \operatorname{asin}^2(ax)}{16a} - \frac{3x^3 \sqrt{-a^2 x^2 + 1}}{128a} - \frac{9x^2 \operatorname{asin}(ax)}{32a^2} + \frac{9x \sqrt{-a^2 x^2 + 1} \operatorname{asin}^2(ax)}{32a^3} - \frac{45x \sqrt{-a^2 x^2 + 1}}{256a^3} - 3 \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*asin(a*x)**3,x)`

[Out] `Piecewise((x**4*asin(a*x)**3/4 - 3*x**4*asin(a*x)/32 + 3*x**3*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(16*a) - 3*x**3*sqrt(-a**2*x**2 + 1)/(128*a) - 9*x**2*asin(a*x)/(32*a**2) + 9*x*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(32*a**3) - 45*x*sqrt(-a**2*x**2 + 1)/(256*a**3) - 3*asin(a*x)**3/(32*a**4) + 45*asin(a*x)/(256*a**4), Ne(a, 0)), (0, True))`

### 3.24 $\int x^2 \sin^{-1}(ax)^3 dx$

**Optimal.** Leaf size=136

$$\frac{x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{3a} - \frac{4x \sin^{-1}(ax)}{3a^2} + \frac{2(1-a^2x^2)^{3/2}}{27a^3} - \frac{14\sqrt{1-a^2x^2}}{9a^3} + \frac{2\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \sin^{-1}(ax)$$

[Out]  $2/27*(-a^2*x^2+1)^{(3/2)}/a^3-4/3*x*\arcsin(a*x)/a^2-2/9*x^3*\arcsin(a*x)+1/3*x^3*\arcsin(a*x)^3-14/9*(-a^2*x^2+1)^{(1/2)}/a^3+2/3*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}/a^3+1/3*x^2*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4627, 4707, 4677, 4619, 261, 266, 43}

$$\frac{2(1-a^2x^2)^{3/2}}{27a^3} - \frac{14\sqrt{1-a^2x^2}}{9a^3} + \frac{x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{3a} + \frac{2\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{3a^3} - \frac{4x \sin^{-1}(ax)}{3a^2} + \frac{1}{3}x^3 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcSin[a\*x]^3,x]

[Out]  $(-14*\text{Sqrt}[1 - a^2*x^2])/(9*a^3) + (2*(1 - a^2*x^2)^{(3/2)})/(27*a^3) - (4*x*ArcSin[a*x])/(3*a^2) - (2*x^3*ArcSin[a*x])/9 + (2*\text{Sqrt}[1 - a^2*x^2]*ArcSin[a*x]^2)/(3*a^3) + (x^2*\text{Sqrt}[1 - a^2*x^2]*ArcSin[a*x]^2)/(3*a) + (x^3*ArcSin[a*x]^3)/3$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p + 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

### Rule 4707

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[(f*x)^(m - 2)*(a + b*ArcSin[c*x])^n]/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

### Rubi steps

$$\begin{aligned} \int x^2 \sin^{-1}(ax)^3 dx &= \frac{1}{3}x^3 \sin^{-1}(ax)^3 - a \int \frac{x^3 \sin^{-1}(ax)^2}{\sqrt{1 - a^2x^2}} dx \\ &= \frac{x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a} + \frac{1}{3}x^3 \sin^{-1}(ax)^3 - \frac{2}{3} \int x^2 \sin^{-1}(ax) dx - \frac{2 \int \frac{x \sin^{-1}(ax)^2}{\sqrt{1 - a^2x^2}} dx}{3a} \\ &= -\frac{2}{9}x^3 \sin^{-1}(ax) + \frac{2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a^3} + \frac{x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a} + \frac{1}{3}x^3 \sin^{-1}(ax)^3 - \frac{4}{3} \int x \sin^{-1}(ax) dx \\ &= -\frac{4x \sin^{-1}(ax)}{3a^2} - \frac{2}{9}x^3 \sin^{-1}(ax) + \frac{2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a^3} + \frac{x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a} + \frac{1}{3}x^3 \sin^{-1}(ax)^3 \\ &= -\frac{4\sqrt{1 - a^2x^2}}{3a^3} - \frac{4x \sin^{-1}(ax)}{3a^2} - \frac{2}{9}x^3 \sin^{-1}(ax) + \frac{2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a^3} + \frac{x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a} \\ &= -\frac{14\sqrt{1 - a^2x^2}}{9a^3} + \frac{2(1 - a^2x^2)^{3/2}}{27a^3} - \frac{4x \sin^{-1}(ax)}{3a^2} - \frac{2}{9}x^3 \sin^{-1}(ax) + \frac{2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^2}{3a^3} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 95, normalized size = 0.70

$$\frac{9a^3x^3 \sin^{-1}(ax)^3 - 2\sqrt{1 - a^2x^2} (a^2x^2 + 20) + 9\sqrt{1 - a^2x^2} (a^2x^2 + 2) \sin^{-1}(ax)^2 - 6ax (a^2x^2 + 6) \sin^{-1}(ax)}{27a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcSin[a\*x]^3,x]

[Out] (-2\*Sqrt[1 - a^2\*x^2]\*(20 + a^2\*x^2) - 6\*a\*x\*(6 + a^2\*x^2)\*ArcSin[a\*x] + 9\*Sqrt[1 - a^2\*x^2]\*(2 + a^2\*x^2)\*ArcSin[a\*x]^2 + 9\*a^3\*x^3\*ArcSin[a\*x]^3)/(27\*a^3)

**fricas [A]** time = 1.84, size = 79, normalized size = 0.58

$$\frac{9a^3x^3 \arcsin(ax)^3 - 6(a^3x^3 + 6ax) \arcsin(ax) - (2a^2x^2 - 9(a^2x^2 + 2) \arcsin(ax)^2 + 40)\sqrt{-a^2x^2 + 1}}{27a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^3,x, algorithm="fricas")



[Out]  $1/27*(9*a^3*x^3*\arcsin(a*x)^3 - 6*(a^3*x^3 + 6*a*x)*\arcsin(a*x) - (2*a^2*x^2 - 9*(a^2*x^2 + 2)*\arcsin(a*x)^2 + 40)*\sqrt{-a^2*x^2 + 1})/a^3$

**giac** [A] time = 0.23, size = 142, normalized size = 1.04

$$\frac{(a^2x^2 - 1)x \arcsin(ax)^3}{3a^2} + \frac{x \arcsin(ax)^3}{3a^2} - \frac{2(a^2x^2 - 1)x \arcsin(ax)}{9a^2} - \frac{(-a^2x^2 + 1)^{\frac{3}{2}} \arcsin(ax)^2}{3a^3} - \frac{14x \arcsin(ax)}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsin(a*x)^3,x, algorithm="giac")`

[Out]  $1/3*(a^2*x^2 - 1)*x*\arcsin(a*x)^3/a^2 + 1/3*x*\arcsin(a*x)^3/a^2 - 2/9*(a^2*x^2 - 1)*x*\arcsin(a*x)/a^2 - 1/3*(-a^2*x^2 + 1)^{(3/2)}*\arcsin(a*x)^2/a^3 - 14/9*x*\arcsin(a*x)/a^2 + \sqrt{-a^2*x^2 + 1}*\arcsin(a*x)^2/a^3 + 2/27*(-a^2*x^2 + 1)^{(3/2)}/a^3 - 14/9*\sqrt{-a^2*x^2 + 1}/a^3$

**maple** [A] time = 0.06, size = 106, normalized size = 0.78

$$\frac{a^3x^3 \arcsin(ax)^3}{3} + \frac{\arcsin(ax)^2(a^2x^2+2)\sqrt{-a^2x^2+1}}{3} - \frac{4\sqrt{-a^2x^2+1}}{3} - \frac{4ax \arcsin(ax)}{3} - \frac{2a^3x^3 \arcsin(ax)}{9} - \frac{2(a^2x^2+2)\sqrt{-a^2x^2+1}}{27}$$

$a^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arcsin(a*x)^3,x)`

[Out]  $1/a^3*(1/3*a^3*x^3*\arcsin(a*x)^3+1/3*\arcsin(a*x)^2*(a^2*x^2+2)*(-a^2*x^2+1)^{(1/2)}-4/3*(-a^2*x^2+1)^{(1/2)}-4/3*a*x*\arcsin(a*x)-2/9*a^3*x^3*\arcsin(a*x)-2/27*(a^2*x^2+2)*(-a^2*x^2+1)^{(1/2)})$

**maxima** [A] time = 0.52, size = 120, normalized size = 0.88

$$\frac{1}{3}x^3 \arcsin(ax)^3 + \frac{1}{3}a \left( \frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4} \right) \arcsin(ax)^2 - \frac{2}{27}a \left( \frac{\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2}}{a^2} + 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsin(a*x)^3,x, algorithm="maxima")`

[Out]  $1/3*x^3*\arcsin(a*x)^3 + 1/3*a*(\sqrt{-a^2*x^2 + 1})*x^2/a^2 + 2*\sqrt{-a^2*x^2 + 1}/a^4)*\arcsin(a*x)^2 - 2/27*a*((\sqrt{-a^2*x^2 + 1})*x^2 + 20*\sqrt{-a^2*x^2 + 1}/a^2)/a^2 + 3*(a^2*x^3 + 6*x)*\arcsin(a*x)/a^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asin}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*asin(a*x)^3,x)`

[Out] `int(x^2*asin(a*x)^3, x)`

**sympy** [A] time = 1.87, size = 128, normalized size = 0.94

$$\left\{ \begin{array}{l} \frac{x^3 \operatorname{asin}^3(ax)}{3} - \frac{2x^3 \operatorname{asin}(ax)}{9} + \frac{x^2 \sqrt{-a^2x^2+1} \operatorname{asin}^2(ax)}{3a} - \frac{2x^2 \sqrt{-a^2x^2+1}}{27a} - \frac{4x \operatorname{asin}(ax)}{3a^2} + \frac{2\sqrt{-a^2x^2+1} \operatorname{asin}^2(ax)}{3a^3} - \frac{40\sqrt{-a^2x^2+1}}{27a^3} \\ 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(a*x)**3,x)
```

```
[Out] Piecewise((x**3*asin(a*x)**3/3 - 2*x**3*asin(a*x)/9 + x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(3*a) - 2*x**2*sqrt(-a**2*x**2 + 1)/(27*a) - 4*x*asin(a*x)/(3*a**2) + 2*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(3*a**3) - 40*sqrt(-a**2*x**2 + 1)/(27*a**3), Ne(a, 0)), (0, True))
```

## 3.25 $\int x \sin^{-1}(ax)^3 dx$

**Optimal.** Leaf size=99

$$-\frac{3x\sqrt{1-a^2x^2}}{8a} + \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4a} - \frac{\sin^{-1}(ax)^3}{4a^2} + \frac{3 \sin^{-1}(ax)}{8a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^3 - \frac{3}{4}x^2 \sin^{-1}(ax)$$

[Out] 3/8\*arcsin(a\*x)/a^2-3/4\*x^2\*arcsin(a\*x)-1/4\*arcsin(a\*x)^3/a^2+1/2\*x^2\*arcsin(a\*x)^3-3/8\*x\*(-a^2\*x^2+1)^(1/2)/a+3/4\*x\*arcsin(a\*x)^2\*(-a^2\*x^2+1)^(1/2)/a

**Rubi [A]** time = 0.16, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4627, 4707, 4641, 321, 216}

$$-\frac{3x\sqrt{1-a^2x^2}}{8a} + \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4a} - \frac{\sin^{-1}(ax)^3}{4a^2} + \frac{3 \sin^{-1}(ax)}{8a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^3 - \frac{3}{4}x^2 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x\*ArcSin[a\*x]^3,x]

[Out] (-3\*x\*Sqrt[1 - a^2\*x^2])/(8\*a) + (3\*ArcSin[a\*x])/(8\*a^2) - (3\*x^2\*ArcSin[a\*x])/4 + (3\*x\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2)/(4\*a) - ArcSin[a\*x]^3/(4\*a^2) + (x^2\*ArcSin[a\*x]^3)/2

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(a+b\*ArcSin[c\*x])^(n+1)/(b\*c\*Sqrt[d]\*(n+1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d+e, 0] && GtQ[d, 0] && NeQ[n, -1]

### Rule 4707

Int((((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m-1)\*Sqrt[d+e\*x^2]\*(a+b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m-1))/(c^2\*m), Int[((f\*x)^(m-2)\*(a+b\*ArcSin[c\*x])^n)/Sqrt[d+e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1-c^2\*x^2])/(c\*m\*Sqrt[d+e\*x^2]), Int[(f\*x)^(m-1)\*(a+b\*ArcSin[c\*x])^(n-1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d+e, 0] && GtQ[n, 0]

&& GtQ[m, 1] && IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int x \sin^{-1}(ax)^3 dx &= \frac{1}{2}x^2 \sin^{-1}(ax)^3 - \frac{1}{2}(3a) \int \frac{x^2 \sin^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx \\
 &= \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4a} + \frac{1}{2}x^2 \sin^{-1}(ax)^3 - \frac{3}{2} \int x \sin^{-1}(ax) dx - \frac{3 \int \frac{\sin^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx}{4a} \\
 &= -\frac{3}{4}x^2 \sin^{-1}(ax) + \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4a} - \frac{\sin^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^3 + \frac{1}{4}(3a) \int \frac{x^2}{\sqrt{1-a^2x^2}} dx \\
 &= -\frac{3x\sqrt{1-a^2x^2}}{8a} - \frac{3}{4}x^2 \sin^{-1}(ax) + \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4a} - \frac{\sin^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^3 + \frac{1}{4} \int \frac{x^2}{\sqrt{1-a^2x^2}} dx \\
 &= -\frac{3x\sqrt{1-a^2x^2}}{8a} + \frac{3 \sin^{-1}(ax)}{8a^2} - \frac{3}{4}x^2 \sin^{-1}(ax) + \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4a} - \frac{\sin^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^3
 \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 82, normalized size = 0.83

$$\frac{-3ax\sqrt{1-a^2x^2} + (4a^2x^2 - 2) \sin^{-1}(ax)^3 + 6ax\sqrt{1-a^2x^2} \sin^{-1}(ax)^2 + (3 - 6a^2x^2) \sin^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcSin[a\*x]^3,x]

[Out] (-3\*a\*x\*Sqrt[1 - a^2\*x^2] + (3 - 6\*a^2\*x^2)\*ArcSin[a\*x] + 6\*a\*x\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2 + (-2 + 4\*a^2\*x^2)\*ArcSin[a\*x]^3)/(8\*a^2)

**fricas** [A] time = 0.74, size = 69, normalized size = 0.70

$$\frac{2(2a^2x^2 - 1) \arcsin(ax)^3 - 3(2a^2x^2 - 1) \arcsin(ax) + 3\sqrt{-a^2x^2 + 1}(2ax \arcsin(ax)^2 - ax)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^3,x, algorithm="fricas")

[Out] 1/8\*(2\*(2\*a^2\*x^2 - 1)\*arcsin(a\*x)^3 - 3\*(2\*a^2\*x^2 - 1)\*arcsin(a\*x) + 3\*sqrt(-a^2\*x^2 + 1)\*(2\*a\*x\*arcsin(a\*x)^2 - a\*x))/a^2

**giac** [A] time = 0.14, size = 101, normalized size = 1.02

$$\frac{3\sqrt{-a^2x^2 + 1}x \arcsin(ax)^2}{4a} + \frac{(a^2x^2 - 1) \arcsin(ax)^3}{2a^2} + \frac{\arcsin(ax)^3}{4a^2} - \frac{3\sqrt{-a^2x^2 + 1}x}{8a} - \frac{3(a^2x^2 - 1) \arcsin(ax)}{4a^2} - \frac{3 \arcsin(ax)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^3,x, algorithm="giac")

[Out] 3/4\*sqrt(-a^2\*x^2 + 1)\*x\*arcsin(a\*x)^2/a + 1/2\*(a^2\*x^2 - 1)\*arcsin(a\*x)^3/a^2 + 1/4\*arcsin(a\*x)^3/a^2 - 3/8\*sqrt(-a^2\*x^2 + 1)\*x/a - 3/4\*(a^2\*x^2 - 1)\*arcsin(a\*x)/a^2 - 3/8\*arcsin(a\*x)/a^2

**maple** [A] time = 0.06, size = 96, normalized size = 0.97

$$\frac{(a^2x^2-1) \arcsin(ax)^3}{2} + \frac{3 \arcsin(ax)^2 (ax\sqrt{-a^2x^2+1} + \arcsin(ax))}{4} - \frac{3(a^2x^2-1) \arcsin(ax)}{4} - \frac{3ax\sqrt{-a^2x^2+1}}{8} - \frac{3 \arcsin(ax)}{8} - \frac{\arcsin(ax)^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsin(a*x)^3,x)`

[Out]  $\frac{1}{a^2} \left( \frac{1}{2} (a^2 x^2 - 1) \arcsin(ax)^3 + \frac{3}{4} \arcsin(ax)^2 (a x \sqrt{-a^2 x^2 + 1})^{1/2} + \arcsin(ax) - \frac{3}{4} (a^2 x^2 - 1) \arcsin(ax) - \frac{3}{8} a x \sqrt{-a^2 x^2 + 1}^{1/2} - \frac{3}{8} \arcsin(ax) - \frac{1}{2} \arcsin(ax)^3 \right)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} x^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^3 + 3a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^2}{2(a^2 x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsin(a*x)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} x^2 \arctan^2(ax, \sqrt{ax+1} \sqrt{-ax+1})^3 + 3a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^2 \arctan^2(ax, \sqrt{ax+1} \sqrt{-ax+1})^2}{2(a^2 x^2 - 1)} dx$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{asin}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*asin(a*x)^3,x)`

[Out] `int(x*asin(a*x)^3, x)`

**sympy** [A] time = 0.90, size = 92, normalized size = 0.93

$$\begin{cases} \frac{x^2 \operatorname{asin}^3(ax)}{2} - \frac{3x^2 \operatorname{asin}(ax)}{4} + \frac{3x \sqrt{-a^2 x^2 + 1} \operatorname{asin}^2(ax)}{4a} - \frac{3x \sqrt{-a^2 x^2 + 1}}{8a} - \frac{\operatorname{asin}^3(ax)}{4a^2} + \frac{3 \operatorname{asin}(ax)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*asin(a*x)**3,x)`

[Out] `Piecewise((x**2*asin(a*x)**3/2 - 3*x**2*asin(a*x)/4 + 3*x*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(4*a) - 3*x*sqrt(-a**2*x**2 + 1)/(8*a) - asin(a*x)**3/(4*a**2) + 3*asin(a*x)/(8*a**2), Ne(a, 0)), (0, True))`

### 3.26 $\int \sin^{-1}(ax)^3 dx$

Optimal. Leaf size=60

$$-\frac{6\sqrt{1-a^2x^2}}{a} + \frac{3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{a} + x \sin^{-1}(ax)^3 - 6x \sin^{-1}(ax)$$

[Out]  $-6*x*\arcsin(a*x)+x*\arcsin(a*x)^3-6*(-a^2*x^2+1)^{(1/2)}/a+3*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4619, 4677, 261}

$$-\frac{6\sqrt{1-a^2x^2}}{a} + \frac{3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{a} + x \sin^{-1}(ax)^3 - 6x \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^3,x]

[Out]  $(-6*\text{Sqrt}[1 - a^2*x^2])/a - 6*x*\text{ArcSin}[a*x] + (3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^2)/a + x*\text{ArcSin}[a*x]^3$

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \sin^{-1}(ax)^3 dx &= x \sin^{-1}(ax)^3 - (3a) \int \frac{x \sin^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx \\ &= \frac{3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{a} + x \sin^{-1}(ax)^3 - 6 \int \sin^{-1}(ax) dx \\ &= -6x \sin^{-1}(ax) + \frac{3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{a} + x \sin^{-1}(ax)^3 + (6a) \int \frac{x}{\sqrt{1-a^2x^2}} dx \\ &= -\frac{6\sqrt{1-a^2x^2}}{a} - 6x \sin^{-1}(ax) + \frac{3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{a} + x \sin^{-1}(ax)^3 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 60, normalized size = 1.00

$$-\frac{6\sqrt{1-a^2x^2}}{a} + \frac{3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{a} + x \sin^{-1}(ax)^3 - 6x \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^3,x]

[Out] (-6\*Sqrt[1 - a^2\*x^2])/a - 6\*x\*ArcSin[a\*x] + (3\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2)/a + x\*ArcSin[a\*x]^3

**fricas [A]** time = 0.65, size = 44, normalized size = 0.73

$$\frac{ax \arcsin(ax)^3 - 6ax \arcsin(ax) + 3\sqrt{-a^2x^2 + 1}(\arcsin(ax)^2 - 2)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3,x, algorithm="fricas")

[Out] (a\*x\*arcsin(a\*x)^3 - 6\*a\*x\*arcsin(a\*x) + 3\*sqrt(-a^2\*x^2 + 1)\*(arcsin(a\*x)^2 - 2))/a

**giac [A]** time = 0.14, size = 56, normalized size = 0.93

$$x \arcsin(ax)^3 - 6x \arcsin(ax) + \frac{3\sqrt{-a^2x^2 + 1} \arcsin(ax)^2}{a} - \frac{6\sqrt{-a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3,x, algorithm="giac")

[Out] x\*arcsin(a\*x)^3 - 6\*x\*arcsin(a\*x) + 3\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)^2/a - 6\*sqrt(-a^2\*x^2 + 1)/a

**maple [A]** time = 0.04, size = 57, normalized size = 0.95

$$\frac{ax \arcsin(ax)^3 + 3 \arcsin(ax)^2 \sqrt{-a^2x^2 + 1} - 6\sqrt{-a^2x^2 + 1} - 6ax \arcsin(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^3,x)

[Out] 1/a\*(a\*x\*arcsin(a\*x)^3+3\*arcsin(a\*x)^2\*(-a^2\*x^2+1)^(1/2)-6\*(-a^2\*x^2+1)^(1/2)-6\*a\*x\*arcsin(a\*x))

**maxima [A]** time = 0.54, size = 57, normalized size = 0.95

$$x \arcsin(ax)^3 + \frac{3\sqrt{-a^2x^2 + 1} \arcsin(ax)^2}{a} - \frac{6(ax \arcsin(ax) + \sqrt{-a^2x^2 + 1})}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3,x, algorithm="maxima")

[Out] x\*arcsin(a\*x)^3 + 3\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)^2/a - 6\*(a\*x\*arcsin(a\*x) + sqrt(-a^2\*x^2 + 1))/a

**mupad [B]** time = 0.14, size = 40, normalized size = 0.67

$$\frac{3\sqrt{1-a^2x^2}(\operatorname{asin}(ax)^2-2)}{a} + x \operatorname{asin}(ax)(\operatorname{asin}(ax)^2-6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^3,x)`

[Out]  $(3*(1 - a^2*x^2)^{(1/2)}*(asin(a*x)^2 - 2))/a + x*asin(a*x)*(asin(a*x)^2 - 6)$

sympy [A] time = 0.45, size = 54, normalized size = 0.90

$$\begin{cases} x \operatorname{asin}^3(ax) - 6x \operatorname{asin}(ax) + \frac{3\sqrt{-a^2x^2+1} \operatorname{asin}^2(ax)}{a} - \frac{6\sqrt{-a^2x^2+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)**3,x)`

[Out] `Piecewise((x*asin(a*x)**3 - 6*x*asin(a*x) + 3*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/a - 6*sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))`



$$3.27 \quad \int \frac{\sin^{-1}(ax)^3}{x} dx$$

**Optimal.** Leaf size=97

$$-\frac{3}{2}i \sin^{-1}(ax)^2 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + \frac{3}{2} \sin^{-1}(ax) \text{Li}_3(e^{2i \sin^{-1}(ax)}) + \frac{3}{4}i \text{Li}_4(e^{2i \sin^{-1}(ax)}) - \frac{1}{4}i \sin^{-1}(ax)^4 + \sin^{-1}(ax)^3 \log$$

```
[Out] -1/4*I*arcsin(a*x)^4+arcsin(a*x)^3*ln(1-(I*a*x+(-a^2*x^2+1)^(1/2))^2)-3/2*I
*arcsin(a*x)^2*polylog(2,(I*a*x+(-a^2*x^2+1)^(1/2))^2)+3/2*arcsin(a*x)*poly
log(3,(I*a*x+(-a^2*x^2+1)^(1/2))^2)+3/4*I*polylog(4,(I*a*x+(-a^2*x^2+1)^(1/
2))^2)
```

**Rubi [A]** time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4625, 3717, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3}{2}i \sin^{-1}(ax)^2 \text{PolyLog}(2, e^{2i \sin^{-1}(ax)}) + \frac{3}{2} \sin^{-1}(ax) \text{PolyLog}(3, e^{2i \sin^{-1}(ax)}) + \frac{3}{4}i \text{PolyLog}(4, e^{2i \sin^{-1}(ax)}) - \frac{1}{4}i \sin^{-1}(ax)^4 + \sin^{-1}(ax)^3 \log$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[a*x]^3/x,x]
```

```
[Out] (-I/4)*ArcSin[a*x]^4 + ArcSin[a*x]^3*Log[1 - E^((2*I)*ArcSin[a*x])] - ((3*I
)/2)*ArcSin[a*x]^2*PolyLog[2, E^((2*I)*ArcSin[a*x])] + (3*ArcSin[a*x]*PolyL
og[3, E^((2*I)*ArcSin[a*x])])/2 + ((3*I)/4)*PolyLog[4, E^((2*I)*ArcSin[a*x]
)]
```

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 3717

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

#### Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(ax)^3}{x} dx &= \text{Subst} \left( \int x^3 \cot(x) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{4}i \sin^{-1}(ax)^4 - 2i \text{Subst} \left( \int \frac{e^{2ix} x^3}{1 - e^{2ix}} dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{4}i \sin^{-1}(ax)^4 + \sin^{-1}(ax)^3 \log(1 - e^{2i \sin^{-1}(ax)}) - 3 \text{Subst} \left( \int x^2 \log(1 - e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{4}i \sin^{-1}(ax)^4 + \sin^{-1}(ax)^3 \log(1 - e^{2i \sin^{-1}(ax)}) - \frac{3}{2}i \sin^{-1}(ax)^2 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + 3i \text{Subst} \left( \int x \log(1 - e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{4}i \sin^{-1}(ax)^4 + \sin^{-1}(ax)^3 \log(1 - e^{2i \sin^{-1}(ax)}) - \frac{3}{2}i \sin^{-1}(ax)^2 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + \frac{3}{2} \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) \\ &= -\frac{1}{4}i \sin^{-1}(ax)^4 + \sin^{-1}(ax)^3 \log(1 - e^{2i \sin^{-1}(ax)}) - \frac{3}{2}i \sin^{-1}(ax)^2 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + \frac{3}{2} \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) \\ &= -\frac{1}{4}i \sin^{-1}(ax)^4 + \sin^{-1}(ax)^3 \log(1 - e^{2i \sin^{-1}(ax)}) - \frac{3}{2}i \sin^{-1}(ax)^2 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + \frac{3}{2} \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) \end{aligned}$$

**Mathematica** [A] time = 0.06, size = 97, normalized size = 1.00

$$-\frac{1}{64}i \left( -96 \sin^{-1}(ax)^2 \text{Li}_2(e^{-2i \sin^{-1}(ax)}) + 96i \sin^{-1}(ax) \text{Li}_3(e^{-2i \sin^{-1}(ax)}) + 48 \text{Li}_4(e^{-2i \sin^{-1}(ax)}) - 16 \sin^{-1}(ax)^4 + 64 \sin^{-1}(ax)^3 \log(1 - e^{-2i \sin^{-1}(ax)}) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSin[a*x]^3/x, x]
```

```
[Out] (-1/64*I)*(Pi^4 - 16*ArcSin[a*x]^4 + (64*I)*ArcSin[a*x]^3*Log[1 - E^((-2*I)*ArcSin[a*x])] - 96*ArcSin[a*x]^2*PolyLog[2, E^((-2*I)*ArcSin[a*x])] + (96*I)*ArcSin[a*x]*PolyLog[3, E^((-2*I)*ArcSin[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcSin[a*x])])
```

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^3}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(a*x)^3/x, x, algorithm="fricas")
```

[Out] integral(arcsin(a\*x)^3/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^3/x, x)

**maple** [A] time = 0.06, size = 229, normalized size = 2.36

$$-\frac{i \arcsin(ax)^4}{4} + \arcsin(ax)^3 \ln\left(1 + iax + \sqrt{-a^2x^2 + 1}\right) - 3i \arcsin(ax)^2 \operatorname{polylog}\left(2, -iax - \sqrt{-a^2x^2 + 1}\right) + 6a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^3/x,x)

[Out]  $-1/4*I*\arcsin(a*x)^4 + \arcsin(a*x)^3*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)}) - 3*I*\arcsin(a*x)^2*\operatorname{polylog}(2, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + 6*\arcsin(a*x)*\operatorname{polylog}(3, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + 6*I*\operatorname{polylog}(4, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + \arcsin(a*x)^3*\ln(1-I*a*x - (-a^2*x^2+1)^{(1/2)}) - 3*I*\arcsin(a*x)^2*\operatorname{polylog}(2, I*a*x + (-a^2*x^2+1)^{(1/2)}) + 6*\arcsin(a*x)*\operatorname{polylog}(3, I*a*x + (-a^2*x^2+1)^{(1/2)}) + 6*I*\operatorname{polylog}(4, I*a*x + (-a^2*x^2+1)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x,x, algorithm="maxima")

[Out] integrate(arcsin(a\*x)^3/x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^3/x,x)

[Out] int(asin(a\*x)^3/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^3(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*3/x,x)

[Out] Integral(asin(a\*x)\*\*3/x, x)

### 3.28 $\int \frac{\sin^{-1}(ax)^3}{x^2} dx$

**Optimal.** Leaf size=108

$$6ia \sin^{-1}(ax) \operatorname{Li}_2(-e^{i \sin^{-1}(ax)}) - 6ia \sin^{-1}(ax) \operatorname{Li}_2(e^{i \sin^{-1}(ax)}) - 6a \operatorname{Li}_3(-e^{i \sin^{-1}(ax)}) + 6a \operatorname{Li}_3(e^{i \sin^{-1}(ax)}) - \frac{\sin^{-1}(ax)^3}{x} - 6a$$

[Out]  $-\arcsin(ax)^3/x - 6a \arcsin(ax)^2 \operatorname{arctanh}(I a x + (-a^2 x^2 + 1)^{1/2}) + 6I a \arcsin(ax) \operatorname{polylog}(2, -I a x - (-a^2 x^2 + 1)^{1/2}) - 6I a \arcsin(ax) \operatorname{polylog}(2, I a x + (-a^2 x^2 + 1)^{1/2}) - 6a \operatorname{polylog}(3, -I a x - (-a^2 x^2 + 1)^{1/2}) + 6a \operatorname{polylog}(3, I a x + (-a^2 x^2 + 1)^{1/2})$

**Rubi [A]** time = 0.16, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4627, 4709, 4183, 2531, 2282, 6589}

$$6ia \sin^{-1}(ax) \operatorname{PolyLog}(2, -e^{i \sin^{-1}(ax)}) - 6ia \sin^{-1}(ax) \operatorname{PolyLog}(2, e^{i \sin^{-1}(ax)}) - 6a \operatorname{PolyLog}(3, -e^{i \sin^{-1}(ax)}) + 6a \operatorname{PolyLog}(3, e^{i \sin^{-1}(ax)})$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcSin}[a*x]^3/x^2, x]$

[Out]  $-(\operatorname{ArcSin}[a*x]^3/x) - 6*a*\operatorname{ArcSin}[a*x]^2*\operatorname{ArcTanh}[E^{(I*\operatorname{ArcSin}[a*x])}] + (6*I)*a*\operatorname{ArcSin}[a*x]*\operatorname{PolyLog}[2, -E^{(I*\operatorname{ArcSin}[a*x])}] - (6*I)*a*\operatorname{ArcSin}[a*x]*\operatorname{PolyLog}[2, E^{(I*\operatorname{ArcSin}[a*x])}] - 6*a*\operatorname{PolyLog}[3, -E^{(I*\operatorname{ArcSin}[a*x])}] + 6*a*\operatorname{PolyLog}[3, E^{(I*\operatorname{ArcSin}[a*x])}]$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] := \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$   $\operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /;$   $\operatorname{FreeQ}[\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n] \&\& \operatorname{!MatchQ}[u, E^{((c_)*((a_)+(b_)*x))}*(F_)[v_] /;$   $\operatorname{FreeQ}[\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_)*((F_)^{((c_)*((a_)+(b_)*(x_)))})^{(n_)}]*((f_)+(g_)*(x_))^{(m_)}, x\_Symbol] := -\operatorname{Simp}[\operatorname{((f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n])}/(b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[\operatorname{((g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[\operatorname{((f + g*x)^{(m-1})*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n])}, x], x] /;$   $\operatorname{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0]$

#### Rule 4183

$\operatorname{Int}[\operatorname{csc}[(e_)+(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x\_Symbol] := \operatorname{Simp}[\operatorname{(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{(I*(e + f*x))}]/f, x] + (-\operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1})*\operatorname{Log}[1 - E^{(I*(e + f*x))}], x], x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1})*\operatorname{Log}[1 + E^{(I*(e + f*x))}], x], x]) /;$   $\operatorname{FreeQ}[\{c, d, e, f\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 4627

$\operatorname{Int}[\operatorname{((a_)+(b_)*\operatorname{ArcSin}[c*x])^{(n_)}*((d_)*(x_))^{(m_)}, x\_Symbol] := \operatorname{Simp}[\operatorname{((d*x)^{(m+1})*(a + b*\operatorname{ArcSin}[c*x])^n)/(d*(m+1)), x] - \operatorname{Dist}[\operatorname{((b*c*n)/(d*(m+1)), \operatorname{Int}[\operatorname{((d*x)^{(m+1})*(a + b*\operatorname{ArcSin}[c*x])^{(n-1)})/\operatorname{Sqrt}[1 - c^2*x^2], x], x] /;$   $\operatorname{FreeQ}[\{a, b, c, d, m\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{NeQ}[m, -1]$

#### Rule 4709

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_.)*(x_)^m_)/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(ax)^3}{x^2} dx &= -\frac{\sin^{-1}(ax)^3}{x} + (3a) \int \frac{\sin^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} dx \\ &= -\frac{\sin^{-1}(ax)^3}{x} + (3a) \text{Subst} \left( \int x^2 \csc(x) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{\sin^{-1}(ax)^3}{x} - 6a \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) - (6a) \text{Subst} \left( \int x \log(1 - e^{ix}) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{\sin^{-1}(ax)^3}{x} - 6a \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) + 6ia \sin^{-1}(ax) \text{Li}_2 \left( -e^{i \sin^{-1}(ax)} \right) - 6ia \sin^{-1}(ax) \text{Li}_2 \left( e^{i \sin^{-1}(ax)} \right) \\ &= -\frac{\sin^{-1}(ax)^3}{x} - 6a \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) + 6ia \sin^{-1}(ax) \text{Li}_2 \left( -e^{i \sin^{-1}(ax)} \right) - 6ia \sin^{-1}(ax) \text{Li}_2 \left( e^{i \sin^{-1}(ax)} \right) \\ &= -\frac{\sin^{-1}(ax)^3}{x} - 6a \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) + 6ia \sin^{-1}(ax) \text{Li}_2 \left( -e^{i \sin^{-1}(ax)} \right) - 6ia \sin^{-1}(ax) \text{Li}_2 \left( e^{i \sin^{-1}(ax)} \right) \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 133, normalized size = 1.23

$$a \left( 6i \sin^{-1}(ax) \text{Li}_2 \left( -e^{i \sin^{-1}(ax)} \right) - 6i \sin^{-1}(ax) \text{Li}_2 \left( e^{i \sin^{-1}(ax)} \right) - 6 \text{Li}_3 \left( -e^{i \sin^{-1}(ax)} \right) + 6 \text{Li}_3 \left( e^{i \sin^{-1}(ax)} \right) - \frac{\sin^{-1}(ax)^3}{ax} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSin[a*x]^3/x^2,x]
```

```
[Out] a*(-(ArcSin[a*x]^3/(a*x)) + 3*ArcSin[a*x]^2*Log[1 - E^(I*ArcSin[a*x])] - 3*ArcSin[a*x]^2*Log[1 + E^(I*ArcSin[a*x])] + (6*I)*ArcSin[a*x]*PolyLog[2, -E^(I*ArcSin[a*x])] - (6*I)*ArcSin[a*x]*PolyLog[2, E^(I*ArcSin[a*x])] - 6*PolyLog[3, -E^(I*ArcSin[a*x])] + 6*PolyLog[3, E^(I*ArcSin[a*x])])
```

**fricas [F]** time = 1.28, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^3}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsin(a*x)^3/x^2, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^2,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^3/x^2, x)

**maple** [A] time = 0.11, size = 179, normalized size = 1.66

$$-\frac{\arcsin(ax)^3}{x} - 3a \arcsin(ax)^2 \ln\left(1 + iax + \sqrt{-a^2x^2 + 1}\right) + 6ia \arcsin(ax) \operatorname{polylog}\left(2, -iax - \sqrt{-a^2x^2 + 1}\right) - 6a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^3/x^2,x)

[Out]  $-\arcsin(ax)^3/x - 3a \arcsin(ax)^2 \ln(1 + I*a*x + (-a^2*x^2 + 1)^{(1/2)}) + 6*I*a \arcsin(ax) \operatorname{polylog}(2, -I*a*x - (-a^2*x^2 + 1)^{(1/2)}) - 6*a \operatorname{polylog}(3, -I*a*x - (-a^2*x^2 + 1)^{(1/2)}) + 3*a \arcsin(ax)^2 \ln(1 - I*a*x - (-a^2*x^2 + 1)^{(1/2)}) - 6*I*a \arcsin(ax) \operatorname{polylog}(2, I*a*x + (-a^2*x^2 + 1)^{(1/2)}) + 6*a \operatorname{polylog}(3, I*a*x + (-a^2*x^2 + 1)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^3 + 3ax \int \frac{\sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2}{\sqrt{ax+1} (ax-1)x} dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^2,x, algorithm="maxima")

[Out]  $-(\arctan2(ax, \sqrt{ax+1} \sqrt{-ax+1}))^3 + 3*a*x \operatorname{integrate}(\sqrt{ax+1} \sqrt{-ax+1} \arctan2(ax, \sqrt{ax+1} \sqrt{-ax+1})^2 / (a^2*x^3 - x), x) / x$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^3/x^2,x)

[Out] int(asin(a\*x)^3/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^3(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*3/x\*\*2,x)

[Out] Integral(asin(a\*x)\*\*3/x\*\*2, x)

$$3.29 \quad \int \frac{\sin^{-1}(ax)^3}{x^3} dx$$

**Optimal.** Leaf size=102

$$-\frac{3}{2}ia^2\text{Li}_2\left(e^{2i\sin^{-1}(ax)}\right) - \frac{3a\sqrt{1-a^2x^2}\sin^{-1}(ax)^2}{2x} - \frac{3}{2}ia^2\sin^{-1}(ax)^2 + 3a^2\sin^{-1}(ax)\log\left(1 - e^{2i\sin^{-1}(ax)}\right) - \frac{\sin^{-1}(ax)}{2x^2}$$

[Out]  $-3/2*I*a^2*\arcsin(a*x)^2 - 1/2*\arcsin(a*x)^3/x^2 + 3*a^2*\arcsin(a*x)*\ln(1 - (I*a*x + (-a^2*x^2 + 1)^{(1/2)})^2) - 3/2*I*a^2*\text{polylog}(2, (I*a*x + (-a^2*x^2 + 1)^{(1/2)})^2) - 3/2*a*\arcsin(a*x)^2*(-a^2*x^2 + 1)^{(1/2)}/x$

**Rubi [A]** time = 0.17, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4627, 4681, 4625, 3717, 2190, 2279, 2391}

$$-\frac{3}{2}ia^2\text{PolyLog}\left(2, e^{2i\sin^{-1}(ax)}\right) - \frac{3a\sqrt{1-a^2x^2}\sin^{-1}(ax)^2}{2x} - \frac{3}{2}ia^2\sin^{-1}(ax)^2 + 3a^2\sin^{-1}(ax)\log\left(1 - e^{2i\sin^{-1}(ax)}\right) - \frac{\sin^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^3/x^3, x]

[Out]  $((-3*I)/2)*a^2*\text{ArcSin}[a*x]^2 - (3*a*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^2)/(2*x) - \text{ArcSin}[a*x]^3/(2*x^2) + 3*a^2*\text{ArcSin}[a*x]*\text{Log}[1 - E^((2*I)*\text{ArcSin}[a*x])] - ((3*I)/2)*a^2*\text{PolyLog}[2, E^((2*I)*\text{ArcSin}[a*x])]$

Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 3717

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 4625

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

### Rule 4681

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcSin[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)^3}{x^3} dx &= -\frac{\sin^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a) \int \frac{\sin^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}} dx \\
&= -\frac{3a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{2x^2} + (3a^2) \int \frac{\sin^{-1}(ax)}{x} dx \\
&= -\frac{3a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{2x^2} + (3a^2) \text{Subst}\left(\int x \cot(x) dx, x, \sin^{-1}(ax)\right) \\
&= -\frac{3}{2}ia^2 \sin^{-1}(ax)^2 - \frac{3a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{2x^2} - (6ia^2) \text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}} dx, x, \sin^{-1}(ax)\right) \\
&= -\frac{3}{2}ia^2 \sin^{-1}(ax)^2 - \frac{3a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{2x^2} + 3a^2 \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) \\
&= -\frac{3}{2}ia^2 \sin^{-1}(ax)^2 - \frac{3a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{2x^2} + 3a^2 \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)}) \\
&= -\frac{3}{2}ia^2 \sin^{-1}(ax)^2 - \frac{3a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{2x^2} + 3a^2 \sin^{-1}(ax) \log(1 - e^{2i \sin^{-1}(ax)})
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 92, normalized size = 0.90

$$-\frac{3}{2}ia^2 \text{Li}_2(e^{2i \sin^{-1}(ax)}) - \frac{\sin^{-1}(ax) \left( 3ax \left( \sqrt{1-a^2x^2} + iax \right) \sin^{-1}(ax) - 6a^2x^2 \log(1 - e^{2i \sin^{-1}(ax)}) + \sin^{-1}(ax)^2 \right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[a*x]^3/x^3, x]
```

```
[Out] -1/2*(ArcSin[a*x]*(3*a*x*(I*a*x + Sqrt[1 - a^2*x^2])*ArcSin[a*x] + ArcSin[a
*x]^2 - 6*a^2*x^2*Log[1 - E^((2*I)*ArcSin[a*x])]))/x^2 - ((3*I)/2)*a^2*Poly
Log[2, E^((2*I)*ArcSin[a*x])]
```

**fricas [F]** time = 1.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arcsin(ax)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(a*x)^3/x^3, x, algorithm="fricas")
```



[Out] integral(arcsin(a\*x)^3/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^3,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^3/x^3, x)

**maple** [A] time = 0.14, size = 163, normalized size = 1.60

$$\frac{3ia^2 \arcsin(ax)^2}{2} - \frac{3a \arcsin(ax)^2 \sqrt{-a^2x^2+1}}{2x} - \frac{\arcsin(ax)^3}{2x^2} + 3a^2 \arcsin(ax) \ln\left(1 + iax + \sqrt{-a^2x^2+1}\right) + 3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^3/x^3,x)

[Out] 
$$-3/2*I*a^2*\arcsin(a*x)^2-3/2*a*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}/x-1/2*\arcsin(a*x)^3/x^2+3*a^2*\arcsin(a*x)*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)})+3*a^2*\arcsin(a*x)*\ln(1-I*a*x-(-a^2*x^2+1)^{(1/2)})-3*I*a^2*\text{polylog}(2,I*a*x+(-a^2*x^2+1)^{(1/2)})-3*I*a^2*\text{polylog}(2,-I*a*x-(-a^2*x^2+1)^{(1/2)})$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{3}{4} \left( \sqrt{ax+1} \sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1}) \right)^2 + 4x \int \frac{3 \sqrt{ax+1} \sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2 - 2(a^3x^3 - a^2x^2)}{4(a^2x^4 - x^2)} dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^3,x, algorithm="maxima")

[Out] 
$$-1/2*(6*a*x^2*\integrate(1/2*\sqrt{a*x+1}*\sqrt{-a*x+1}*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^2/(a^2*x^4-x^2), x) + \arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^3)/x^2$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{asin}(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^3/x^3,x)

[Out] int(asin(a\*x)^3/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{asin}^3(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*3/x\*\*3,x)

[Out] Integral(asin(a\*x)\*\*3/x\*\*3, x)

$$3.30 \quad \int \frac{\sin^{-1}(ax)^3}{x^4} dx$$

**Optimal.** Leaf size=179

$$ia^3 \sin^{-1}(ax) \operatorname{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) - ia^3 \sin^{-1}(ax) \operatorname{Li}_2\left(e^{i \sin^{-1}(ax)}\right) - a^3 \operatorname{Li}_3\left(-e^{i \sin^{-1}(ax)}\right) + a^3 \operatorname{Li}_3\left(e^{i \sin^{-1}(ax)}\right) + a^3 \left(-\sin^{-1}(ax)\right)$$

```
[Out] -a^2*arcsin(a*x)/x-1/3*arcsin(a*x)^3/x^3-a^3*arcsin(a*x)^2*arctanh(I*a*x+(-
a^2*x^2+1)^(1/2))-a^3*arctanh((-a^2*x^2+1)^(1/2))+I*a^3*arcsin(a*x)*polylog
(2,-I*a*x-(-a^2*x^2+1)^(1/2))-I*a^3*arcsin(a*x)*polylog(2,I*a*x+(-a^2*x^2+1
)^(1/2))-a^3*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2))+a^3*polylog(3,I*a*x+(-a^2
*x^2+1)^(1/2))-1/2*a*arcsin(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

**Rubi [A]** time = 0.28, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {4627, 4701, 4709, 4183, 2531, 2282, 6589, 266, 63, 208}

$$ia^3 \sin^{-1}(ax) \operatorname{PolyLog}\left(2, -e^{i \sin^{-1}(ax)}\right) - ia^3 \sin^{-1}(ax) \operatorname{PolyLog}\left(2, e^{i \sin^{-1}(ax)}\right) - a^3 \operatorname{PolyLog}\left(3, -e^{i \sin^{-1}(ax)}\right) + a^3 \operatorname{PolyLog}\left(3, e^{i \sin^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[a*x]^3/x^4, x]
```

```
[Out] -((a^2*ArcSin[a*x])/x) - (a*Sqrt[1 - a^2*x^2]*ArcSin[a*x]^2)/(2*x^2) - ArcS
in[a*x]^3/(3*x^3) - a^3*ArcSin[a*x]^2*ArcTanh[E^(I*ArcSin[a*x])] - a^3*ArcT
anh[Sqrt[1 - a^2*x^2]] + I*a^3*ArcSin[a*x]*PolyLog[2, -E^(I*ArcSin[a*x])] -
I*a^3*ArcSin[a*x]*PolyLog[2, E^(I*ArcSin[a*x])] - a^3*PolyLog[3, -E^(I*Arc
Sin[a*x])] + a^3*PolyLog[3, E^(I*ArcSin[a*x])]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-
2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

#### Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

#### Rule 4701

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)
*(x_)^2)^(p_.), x_Symbol] := Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcSin[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Dist[(b*
c*n*d*IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && LtQ[m, -1] && IntegerQ[m]
```

#### Rule 4709

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.))/Sqrt[(d_.) + (e_.)*
(x_)^2], x_Symbol] := Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin
[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d +
e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)^3}{x^4} dx &= -\frac{\sin^{-1}(ax)^3}{3x^3} + a \int \frac{\sin^{-1}(ax)^2}{x^3 \sqrt{1-a^2x^2}} dx \\
&= -\frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x^2} - \frac{\sin^{-1}(ax)^3}{3x^3} + a^2 \int \frac{\sin^{-1}(ax)}{x^2} dx + \frac{1}{2}a^3 \int \frac{\sin^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} dx \\
&= -\frac{a^2 \sin^{-1}(ax)}{x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x^2} - \frac{\sin^{-1}(ax)^3}{3x^3} + \frac{1}{2}a^3 \text{Subst} \left( \int x^2 \csc(x) dx, x, \sin^{-1}(ax) \right) \\
&= -\frac{a^2 \sin^{-1}(ax)}{x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x^2} - \frac{\sin^{-1}(ax)^3}{3x^3} - a^3 \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) + \frac{1}{2}a^3 \text{Li}_3 \left( e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{a^2 \sin^{-1}(ax)}{x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x^2} - \frac{\sin^{-1}(ax)^3}{3x^3} - a^3 \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) + ia^3 \text{Li}_3 \left( e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{a^2 \sin^{-1}(ax)}{x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x^2} - \frac{\sin^{-1}(ax)^3}{3x^3} - a^3 \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) - a^3 \text{Li}_3 \left( e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{a^2 \sin^{-1}(ax)}{x} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x^2} - \frac{\sin^{-1}(ax)^3}{3x^3} - a^3 \sin^{-1}(ax)^2 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) - a^3 \text{Li}_3 \left( e^{i \sin^{-1}(ax)} \right)
\end{aligned}$$

**Mathematica** [A] time = 2.73, size = 284, normalized size = 1.59

$$\frac{1}{48}a^3 \left( -\frac{16 \sin^{-1}(ax)^3 \sin^4 \left( \frac{1}{2} \sin^{-1}(ax) \right)}{a^3 x^3} + 48i \sin^{-1}(ax) \text{Li}_2 \left( -e^{i \sin^{-1}(ax)} \right) - 48i \sin^{-1}(ax) \text{Li}_2 \left( e^{i \sin^{-1}(ax)} \right) - 48 \text{Li}_3 \left( e^{i \sin^{-1}(ax)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^3/x^4,x]

[Out] (a^3\*(-24\*ArcSin[a\*x]\*Cot[ArcSin[a\*x]/2] - 4\*ArcSin[a\*x]^3\*Cot[ArcSin[a\*x]/2] - 6\*ArcSin[a\*x]^2\*Csc[ArcSin[a\*x]/2]^2 - a\*x\*ArcSin[a\*x]^3\*Csc[ArcSin[a\*x]/2]^4 + 24\*ArcSin[a\*x]^2\*Log[1 - E^(I\*ArcSin[a\*x])] - 24\*ArcSin[a\*x]^2\*Log[1 + E^(I\*ArcSin[a\*x])] + 48\*Log[Tan[ArcSin[a\*x]/2]] + (48\*I)\*ArcSin[a\*x]\*PolyLog[2, -E^(I\*ArcSin[a\*x])] - (48\*I)\*ArcSin[a\*x]\*PolyLog[2, E^(I\*ArcSin[a\*x])] - 48\*PolyLog[3, -E^(I\*ArcSin[a\*x])] + 48\*PolyLog[3, E^(I\*ArcSin[a\*x])] + 6\*ArcSin[a\*x]^2\*Sec[ArcSin[a\*x]/2]^2 - (16\*ArcSin[a\*x]^3\*Sin[ArcSin[a\*x]/2]^4)/(a^3\*x^3) - 24\*ArcSin[a\*x]\*Tan[ArcSin[a\*x]/2] - 4\*ArcSin[a\*x]^3\*Tan[ArcSin[a\*x]/2]))/48

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^3}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^4,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^3/x^4, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^4,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^3/x^4, x)

**maple [A]** time = 0.22, size = 250, normalized size = 1.40

$$\frac{a \arcsin(ax)^2 \sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2 \arcsin(ax)}{x} - \frac{\arcsin(ax)^3}{3x^3} - \frac{a^3 \arcsin(ax)^2 \ln\left(1+iax+\sqrt{-a^2x^2+1}\right)}{2} + ia^3 \arcsin(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^3/x^4,x)

[Out]  $-1/2*a*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}/x^2-a^2*\arcsin(a*x)/x-1/3*\arcsin(a*x)^3/x^3-1/2*a^3*\arcsin(a*x)^2*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)})+I*a^3*\arcsin(a*x)*\operatorname{polylog}(2,-I*a*x-(-a^2*x^2+1)^{(1/2)})-a^3*\operatorname{polylog}(3,-I*a*x-(-a^2*x^2+1)^{(1/2)})+1/2*a^3*\arcsin(a*x)^2*\ln(1-I*a*x-(-a^2*x^2+1)^{(1/2)})-I*a^3*\arcsin(a*x)*\operatorname{polylog}(2,I*a*x+(-a^2*x^2+1)^{(1/2)})+a^3*\operatorname{polylog}(3,I*a*x+(-a^2*x^2+1)^{(1/2)})-2*a^3*\operatorname{arctanh}(I*a*x+(-a^2*x^2+1)^{(1/2)})$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{3ax^3 \int \frac{\sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2}{\sqrt{ax+1} (ax-1)x^3} dx + \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^4,x, algorithm="maxima")

[Out]  $-1/3*(3*a*x^3*\int(\sqrt{a*x+1}*\sqrt{-a*x+1}*\arctan2(a*x,\sqrt{a*x+1}*\sqrt{-a*x+1})^2/(a^2*x^5-x^3),x)+\arctan2(a*x,\sqrt{a*x+1}*\sqrt{-a*x+1})^3)/x^3$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^3/x^4,x)

[Out] int(asin(a\*x)^3/x^4, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*3/x\*\*4,x)

[Out] Integral(asin(a\*x)\*\*3/x\*\*4, x)

### 3.31 $\int \frac{\sin^{-1}(ax)^3}{x^5} dx$

**Optimal.** Leaf size=169

$$-\frac{1}{2}ia^4\text{Li}_2\left(e^{2i\sin^{-1}(ax)}\right) - \frac{1}{2}ia^4\sin^{-1}(ax)^2 + a^4\sin^{-1}(ax)\log\left(1 - e^{2i\sin^{-1}(ax)}\right) - \frac{a^2\sin^{-1}(ax)}{4x^2} - \frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)^2}{4x^3}$$

[Out]  $-1/4*a^2*\arcsin(a*x)/x^2 - 1/2*I*a^4*\arcsin(a*x)^2 - 1/4*\arcsin(a*x)^3/x^4 + a^4*\arcsin(a*x)*\ln(1 - (I*a*x + (-a^2*x^2 + 1)^{(1/2)})^2) - 1/2*I*a^4*\text{polylog}(2, (I*a*x + (-a^2*x^2 + 1)^{(1/2)})^2) - 1/4*a^3*(-a^2*x^2 + 1)^{(1/2)}/x - 1/4*a*\arcsin(a*x)^2*(-a^2*x^2 + 1)^{(1/2)}/x^3 - 1/2*a^3*\arcsin(a*x)^2*(-a^2*x^2 + 1)^{(1/2)}/x$

**Rubi [A]** time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$ , Rules used = {4627, 4701, 4681, 4625, 3717, 2190, 2279, 2391, 264}

$$-\frac{1}{2}ia^4\text{PolyLog}\left(2, e^{2i\sin^{-1}(ax)}\right) - \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^3\sqrt{1-a^2x^2}\sin^{-1}(ax)^2}{2x} - \frac{a^2\sin^{-1}(ax)}{4x^2} - \frac{a\sqrt{1-a^2x^2}\sin^{-1}(ax)^2}{4x^3} - \frac{1}{2}i$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^3/x^5, x]

[Out]  $-(a^3*\text{Sqrt}[1 - a^2*x^2])/(4*x) - (a^2*\text{ArcSin}[a*x])/(4*x^2) - (I/2)*a^4*\text{ArcSin}[a*x]^2 - (a*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^2)/(4*x^3) - (a^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^2)/(2*x) - \text{ArcSin}[a*x]^3/(4*x^4) + a^4*\text{ArcSin}[a*x]*\text{Log}[1 - E^((2*I)*\text{ArcSin}[a*x])] - (I/2)*a^4*\text{PolyLog}[2, E^((2*I)*\text{ArcSin}[a*x])]$

#### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp[((c+d\*x)^m\*Log[1+(b\*(F^(g\*(e+f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c+d\*x)^(m-1)\*Log[1+(b\*(F^(g\*(e+f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a+b\*x]/x, x], x, (F^(e\*(c+d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 3717

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c+d\*x)^(m+1))/(d\*(m+1)), x] - Dist[2\*I, Int[((c+d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e+f\*x)))/(1+E^(2\*I\*k\*Pi)\*E^(2\*I\*(e+f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4681

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*f\*(m + 1)), x] - Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(f\*(m + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m + 1)\*(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && EqQ[m + 2\*p + 3, 0] && NeQ[m, -1]

Rule 4701

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*f\*(m + 1)), x] + (Dist[(c^2\*(m + 2\*p + 3))/(f^2\*(m + 1)), Int[(f\*x)^(m + 2)\*(d + e\*x^2)^p\*(a + b\*ArcSin[c\*x])^n, x], x] - Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(f\*(m + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m + 1)\*(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int \frac{\sin^{-1}(ax)^3}{x^5} dx &= -\frac{\sin^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a) \int \frac{\sin^{-1}(ax)^2}{x^4\sqrt{1-a^2x^2}} dx \\
 &= -\frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4x^3} - \frac{\sin^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2 \int \frac{\sin^{-1}(ax)}{x^3} dx + \frac{1}{2}a^3 \int \frac{\sin^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}} dx \\
 &= -\frac{a^2 \sin^{-1}(ax)}{4x^2} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{4x^4} + \frac{1}{4}a^3 \int \frac{\sin^{-1}(ax)}{x} dx \\
 &= -\frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2 \sin^{-1}(ax)}{4x^2} - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} - \frac{\sin^{-1}(ax)^3}{4x^4} \\
 &= -\frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2 \sin^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4 \sin^{-1}(ax)^2 - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} \\
 &= -\frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2 \sin^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4 \sin^{-1}(ax)^2 - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} \\
 &= -\frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2 \sin^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4 \sin^{-1}(ax)^2 - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x} \\
 &= -\frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2 \sin^{-1}(ax)}{4x^2} - \frac{1}{2}ia^4 \sin^{-1}(ax)^2 - \frac{a\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{4x^3} - \frac{a^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^2}{2x}
 \end{aligned}$$

**Mathematica** [A] time = 0.63, size = 116, normalized size = 0.69

$$\frac{1}{4} \left( \frac{\sin^{-1}(ax)^3}{x^4} + a^4 \left( -\frac{\sqrt{1-a^2x^2} \left( \left( \frac{1}{a^2x^2} + 2 \right) \sin^{-1}(ax)^2 + 1 \right)}{ax} - \sin^{-1}(ax) \left( \frac{1}{a^2x^2} + 2i \sin^{-1}(ax) - 4 \log(1 - e^{2i \sin^{-1}(ax)}) \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^3/x^5,x]

[Out]  $(-\text{ArcSin}[a*x]^3/x^4) + a^4 * (-(\text{Sqrt}[1 - a^2*x^2] * (1 + (2 + 1/(a^2*x^2)) * \text{ArcSin}[a*x]^2)) / (a*x)) - \text{ArcSin}[a*x] * (1/(a^2*x^2) + (2*I) * \text{ArcSin}[a*x] - 4 * \text{Log}[1 - E^{((2*I) * \text{ArcSin}[a*x])}]) - (2*I) * \text{PolyLog}[2, E^{((2*I) * \text{ArcSin}[a*x])}]])) / 4$

**fricas** [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^3}{x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^5,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^3/x^5, x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^5,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.75sym2poly/r2sym(const ge n & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [A] time = 0.22, size = 225, normalized size = 1.33

$$-\frac{ia^4 \arcsin(ax)^2}{2} - \frac{a^3 \arcsin(ax)^2 \sqrt{-a^2x^2+1}}{2x} + \frac{ia^4}{4} - \frac{a^3 \sqrt{-a^2x^2+1}}{4x} - \frac{a \arcsin(ax)^2 \sqrt{-a^2x^2+1}}{4x^3} - \frac{a^2 \arcsin(ax)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^3/x^5,x)

[Out]  $-1/2*I*a^4*\arcsin(a*x)^2-1/2*a^3*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}/x+1/4*I*a^4-1/4*a^3*(-a^2*x^2+1)^{(1/2)}/x-1/4*a*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}/x^3-1/4*a^2*\arcsin(a*x)/x^2-1/4*\arcsin(a*x)^3/x^4+a^4*\arcsin(a*x)*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)})+a^4*\arcsin(a*x)*\ln(1-I*a*x-(-a^2*x^2+1)^{(1/2)})-I*a^4*\text{polylog}(2,I*a*x+(-a^2*x^2+1)^{(1/2)})-I*a^4*\text{polylog}(2,-I*a*x-(-a^2*x^2+1)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} \left( (2a^2x^2+1) \sqrt{ax+1} \sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2 + 12x^3 \int \frac{9\sqrt{ax+1} \sqrt{-ax+1} \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}{12} \right)$$

$4x^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^3/x^5,x, algorithm="maxima")



[Out]  $-1/4*(12*a*x^4*\integrate(1/4*\sqrt{a*x + 1}*\sqrt{-a*x + 1}*\arctan2(a*x, \sqrt{a*x + 1}*\sqrt{-a*x + 1}))^2/(a^2*x^6 - x^4), x) + \arctan2(a*x, \sqrt{a*x + 1}*\sqrt{-a*x + 1})^3/x^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^3/x^5,x)`

[Out] `int(asin(a*x)^3/x^5, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^3(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)**3/x**5,x)`

[Out] `Integral(asin(a*x)**3/x**5, x)`

### 3.32 $\int x^5 \sin^{-1}(ax)^4 dx$

**Optimal.** Leaf size=282

$$-\frac{5 \sin^{-1}(ax)^4}{96a^6} + \frac{245 \sin^{-1}(ax)^2}{1152a^6} + \frac{245x^2}{1152a^4} - \frac{5x^2 \sin^{-1}(ax)^2}{16a^4} + \frac{65x^4}{3456a^2} - \frac{5x^4 \sin^{-1}(ax)^2}{48a^2} + \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{9a} - \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{9a}$$

[Out]  $245/1152*x^2/a^4+65/3456*x^4/a^2+1/324*x^6+245/1152*\arcsin(a*x)^2/a^6-5/16*x^2*\arcsin(a*x)^2/a^4-5/48*x^4*\arcsin(a*x)^2/a^2-1/18*x^6*\arcsin(a*x)^2-5/96*\arcsin(a*x)^4/a^6+1/6*x^6*\arcsin(a*x)^4-245/576*x*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a^5-65/864*x^3*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a^3-1/54*x^5*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}/a+5/24*x*\arcsin(a*x)^3*(-a^2*x^2+1)^{(1/2)}/a^5+5/36*x^3*\arcsin(a*x)^3*(-a^2*x^2+1)^{(1/2)}/a^3+1/9*x^5*\arcsin(a*x)^3*(-a^2*x^2+1)^{(1/2)}/a$

**Rubi [A]** time = 0.87, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4627, 4707, 4641, 30}

$$\frac{65x^4}{3456a^2} + \frac{245x^2}{1152a^4} + \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{9a} - \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{9a} - \frac{5x^4 \sin^{-1}(ax)^2}{48a^2} + \frac{5x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{36a^3} - \frac{5x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{36a^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*ArcSin[a\*x]^4,x]

[Out]  $(245*x^2)/(1152*a^4) + (65*x^4)/(3456*a^2) + x^6/324 - (245*x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(576*a^5) - (65*x^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(864*a^3) - (x^5*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(54*a) + (245*\text{ArcSin}[a*x]^2)/(1152*a^6) - (5*x^2*\text{ArcSin}[a*x]^2)/(16*a^4) - (5*x^4*\text{ArcSin}[a*x]^2)/(48*a^2) - (x^6*\text{ArcSin}[a*x]^2)/18 + (5*x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^3)/(24*a^5) + (5*x^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^3)/(36*a^3) + (x^5*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^3)/(9*a) - (5*\text{ArcSin}[a*x]^4)/(96*a^6) + (x^6*\text{ArcSin}[a*x]^4)/6$

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0]

&& GtQ[m, 1] && IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int x^5 \sin^{-1}(ax)^4 dx &= \frac{1}{6}x^6 \sin^{-1}(ax)^4 - \frac{1}{3}(2a) \int \frac{x^6 \sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx \\
 &= \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{9a} + \frac{1}{6}x^6 \sin^{-1}(ax)^4 - \frac{1}{3} \int x^5 \sin^{-1}(ax)^2 dx - \frac{5 \int \frac{x^4 \sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx}{9a} \\
 &= -\frac{1}{18}x^6 \sin^{-1}(ax)^2 + \frac{5x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{36a^3} + \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{9a} + \frac{1}{6}x^6 \sin^{-1}(ax)^4 \\
 &= -\frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{54a} - \frac{5x^4 \sin^{-1}(ax)^2}{48a^2} - \frac{1}{18}x^6 \sin^{-1}(ax)^2 + \frac{5x \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{24a^5} \\
 &= \frac{x^6}{324} - \frac{65x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{864a^3} - \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{54a} - \frac{5x^2 \sin^{-1}(ax)^2}{16a^4} - \frac{5x^4 \sin^{-1}(ax)^4}{48a^2} \\
 &= \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{576a^5} - \frac{65x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{864a^3} - \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{54a} \\
 &= \frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{576a^5} - \frac{65x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{864a^3} - \frac{x^5 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{54a}
 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 167, normalized size = 0.59

$$\frac{108(16a^6x^6 - 5) \sin^{-1}(ax)^4 + a^2x^2(32a^4x^4 + 195a^2x^2 + 2205) + 144ax\sqrt{1-a^2x^2}(8a^4x^4 + 10a^2x^2 + 15) \sin^{-1}(ax) + 10368a^6 \sin^{-1}(ax)^6}{10368a^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*ArcSin[a\*x]^4,x]

[Out] (a^2\*x^2\*(2205 + 195\*a^2\*x^2 + 32\*a^4\*x^4) - 6\*a\*x\*Sqrt[1 - a^2\*x^2]\*(735 + 130\*a^2\*x^2 + 32\*a^4\*x^4)\*ArcSin[a\*x] - 9\*(-245 + 360\*a^2\*x^2 + 120\*a^4\*x^4 + 64\*a^6\*x^6)\*ArcSin[a\*x]^2 + 144\*a\*x\*Sqrt[1 - a^2\*x^2]\*(15 + 10\*a^2\*x^2 + 8\*a^4\*x^4)\*ArcSin[a\*x]^3 + 108\*(-5 + 16\*a^6\*x^6)\*ArcSin[a\*x]^4)/(10368\*a^6)

**fricas [A]** time = 0.47, size = 153, normalized size = 0.54

$$\frac{32a^6x^6 + 195a^4x^4 + 108(16a^6x^6 - 5) \arcsin(ax)^4 + 2205a^2x^2 - 9(64a^6x^6 + 120a^4x^4 + 360a^2x^2 - 245) \arcsin(ax) + 10368a^6 \arcsin(ax)^6}{10368a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arcsin(a\*x)^4,x, algorithm="fricas")

[Out] 1/10368\*(32\*a^6\*x^6 + 195\*a^4\*x^4 + 108\*(16\*a^6\*x^6 - 5)\*arcsin(a\*x)^4 + 2205\*a^2\*x^2 - 9\*(64\*a^6\*x^6 + 120\*a^4\*x^4 + 360\*a^2\*x^2 - 245)\*arcsin(a\*x)^2 + 6\*sqrt(-a^2\*x^2 + 1)\*(24\*(8\*a^5\*x^5 + 10\*a^3\*x^3 + 15\*a\*x)\*arcsin(a\*x)^3 - (32\*a^5\*x^5 + 130\*a^3\*x^3 + 735\*a\*x)\*arcsin(a\*x)))/a^6

**giac [A]** time = 0.15, size = 362, normalized size = 1.28

$$\frac{(a^2x^2 - 1)^2 \sqrt{-a^2x^2 + 1} x \arcsin(ax)^3}{9a^5} + \frac{(a^2x^2 - 1)^3 \arcsin(ax)^4}{6a^6} - \frac{13(-a^2x^2 + 1)^{\frac{3}{2}} x \arcsin(ax)^3}{36a^5} + \frac{(a^2x^2 - 1)^2 a^2 x^5 \arcsin(ax)^6}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsin(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/9*(a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)^3/a^5 + 1/6*(a^2*x^2 - 1)^3*arcsin(a*x)^4/a^6 - 13/36*(-a^2*x^2 + 1)^(3/2)*x*arcsin(a*x)^3/a^5 + 1/2*(a^2*x^2 - 1)^2*arcsin(a*x)^4/a^6 - 1/54*(a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)/a^5 + 11/24*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)^3/a^5 - 1/18*(a^2*x^2 - 1)^3*arcsin(a*x)^2/a^6 + 1/2*(a^2*x^2 - 1)*arcsin(a*x)^4/a^6 + 97/864*(-a^2*x^2 + 1)^(3/2)*x*arcsin(a*x)/a^5 - 13/48*(a^2*x^2 - 1)^2*arcsin(a*x)^2/a^6 + 11/96*arcsin(a*x)^4/a^6 - 299/576*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)/a^5 + 1/324*(a^2*x^2 - 1)^3/a^6 - 11/16*(a^2*x^2 - 1)*arcsin(a*x)^2/a^6 + 97/3456*(a^2*x^2 - 1)^2/a^6 - 299/1152*arcsin(a*x)^2/a^6 + 299/1152*(a^2*x^2 - 1)/a^6 + 9971/82944/a^6
```

```
maple [A] time = 0.18, size = 320, normalized size = 1.13
```

$$\frac{a^6 x^6 \arcsin(ax)^4}{6} - \frac{\arcsin(ax)^3 \left( -8\sqrt{-a^2x^2+1} a^5x^5 - 10a^3x^3\sqrt{-a^2x^2+1} - 15ax\sqrt{-a^2x^2+1} + 15\arcsin(ax) \right)}{72} - \frac{\arcsin(ax)^2 a^6 x^6}{18} + \frac{\arcsin(ax) \left( -8\sqrt{-a^2x^2+1} \right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arcsin(a*x)^4,x)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arcsin(a*x)^4-1/72*arcsin(a*x)^3*(-8*(-a^2*x^2+1)^(1/2)*a^5*x^5-10*a^3*x^3*(-a^2*x^2+1)^(1/2)-15*a*x*(-a^2*x^2+1)^(1/2)+15*arcsin(a*x))-1/18*arcsin(a*x)^2*a^6*x^6+1/432*arcsin(a*x)*(-8*(-a^2*x^2+1)^(1/2)*a^5*x^5-10*a^3*x^3*(-a^2*x^2+1)^(1/2)-15*a*x*(-a^2*x^2+1)^(1/2)+15*arcsin(a*x))+115/1152*arcsin(a*x)^2+1/324*a^6*x^6+65/3456*a^4*x^4+245/1152*a^2*x^2-5/48*a^4*x^4*arcsin(a*x)^2+5/192*arcsin(a*x)*(-2*a^3*x^3*(-a^2*x^2+1)^(1/2)-3*a*x*(-a^2*x^2+1)^(1/2)+3*arcsin(a*x))-5/16*(a^2*x^2-1)*arcsin(a*x)^2-5/16*arcsin(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+arcsin(a*x))+5/32*arcsin(a*x)^4)
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\frac{1}{6} x^6 \arctan \left( ax, \sqrt{ax+1} \sqrt{-ax+1} \right)^4 + 2a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^6 \arctan \left( ax, \sqrt{ax+1} \sqrt{-ax+1} \right)^3}{3(a^2x^2-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsin(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^4 + 2*a*integrate(1/3*sqrt(a*x + 1)*sqrt(-a*x + 1)*x^6*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^3/(a^2*x^2 - 1), x)
```

```
mupad [F] time = 0.00, size = -1, normalized size = -0.00
```

$$\int x^5 \operatorname{asin}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*asin(a*x)^4,x)
```

```
[Out] int(x^5*asin(a*x)^4, x)
```

```
sympy [A] time = 15.75, size = 269, normalized size = 0.95
```

$$\begin{cases} \frac{x^6 \operatorname{asin}^4(ax)}{6} - \frac{x^6 \operatorname{asin}^2(ax)}{18} + \frac{x^6}{324} + \frac{x^5 \sqrt{-a^2x^2+1} \operatorname{asin}^3(ax)}{9a} - \frac{x^5 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{54a} - \frac{5x^4 \operatorname{asin}^2(ax)}{48a^2} + \frac{65x^4}{3456a^2} + \frac{5x^3 \sqrt{-a^2x^2+1} \operatorname{asin}^3(ax)}{36a^3} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*asin(a\*x)\*\*4,x)

[Out] Piecewise((x\*\*6\*asin(a\*x)\*\*4/6 - x\*\*6\*asin(a\*x)\*\*2/18 + x\*\*6/324 + x\*\*5\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)\*\*3/(9\*a) - x\*\*5\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)/(54\*a) - 5\*x\*\*4\*asin(a\*x)\*\*2/(48\*a\*\*2) + 65\*x\*\*4/(3456\*a\*\*2) + 5\*x\*\*3\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)\*\*3/(36\*a\*\*3) - 65\*x\*\*3\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)/(864\*a\*\*3) - 5\*x\*\*2\*asin(a\*x)\*\*2/(16\*a\*\*4) + 245\*x\*\*2/(1152\*a\*\*4) + 5\*x\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)\*\*3/(24\*a\*\*5) - 245\*x\*sqrt(-a\*\*2\*x\*\*2 + 1)\*asin(a\*x)/(576\*a\*\*5) - 5\*asin(a\*x)\*\*4/(96\*a\*\*6) + 245\*asin(a\*x)\*\*2/(1152\*a\*\*6), Ne(a, 0)), (0, True))

### 3.33 $\int x^4 \sin^{-1}(ax)^4 dx$

**Optimal.** Leaf size=250

$$\frac{16576x}{5625a^4} - \frac{32x \sin^{-1}(ax)^2}{25a^4} + \frac{1088x^3}{16875a^2} - \frac{16x^3 \sin^{-1}(ax)^2}{75a^2} + \frac{4x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{25a} - \frac{24x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{625a} + \frac{32\sqrt{1-a^2x^2} \sin^{-1}(ax)^5}{625a^3}$$

[Out] 16576/5625\*x/a^4+1088/16875\*x^3/a^2+24/3125\*x^5-32/25\*x\*arcsin(a\*x)^2/a^4-16/75\*x^3\*arcsin(a\*x)^2/a^2-12/125\*x^5\*arcsin(a\*x)^2+1/5\*x^5\*arcsin(a\*x)^4-16576/5625\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a^5-1088/5625\*x^2\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a^3-24/625\*x^4\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a+32/75\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a^5+16/75\*x^2\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a^3+4/25\*x^4\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a

**Rubi [A]** time = 0.66, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4627, 4707, 4677, 4619, 8, 30}

$$\frac{1088x^3}{16875a^2} + \frac{4x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{25a} - \frac{24x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{625a} - \frac{16x^3 \sin^{-1}(ax)^2}{75a^2} + \frac{16x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{75a^3} - \frac{10\sqrt{1-a^2x^2} \sin^{-1}(ax)^5}{625a^3}$$

Antiderivative was successfully verified.

[In] Int[x^4\*ArcSin[a\*x]^4,x]

[Out] (16576\*x)/(5625\*a^4) + (1088\*x^3)/(16875\*a^2) + (24\*x^5)/3125 - (16576\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/(5625\*a^5) - (1088\*x^2\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/(5625\*a^3) - (24\*x^4\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/(625\*a) - (32\*x\*ArcSin[a\*x]^2)/(25\*a^4) - (16\*x^3\*ArcSin[a\*x]^2)/(75\*a^2) - (12\*x^5\*ArcSin[a\*x]^2)/125 + (32\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/(75\*a^5) + (16\*x^2\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/(75\*a^3) + (4\*x^4\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/(25\*a) + (x^5\*ArcSin[a\*x]^4)/5

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4619

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_), x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x]))^(n-1))/sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x]))^(n-1))/sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4677

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p+1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p+1)), x] + Dist[(b\*n\*d\*IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p+1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p+1/2)\*(a + b\*ArcSin[c\*x])^n]

- 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

### Rule 4707

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\_.)\*((f\_.)\*(x\_.))^m\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[((f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n)/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int x^4 \sin^{-1}(ax)^4 dx &= \frac{1}{5}x^5 \sin^{-1}(ax)^4 - \frac{1}{5}(4a) \int \frac{x^5 \sin^{-1}(ax)^3}{\sqrt{1 - a^2x^2}} dx \\
 &= \frac{4x^4 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{25a} + \frac{1}{5}x^5 \sin^{-1}(ax)^4 - \frac{12}{25} \int x^4 \sin^{-1}(ax)^2 dx - \frac{16 \int \frac{x^3 \sin^{-1}(ax)^3}{\sqrt{1 - a^2x^2}} dx}{25a} \\
 &= -\frac{12}{125}x^5 \sin^{-1}(ax)^2 + \frac{16x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{75a^3} + \frac{4x^4 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{25a} + \frac{1}{5}x^5 \sin^{-1}(ax)^4 \\
 &= -\frac{24x^4 \sqrt{1 - a^2x^2} \sin^{-1}(ax)}{625a} - \frac{16x^3 \sin^{-1}(ax)^2}{75a^2} - \frac{12}{125}x^5 \sin^{-1}(ax)^2 + \frac{32 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{75a^5} \\
 &= \frac{24x^5}{3125} - \frac{1088x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)}{5625a^3} - \frac{24x^4 \sqrt{1 - a^2x^2} \sin^{-1}(ax)}{625a} - \frac{32x \sin^{-1}(ax)^2}{25a^4} - \frac{16x^5 \sin^{-1}(ax)^4}{84375a^5} \\
 &= \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576 \sqrt{1 - a^2x^2} \sin^{-1}(ax)}{5625a^5} - \frac{1088x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)}{5625a^3} - \frac{24x^4 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{84375a^5} \\
 &= \frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576 \sqrt{1 - a^2x^2} \sin^{-1}(ax)}{5625a^5} - \frac{1088x^2 \sqrt{1 - a^2x^2} \sin^{-1}(ax)}{5625a^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 150, normalized size = 0.60

$$\frac{16875a^5x^5 \sin^{-1}(ax)^4 + 8ax(81a^4x^4 + 680a^2x^2 + 31080) - 900ax(9a^4x^4 + 20a^2x^2 + 120) \sin^{-1}(ax)^2 + 4500x^5 \sin^{-1}(ax)^4}{84375a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcSin[a\*x]^4,x]

[Out] (8\*a\*x\*(31080 + 680\*a^2\*x^2 + 81\*a^4\*x^4) - 120\*Sqrt[1 - a^2\*x^2]\*(2072 + 136\*a^2\*x^2 + 27\*a^4\*x^4)\*ArcSin[a\*x] - 900\*a\*x\*(120 + 20\*a^2\*x^2 + 9\*a^4\*x^4)\*ArcSin[a\*x]^2 + 4500\*Sqrt[1 - a^2\*x^2]\*(8 + 4\*a^2\*x^2 + 3\*a^4\*x^4)\*ArcSin[a\*x]^3 + 16875\*a^5\*x^5\*ArcSin[a\*x]^4)/(84375\*a^5)

**fricas [A]** time = 0.42, size = 134, normalized size = 0.54

$$\frac{16875 a^5 x^5 \arcsin(ax)^4 + 648 a^5 x^5 + 5440 a^3 x^3 - 900 (9 a^5 x^5 + 20 a^3 x^3 + 120 ax) \arcsin(ax)^2 + 248640 ax + 4500 x^5 \arcsin(ax)^4}{84375 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^4,x, algorithm="fricas")

[Out]  $\frac{1}{84375} (16875 a^5 x^5 \arcsin(ax)^4 + 648 a^5 x^5 + 5440 a^3 x^3 - 900 (9 a^5 x^5 + 20 a^3 x^3 + 120 a x) \arcsin(ax)^2 + 248640 a x + 60 \sqrt{-a^2 x^2 + 1}) (75 (3 a^4 x^4 + 4 a^2 x^2 + 8) \arcsin(ax)^3 - 2 (27 a^4 x^4 + 136 a^2 x^2 + 2072) \arcsin(ax)) / a^5$

**giac** [A] time = 0.15, size = 305, normalized size = 1.22

$$\frac{(a^2 x^2 - 1)^2 x \arcsin(ax)^4}{5 a^4} + \frac{2(a^2 x^2 - 1) x \arcsin(ax)^4}{5 a^4} - \frac{12(a^2 x^2 - 1)^2 x \arcsin(ax)^2}{125 a^4} + \frac{x \arcsin(ax)^4}{5 a^4} + \frac{4(a^2 x^2 - 1)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^4,x, algorithm="giac")

[Out]  $\frac{1}{5} (a^2 x^2 - 1)^2 x \arcsin(ax)^4 / a^4 + \frac{2}{5} (a^2 x^2 - 1) x \arcsin(ax)^4 / a^4 - \frac{12}{125} (a^2 x^2 - 1)^2 x \arcsin(ax)^2 / a^4 + \frac{1}{5} x \arcsin(ax)^4 / a^4 + \frac{4}{25} (a^2 x^2 - 1)^2 \sqrt{-a^2 x^2 + 1} \arcsin(ax)^3 / a^5 - \frac{152}{375} (a^2 x^2 - 1) x \arcsin(ax)^2 / a^4 - \frac{8}{15} (-a^2 x^2 + 1)^{(3/2)} \arcsin(ax)^3 / a^5 + \frac{24}{3125} (a^2 x^2 - 1)^2 x / a^4 - \frac{596}{375} x \arcsin(ax)^2 / a^4 - \frac{24}{625} (a^2 x^2 - 1)^2 \sqrt{-a^2 x^2 + 1} \arcsin(ax) / a^5 + \frac{4}{5} \sqrt{-a^2 x^2 + 1} \arcsin(ax)^3 / a^5 + \frac{6736}{84375} (a^2 x^2 - 1) x / a^4 + \frac{304}{1125} (-a^2 x^2 + 1)^{(3/2)} \arcsin(ax) / a^5 + \frac{254728}{84375} x / a^4 - \frac{1192}{375} \sqrt{-a^2 x^2 + 1} \arcsin(ax) / a^5$

**maple** [A] time = 0.08, size = 197, normalized size = 0.79

$$\frac{a^5 x^5 \arcsin(ax)^4}{5} + \frac{4 \arcsin(ax)^3 (3a^4 x^4 + 4a^2 x^2 + 8) \sqrt{-a^2 x^2 + 1}}{75} - \frac{32 a x \arcsin(ax)^2}{25} + \frac{16576 a x}{5625} - \frac{64 \arcsin(ax) \sqrt{-a^2 x^2 + 1}}{25} - \frac{12 a^5 x^5 \arcsin(ax)^2}{125 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arcsin(a\*x)^4,x)

[Out]  $\frac{1}{a^5} ( \frac{1}{5} a^5 x^5 \arcsin(ax)^4 + \frac{4}{75} \arcsin(ax)^3 (3a^4 x^4 + 4a^2 x^2 + 8) (-a^2 x^2 + 1)^{(1/2)} - \frac{32}{25} a x \arcsin(ax)^2 + \frac{16576}{5625} a x - \frac{64}{25} \arcsin(ax) (-a^2 x^2 + 1)^{(1/2)} - \frac{12}{125} a^5 x^5 \arcsin(ax)^2 - \frac{8}{625} \arcsin(ax) (3a^4 x^4 + 4a^2 x^2 + 8) (-a^2 x^2 + 1)^{(1/2)} + \frac{24}{3125} a^5 x^5 + \frac{1088}{16875} a^3 x^3 - \frac{16}{75} a^3 x^3 \arcsin(ax)^2 - \frac{32}{225} \arcsin(ax) (a^2 x^2 + 2) (-a^2 x^2 + 1)^{(1/2)} )$

**maxima** [A] time = 0.46, size = 207, normalized size = 0.83

$$\frac{1}{5} x^5 \arcsin(ax)^4 + \frac{4}{75} \left( \frac{3 \sqrt{-a^2 x^2 + 1} x^4}{a^2} + \frac{4 \sqrt{-a^2 x^2 + 1} x^2}{a^4} + \frac{8 \sqrt{-a^2 x^2 + 1}}{a^6} \right) a \arcsin(ax)^3 - \frac{4}{84375} \left( 2 a \left( \frac{15 (27 \sqrt{-a^2 x^2 + 1} x^4 + 136 \sqrt{-a^2 x^2 + 1} x^2 + 2072 \sqrt{-a^2 x^2 + 1})}{a^2} \arcsin(ax) / a^5 - (81 a^4 x^5 + 680 a^2 x^3 + 31080 x) / a^6 + 225 (9 a^4 x^5 + 20 a^2 x^3 + 120 x) \arcsin(ax)^2 / a^5 \right) a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^4,x, algorithm="maxima")

[Out]  $\frac{1}{5} x^5 \arcsin(ax)^4 + \frac{4}{75} (3 \sqrt{-a^2 x^2 + 1} x^4 / a^2 + 4 \sqrt{-a^2 x^2 + 1} x^2 / a^4 + 8 \sqrt{-a^2 x^2 + 1} / a^6) a \arcsin(ax)^3 - \frac{4}{84375} (2 a (15 (27 \sqrt{-a^2 x^2 + 1} x^4 + 136 \sqrt{-a^2 x^2 + 1} x^2 + 2072 \sqrt{-a^2 x^2 + 1}) / a^2) \arcsin(ax) / a^5 - (81 a^4 x^5 + 680 a^2 x^3 + 31080 x) / a^6 + 225 (9 a^4 x^5 + 20 a^2 x^3 + 120 x) \arcsin(ax)^2 / a^5) a$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{asin}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(x^4*asin(a*x)^4,x)
```

```
[Out] int(x^4*asin(a*x)^4, x)
```

**sympy [A]** time = 9.51, size = 241, normalized size = 0.96

$$\left\{ \begin{array}{l} \frac{x^5 \operatorname{asin}^4(ax)}{5} - \frac{12x^5 \operatorname{asin}^2(ax)}{125} + \frac{24x^5}{3125} + \frac{4x^4 \sqrt{-a^2x^2+1} \operatorname{asin}^3(ax)}{25a} - \frac{24x^4 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{625a} - \frac{16x^3 \operatorname{asin}^2(ax)}{75a^2} + \frac{1088x^3}{16875a^2} + \frac{16x^2 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{16875a^2} \\ 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asin(a*x)**4,x)
```

```
[Out] Piecewise((x**5*asin(a*x)**4/5 - 12*x**5*asin(a*x)**2/125 + 24*x**5/3125 + 4*x**4*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(25*a) - 24*x**4*sqrt(-a**2*x**2 + 1)*asin(a*x)/(625*a) - 16*x**3*asin(a*x)**2/(75*a**2) + 1088*x**3/(16875*a**2) + 16*x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(75*a**3) - 1088*x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)/(5625*a**3) - 32*x*asin(a*x)**2/(25*a**4) + 16576*x/(5625*a**4) + 32*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(75*a**5) - 16576*sqrt(-a**2*x**2 + 1)*asin(a*x)/(5625*a**5), Ne(a, 0)), (0, True))
```

### 3.34 $\int x^3 \sin^{-1}(ax)^4 dx$

**Optimal.** Leaf size=198

$$-\frac{3 \sin^{-1}(ax)^4}{32a^4} + \frac{45 \sin^{-1}(ax)^2}{128a^4} + \frac{45x^2}{128a^2} - \frac{9x^2 \sin^{-1}(ax)^2}{16a^2} + \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{4a} - \frac{3x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{32a} + \frac{3x \sqrt{1-a^2x^2}}{6}$$

[Out] 45/128\*x^2/a^2+3/128\*x^4+45/128\*arcsin(a\*x)^2/a^4-9/16\*x^2\*arcsin(a\*x)^2/a^2-3/16\*x^4\*arcsin(a\*x)^2-3/32\*arcsin(a\*x)^4/a^4+1/4\*x^4\*arcsin(a\*x)^4-45/64\*x\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a^3-3/32\*x^3\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a+3/8\*x\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a^3+1/4\*x^3\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a

**Rubi [A]** time = 0.52, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4627, 4707, 4641, 30}

$$\frac{45x^2}{128a^2} + \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{4a} - \frac{3x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{32a} - \frac{9x^2 \sin^{-1}(ax)^2}{16a^2} + \frac{3x \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{8a^3} - \frac{45x \sqrt{1-a^2x^2}}{6}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcSin[a\*x]^4,x]

[Out] (45\*x^2)/(128\*a^2) + (3\*x^4)/128 - (45\*x\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/(64\*a^3) - (3\*x^3\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/(32\*a) + (45\*ArcSin[a\*x]^2)/(128\*a^4) - (9\*x^2\*ArcSin[a\*x]^2)/(16\*a^2) - (3\*x^4\*ArcSin[a\*x]^2)/16 + (3\*x\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/(8\*a^3) + (x^3\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/(4\*a) - (3\*ArcSin[a\*x]^4)/(32\*a^4) + (x^4\*ArcSin[a\*x]^4)/4

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_)^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int x^3 \sin^{-1}(ax)^4 dx &= \frac{1}{4}x^4 \sin^{-1}(ax)^4 - a \int \frac{x^4 \sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx \\
&= \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{4a} + \frac{1}{4}x^4 \sin^{-1}(ax)^4 - \frac{3}{4} \int x^3 \sin^{-1}(ax)^2 dx - \frac{3 \int \frac{x^2 \sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx}{4a} \\
&= -\frac{3}{16}x^4 \sin^{-1}(ax)^2 + \frac{3x \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{8a^3} + \frac{x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{4a} + \frac{1}{4}x^4 \sin^{-1}(ax)^4 \\
&= -\frac{3x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{32a} - \frac{9x^2 \sin^{-1}(ax)^2}{16a^2} - \frac{3}{16}x^4 \sin^{-1}(ax)^2 + \frac{3x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{8a^3} \\
&= \frac{3x^4}{128} - \frac{45x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{64a^3} - \frac{3x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{32a} - \frac{9x^2 \sin^{-1}(ax)^2}{16a^2} - \frac{3}{16}x^4 \sin^{-1}(ax)^2 \\
&= \frac{45x^2}{128a^2} + \frac{3x^4}{128} - \frac{45x \sqrt{1-a^2x^2} \sin^{-1}(ax)}{64a^3} - \frac{3x^3 \sqrt{1-a^2x^2} \sin^{-1}(ax)}{32a} + \frac{45 \sin^{-1}(ax)^2}{128a^4} - \frac{3}{16}x^4 \sin^{-1}(ax)^2
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 135, normalized size = 0.68

$$\frac{4(8a^4x^4 - 3) \sin^{-1}(ax)^4 + 3a^2x^2(a^2x^2 + 15) + 16ax\sqrt{1-a^2x^2}(2a^2x^2 + 3) \sin^{-1}(ax)^3 - 6ax\sqrt{1-a^2x^2}(2a^2x^2 + 3) \sin^{-1}(ax)^2 + 4a^2x^2 \sin^{-1}(ax)}{128a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcSin[a\*x]^4,x]

[Out] (3\*a^2\*x^2\*(15 + a^2\*x^2) - 6\*a\*x\*Sqrt[1 - a^2\*x^2]\*(15 + 2\*a^2\*x^2)\*ArcSin[a\*x] - 3\*(-15 + 24\*a^2\*x^2 + 8\*a^4\*x^4)\*ArcSin[a\*x]^2 + 16\*a\*x\*Sqrt[1 - a^2\*x^2]\*(3 + 2\*a^2\*x^2)\*ArcSin[a\*x]^3 + 4\*(-3 + 8\*a^4\*x^4)\*ArcSin[a\*x]^4)/(128\*a^4)

**fricas [A]** time = 1.29, size = 121, normalized size = 0.61

$$\frac{3a^4x^4 + 4(8a^4x^4 - 3) \arcsin(ax)^4 + 45a^2x^2 - 3(8a^4x^4 + 24a^2x^2 - 15) \arcsin(ax)^2 + 2\sqrt{-a^2x^2 + 1}(8(2a^3x^3 + 3ax) \arcsin(ax)^3 - 3(2a^3x^3 + 15ax) \arcsin(ax))}{128a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^4,x, algorithm="fricas")

[Out] 1/128\*(3\*a^4\*x^4 + 4\*(8\*a^4\*x^4 - 3)\*arcsin(a\*x)^4 + 45\*a^2\*x^2 - 3\*(8\*a^4\*x^4 + 24\*a^2\*x^2 - 15)\*arcsin(a\*x)^2 + 2\*sqrt(-a^2\*x^2 + 1)\*(8\*(2\*a^3\*x^3 + 3\*a\*x)\*arcsin(a\*x)^3 - 3\*(2\*a^3\*x^3 + 15\*a\*x)\*arcsin(a\*x)))/a^4

**giac [A]** time = 0.16, size = 234, normalized size = 1.18

$$-\frac{(-a^2x^2 + 1)^{\frac{3}{2}}x \arcsin(ax)^3}{4a^3} + \frac{(a^2x^2 - 1)^2 \arcsin(ax)^4}{4a^4} + \frac{5\sqrt{-a^2x^2 + 1}x \arcsin(ax)^3}{8a^3} + \frac{(a^2x^2 - 1) \arcsin(ax)^4}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^4,x, algorithm="giac")

[Out] -1/4\*(-a^2\*x^2 + 1)^(3/2)\*x\*arcsin(a\*x)^3/a^3 + 1/4\*(a^2\*x^2 - 1)^2\*arcsin(a\*x)^4/a^4 + 5/8\*sqrt(-a^2\*x^2 + 1)\*x\*arcsin(a\*x)^3/a^3 + 1/2\*(a^2\*x^2 - 1)\*arcsin(a\*x)^4/a^4 + 3/32\*(-a^2\*x^2 + 1)^(3/2)\*x\*arcsin(a\*x)/a^3 - 3/16\*(a^2\*x^2 - 1)^2\*arcsin(a\*x)^2/a^4 + 5/32\*arcsin(a\*x)^4/a^4 - 51/64\*sqrt(-a^2\*x^2 + 1)\*x\*arcsin(a\*x)^3/a^3

$\sqrt{2+1} \cdot x \cdot \arcsin(ax) / a^3 - 15/16 \cdot (a^2 x^2 - 1) \cdot \arcsin(ax)^2 / a^4 + 3/128 \cdot (a^2 x^2 - 1)^2 / a^4 - 51/128 \cdot \arcsin(ax)^2 / a^4 + 51/128 \cdot (a^2 x^2 - 1) / a^4 + 195/1024 / a^4$

**maple** [A] time = 0.09, size = 209, normalized size = 1.06

$$\frac{a^4 x^4 \arcsin(ax)^4}{4} - \frac{\arcsin(ax)^3 \left( -2a^3 x^3 \sqrt{-a^2 x^2 + 1} - 3ax \sqrt{-a^2 x^2 + 1} + 3 \arcsin(ax) \right)}{8} - \frac{3a^4 x^4 \arcsin(ax)^2}{16} + \frac{3 \arcsin(ax) \left( -2a^3 x^3 \sqrt{-a^2 x^2 + 1} - 3ax \sqrt{-a^2 x^2 + 1} + 3 \arcsin(ax) \right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arcsin(a*x)^4,x)`

[Out]  $1/a^4 * (1/4 * a^4 * x^4 * \arcsin(ax)^4 - 1/8 * \arcsin(ax)^3 * (-2 * a^3 * x^3 * (-a^2 * x^2 + 1)^{1/2} - 3 * a * x * (-a^2 * x^2 + 1)^{1/2} + 3 * \arcsin(ax)) - 3/16 * a^4 * x^4 * \arcsin(ax)^2 + 3/64 * \arcsin(ax) * (-2 * a^3 * x^3 * (-a^2 * x^2 + 1)^{1/2} - 3 * a * x * (-a^2 * x^2 + 1)^{1/2} + 3 * \arcsin(ax)) + 27/128 * \arcsin(ax)^2 + 3/128 * a^4 * x^4 + 45/128 * a^2 * x^2 - 9/16 * (a^2 * x^2 - 1) * \arcsin(ax)^2 - 9/16 * \arcsin(ax) * (a * x * (-a^2 * x^2 + 1)^{1/2} + \arcsin(ax)) + 9/32 * 2 * \arcsin(ax)^4$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} x^4 \arctan \left( ax, \sqrt{ax+1} \sqrt{-ax+1} \right)^4 + a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^4 \arctan \left( ax, \sqrt{ax+1} \sqrt{-ax+1} \right)^3}{a^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(a*x)^4,x, algorithm="maxima")`

[Out]  $1/4 * x^4 * \arctan^2(ax, \sqrt{ax+1} * \sqrt{-ax+1})^4 + a * \int (\sqrt{ax+1} * \sqrt{-ax+1} * x^4 * \arctan^2(ax, \sqrt{ax+1} * \sqrt{-ax+1}))^3 / (a^2 * x^2 - 1), x$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asin}(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*asin(a*x)^4,x)`

[Out] `int(x^3*asin(a*x)^4, x)`

**sympy** [A] time = 6.10, size = 190, normalized size = 0.96

$$\left\{ \begin{array}{l} \frac{x^4 \operatorname{asin}^4(ax)}{4} - \frac{3x^4 \operatorname{asin}^2(ax)}{16} + \frac{3x^4}{128} + \frac{x^3 \sqrt{-a^2 x^2 + 1} \operatorname{asin}^3(ax)}{4a} - \frac{3x^3 \sqrt{-a^2 x^2 + 1} \operatorname{asin}(ax)}{32a} - \frac{9x^2 \operatorname{asin}^2(ax)}{16a^2} + \frac{45x^2}{128a^2} + \frac{3x \sqrt{-a^2 x^2 + 1} \operatorname{asin}^3(ax)}{8a^3} \\ 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*asin(a*x)**4,x)`

[Out] `Piecewise((x**4*asin(a*x)**4/4 - 3*x**4*asin(a*x)**2/16 + 3*x**4/128 + x**3*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(4*a) - 3*x**3*sqrt(-a**2*x**2 + 1)*asin(a*x)/(32*a) - 9*x**2*asin(a*x)**2/(16*a**2) + 45*x**2/(128*a**2) + 3*x*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(8*a**3) - 45*x*sqrt(-a**2*x**2 + 1)*asin(a*x)/(64*a**3) - 3*asin(a*x)**4/(32*a**4) + 45*asin(a*x)**2/(128*a**4), Ne(a, 0)), (0, True))`

### 3.35 $\int x^2 \sin^{-1}(ax)^4 dx$

**Optimal.** Leaf size=166

$$\frac{4x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)^3}{9a} - \frac{8x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)}{27a} + \frac{160x}{27a^2} - \frac{8x\sin^{-1}(ax)^2}{3a^2} + \frac{8\sqrt{1-a^2x^2}\sin^{-1}(ax)^3}{9a^3} - \frac{160\sqrt{1-a^2x^2}\sin^{-1}(ax)}{27a^3}$$

[Out] 160/27\*x/a^2+8/81\*x^3-8/3\*x\*arcsin(a\*x)^2/a^2-4/9\*x^3\*arcsin(a\*x)^2+1/3\*x^3\*arcsin(a\*x)^4-160/27\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a^3-8/27\*x^2\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a+8/9\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a^3+4/9\*x^2\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a

**Rubi [A]** time = 0.35, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4627, 4707, 4677, 4619, 8, 30}

$$\frac{4x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)^3}{9a} + \frac{8\sqrt{1-a^2x^2}\sin^{-1}(ax)^3}{9a^3} - \frac{8x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)}{27a} - \frac{160\sqrt{1-a^2x^2}\sin^{-1}(ax)}{27a^3} + \frac{160x}{27a^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcSin[a\*x]^4,x]

[Out] (160\*x)/(27\*a^2) + (8\*x^3)/81 - (160\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/(27\*a^3) - (8\*x^2\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/(27\*a) - (8\*x\*ArcSin[a\*x]^2)/(3\*a^2) - (4\*x^3\*ArcSin[a\*x]^2)/9 + (8\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/(9\*a^3) + (4\*x^2\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/(9\*a) + (x^3\*ArcSin[a\*x]^4)/3

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4619

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_.))^n\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4677

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_.))^n\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n)*((f_.)*(x_))^m)/Sqrt[(d_
+ (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

### Rubi steps

$$\begin{aligned} \int x^2 \sin^{-1}(ax)^4 dx &= \frac{1}{3}x^3 \sin^{-1}(ax)^4 - \frac{1}{3}(4a) \int \frac{x^3 \sin^{-1}(ax)^3}{\sqrt{1 - a^2x^2}} dx \\ &= \frac{4x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{9a} + \frac{1}{3}x^3 \sin^{-1}(ax)^4 - \frac{4}{3} \int x^2 \sin^{-1}(ax)^2 dx - \frac{8 \int \frac{x \sin^{-1}(ax)^3}{\sqrt{1 - a^2x^2}} dx}{9a} \\ &= -\frac{4}{9}x^3 \sin^{-1}(ax)^2 + \frac{8\sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{9a^3} + \frac{4x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{9a} + \frac{1}{3}x^3 \sin^{-1}(ax)^4 - \\ &= -\frac{8x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)}{27a} - \frac{8x \sin^{-1}(ax)^2}{3a^2} - \frac{4}{9}x^3 \sin^{-1}(ax)^2 + \frac{8\sqrt{1 - a^2x^2} \sin^{-1}(ax)^3}{9a^3} + \frac{4x^3 \sin^{-1}(ax)^4}{9} \\ &= \frac{8x^3}{81} - \frac{160\sqrt{1 - a^2x^2} \sin^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)}{27a} - \frac{8x \sin^{-1}(ax)^2}{3a^2} - \frac{4}{9}x^3 \sin^{-1}(ax)^2 \\ &= \frac{160x}{27a^2} + \frac{8x^3}{81} - \frac{160\sqrt{1 - a^2x^2} \sin^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)}{27a} - \frac{8x \sin^{-1}(ax)^2}{3a^2} - \frac{4}{9}x^3 \sin^{-1}(ax)^2 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 114, normalized size = 0.69

$$\frac{27a^3x^3 \sin^{-1}(ax)^4 + 8ax(a^2x^2 + 60) + 36\sqrt{1 - a^2x^2}(a^2x^2 + 2) \sin^{-1}(ax)^3 - 36ax(a^2x^2 + 6) \sin^{-1}(ax)^2 - 24\sqrt{1 - a^2x^2}}{81a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcSin[a\*x]^4,x]

[Out] (8\*a\*x\*(60 + a^2\*x^2) - 24\*Sqrt[1 - a^2\*x^2]\*(20 + a^2\*x^2)\*ArcSin[a\*x] - 3\*6\*a\*x\*(6 + a^2\*x^2)\*ArcSin[a\*x]^2 + 36\*Sqrt[1 - a^2\*x^2]\*(2 + a^2\*x^2)\*ArcSin[a\*x]^3 + 27\*a^3\*x^3\*ArcSin[a\*x]^4)/(81\*a^3)

**fricas [A]** time = 0.63, size = 99, normalized size = 0.60

$$\frac{27a^3x^3 \arcsin(ax)^4 + 8a^3x^3 - 36(a^3x^3 + 6ax) \arcsin(ax)^2 + 480ax + 12\sqrt{-a^2x^2 + 1}(3(a^2x^2 + 2) \arcsin(ax))^3}{81a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^4,x, algorithm="fricas")

[Out] 1/81\*(27\*a^3\*x^3\*arcsin(a\*x)^4 + 8\*a^3\*x^3 - 36\*(a^3\*x^3 + 6\*a\*x)\*arcsin(a\*x)^2 + 480\*a\*x + 12\*sqrt(-a^2\*x^2 + 1)\*(3\*(a^2\*x^2 + 2)\*arcsin(a\*x))^3 - 2\*(a^2\*x^2 + 20)\*arcsin(a\*x))/a^3

**giac [A]** time = 0.18, size = 176, normalized size = 1.06

$$\frac{(a^2x^2 - 1)x \arcsin(ax)^4}{3a^2} + \frac{x \arcsin(ax)^4}{3a^2} - \frac{4(a^2x^2 - 1)x \arcsin(ax)^2}{9a^2} - \frac{4(-a^2x^2 + 1)^{\frac{3}{2}} \arcsin(ax)^3}{9a^3} - \frac{28x \arcsin(ax)}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^4,x, algorithm="giac")

[Out]  $\frac{1}{3}(a^2x^2 - 1)x \arcsin(ax)^4/a^2 + \frac{1}{3}x \arcsin(ax)^4/a^2 - \frac{4}{9}(a^2x^2 - 1)x \arcsin(ax)^2/a^2 - \frac{4}{9}(-a^2x^2 + 1)^{3/2} \arcsin(ax)^3/a^3 - \frac{28}{9}x \arcsin(ax)^2/a^2 + \frac{4}{3}\sqrt{-a^2x^2 + 1} \arcsin(ax)^3/a^3 + \frac{8}{81}(a^2x^2 - 1)x/a^2 + \frac{8}{27}(-a^2x^2 + 1)^{3/2} \arcsin(ax)/a^3 + \frac{488}{81}x/a^2 - \frac{56}{9}\sqrt{-a^2x^2 + 1} \arcsin(ax)/a^3$

**maple [A]** time = 0.07, size = 130, normalized size = 0.78

$$\frac{a^3 x^3 \arcsin(ax)^4}{3} + \frac{4 \arcsin(ax)^3 (a^2 x^2 + 2) \sqrt{-a^2 x^2 + 1}}{9} - \frac{8 a x \arcsin(ax)^2}{3} + \frac{160 a x}{27} - \frac{16 \arcsin(ax) \sqrt{-a^2 x^2 + 1}}{3} - \frac{4 a^3 x^3 \arcsin(ax)^2}{9} - \frac{8 \arcsin(ax)}{9} + \frac{8 \arcsin(ax)^3}{9} - \frac{8 \arcsin(ax)^5}{9} - \frac{8 \arcsin(ax)^7}{9} - \frac{8 \arcsin(ax)^9}{9} - \frac{8 \arcsin(ax)^{11}}{9} - \frac{8 \arcsin(ax)^{13}}{9} - \frac{8 \arcsin(ax)^{15}}{9} - \frac{8 \arcsin(ax)^{17}}{9} - \frac{8 \arcsin(ax)^{19}}{9} - \frac{8 \arcsin(ax)^{21}}{9} - \frac{8 \arcsin(ax)^{23}}{9} - \frac{8 \arcsin(ax)^{25}}{9} - \frac{8 \arcsin(ax)^{27}}{9} - \frac{8 \arcsin(ax)^{29}}{9} - \frac{8 \arcsin(ax)^{31}}{9} - \frac{8 \arcsin(ax)^{33}}{9} - \frac{8 \arcsin(ax)^{35}}{9} - \frac{8 \arcsin(ax)^{37}}{9} - \frac{8 \arcsin(ax)^{39}}{9} - \frac{8 \arcsin(ax)^{41}}{9} - \frac{8 \arcsin(ax)^{43}}{9} - \frac{8 \arcsin(ax)^{45}}{9} - \frac{8 \arcsin(ax)^{47}}{9} - \frac{8 \arcsin(ax)^{49}}{9} - \frac{8 \arcsin(ax)^{51}}{9} - \frac{8 \arcsin(ax)^{53}}{9} - \frac{8 \arcsin(ax)^{55}}{9} - \frac{8 \arcsin(ax)^{57}}{9} - \frac{8 \arcsin(ax)^{59}}{9} - \frac{8 \arcsin(ax)^{61}}{9} - \frac{8 \arcsin(ax)^{63}}{9} - \frac{8 \arcsin(ax)^{65}}{9} - \frac{8 \arcsin(ax)^{67}}{9} - \frac{8 \arcsin(ax)^{69}}{9} - \frac{8 \arcsin(ax)^{71}}{9} - \frac{8 \arcsin(ax)^{73}}{9} - \frac{8 \arcsin(ax)^{75}}{9} - \frac{8 \arcsin(ax)^{77}}{9} - \frac{8 \arcsin(ax)^{79}}{9} - \frac{8 \arcsin(ax)^{81}}{9} - \frac{8 \arcsin(ax)^{83}}{9} - \frac{8 \arcsin(ax)^{85}}{9} - \frac{8 \arcsin(ax)^{87}}{9} - \frac{8 \arcsin(ax)^{89}}{9} - \frac{8 \arcsin(ax)^{91}}{9} - \frac{8 \arcsin(ax)^{93}}{9} - \frac{8 \arcsin(ax)^{95}}{9} - \frac{8 \arcsin(ax)^{97}}{9} - \frac{8 \arcsin(ax)^{99}}{9} - \frac{8 \arcsin(ax)^{101}}{9} - \frac{8 \arcsin(ax)^{103}}{9} - \frac{8 \arcsin(ax)^{105}}{9} - \frac{8 \arcsin(ax)^{107}}{9} - \frac{8 \arcsin(ax)^{109}}{9} - \frac{8 \arcsin(ax)^{111}}{9} - \frac{8 \arcsin(ax)^{113}}{9} - \frac{8 \arcsin(ax)^{115}}{9} - \frac{8 \arcsin(ax)^{117}}{9} - \frac{8 \arcsin(ax)^{119}}{9} - \frac{8 \arcsin(ax)^{121}}{9} - \frac{8 \arcsin(ax)^{123}}{9} - \frac{8 \arcsin(ax)^{125}}{9} - \frac{8 \arcsin(ax)^{127}}{9} - \frac{8 \arcsin(ax)^{129}}{9} - \frac{8 \arcsin(ax)^{131}}{9} - \frac{8 \arcsin(ax)^{133}}{9} - \frac{8 \arcsin(ax)^{135}}{9} - \frac{8 \arcsin(ax)^{137}}{9} - \frac{8 \arcsin(ax)^{139}}{9} - \frac{8 \arcsin(ax)^{141}}{9} - \frac{8 \arcsin(ax)^{143}}{9} - \frac{8 \arcsin(ax)^{145}}{9} - \frac{8 \arcsin(ax)^{147}}{9} - \frac{8 \arcsin(ax)^{149}}{9} - \frac{8 \arcsin(ax)^{151}}{9} - \frac{8 \arcsin(ax)^{153}}{9} - \frac{8 \arcsin(ax)^{155}}{9} - \frac{8 \arcsin(ax)^{157}}{9} - \frac{8 \arcsin(ax)^{159}}{9} - \frac{8 \arcsin(ax)^{161}}{9} - \frac{8 \arcsin(ax)^{163}}{9} - \frac{8 \arcsin(ax)^{165}}{9} - \frac{8 \arcsin(ax)^{167}}{9} - \frac{8 \arcsin(ax)^{169}}{9} - \frac{8 \arcsin(ax)^{171}}{9} - \frac{8 \arcsin(ax)^{173}}{9} - \frac{8 \arcsin(ax)^{175}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arcsin(a\*x)^4,x)

[Out]  $\frac{1}{a^3} \left( \frac{1}{3} a^3 x^3 \arcsin(ax)^4 + \frac{4}{9} a^3 x \arcsin(ax)^3 (a^2 x^2 + 2) (-a^2 x^2 + 1)^{1/2} - \frac{8}{3} a^3 x \arcsin(ax)^2 + \frac{160}{27} a^3 x - \frac{16}{3} a^3 \arcsin(ax) (-a^2 x^2 + 1)^{1/2} - \frac{4}{9} a^3 x^3 \arcsin(ax)^2 - \frac{8}{27} a^3 \arcsin(ax) (a^2 x^2 + 2) (-a^2 x^2 + 1)^{1/2} + \frac{8}{81} a^3 x^3 \right)$

**maxima [A]** time = 0.51, size = 147, normalized size = 0.89

$$\frac{1}{3} x^3 \arcsin(ax)^4 + \frac{4}{9} a \left( \frac{\sqrt{-a^2 x^2 + 1} x^2}{a^2} + \frac{2 \sqrt{-a^2 x^2 + 1}}{a^4} \right) \arcsin(ax)^3 - \frac{4}{81} \left( 2 a \left( \frac{3 \left( \sqrt{-a^2 x^2 + 1} x^2 + \frac{20 \sqrt{-a^2 x^2 + 1}}{a^2} \right)}{a^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^4,x, algorithm="maxima")

[Out]  $\frac{1}{3} x^3 \arcsin(ax)^4 + \frac{4}{9} a \left( \sqrt{-a^2 x^2 + 1} x^2/a^2 + 2 \sqrt{-a^2 x^2 + 1}/a^4 \right) \arcsin(ax)^3 - \frac{4}{81} \left( 2 a \left( 3 \left( \sqrt{-a^2 x^2 + 1} x^2 + \frac{20 \sqrt{-a^2 x^2 + 1}}{a^2} \right) \arcsin(ax)/a^3 - (a^2 x^3 + 60 x)/a^4 \right) + 9(a^2 x^3 + 6 x) \arcsin(ax)^2/a^3 \right) a$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asin}(a x)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*asin(a\*x)^4,x)

[Out] int(x^2\*asin(a\*x)^4, x)

**sympy [A]** time = 3.31, size = 158, normalized size = 0.95

$$\left\{ \begin{array}{l} \frac{x^3 \operatorname{asin}^4(ax)}{3} - \frac{4x^3 \operatorname{asin}^2(ax)}{9} + \frac{8x^3}{81} + \frac{4x^2 \sqrt{-a^2 x^2 + 1} \operatorname{asin}^3(ax)}{9a} - \frac{8x^2 \sqrt{-a^2 x^2 + 1} \operatorname{asin}(ax)}{27a} - \frac{8x \operatorname{asin}^2(ax)}{3a^2} + \frac{160x}{27a^2} + \frac{8 \sqrt{-a^2 x^2 + 1} \operatorname{asin}^3(ax)}{9a^3} \\ 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*asin(a\*x)\*\*4,x)

[Out]  $\operatorname{Piecewise}\left(\left(x^3 \operatorname{asin}(a x)^4/3 - 4 x^3 \operatorname{asin}(a x)^2/9 + 8 x^3/81 + 4 x^2 \sqrt{-a^2 x^2 + 1} \operatorname{asin}(a x)^3/(9 a) - 8 x^2 \sqrt{-a^2 x^2 + 1} \operatorname{asin}(a x)/27 a - 8 x \operatorname{asin}(a x)^2/a^2 + 160 x/27 a^2 + 8 \sqrt{-a^2 x^2 + 1} \operatorname{asin}(a x)^3/9 a^3\right), 0\right)$

```
(a*x)/(27*a) - 8*x*asin(a*x)**2/(3*a**2) + 160*x/(27*a**2) + 8*sqrt(-a**2*x  
**2 + 1)*asin(a*x)**3/(9*a**3) - 160*sqrt(-a**2*x**2 + 1)*asin(a*x)/(27*a**  
3), Ne(a, 0)), (0, True))
```



### 3.36 $\int x \sin^{-1}(ax)^4 dx$

**Optimal.** Leaf size=111

$$\frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} - \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} - \frac{\sin^{-1}(ax)^4}{4a^2} + \frac{3 \sin^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^4 - \frac{3}{2}x^2 \sin^{-1}(ax)^2 + \frac{1}{2}x^2 \sin^{-1}(ax)$$

[Out]  $\frac{3}{4}x^2 + \frac{3}{4}\arcsin(ax)^2/a^2 - \frac{3}{2}x^2\arcsin(ax)^2 - \frac{1}{4}\arcsin(ax)^4/a^2 + \frac{1}{2}x^2\arcsin(ax)^4 - \frac{3}{2}x^2\arcsin(ax)^2 + \frac{1}{2}x^2\arcsin(ax)$

**Rubi [A]** time = 0.24, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 4707, 4641, 30}

$$\frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} - \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} - \frac{\sin^{-1}(ax)^4}{4a^2} + \frac{3 \sin^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^4 - \frac{3}{2}x^2 \sin^{-1}(ax)^2 + \frac{1}{2}x^2 \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x\*ArcSin[a\*x]^4,x]

[Out]  $\frac{(3x^2)/4 - (3x\sqrt{1-a^2x^2}\text{ArcSin}[a*x])/(2a) + (3\text{ArcSin}[a*x]^2)/(4a^2) - (3x^2\text{ArcSin}[a*x]^2)/2 + (x\sqrt{1-a^2x^2}\text{ArcSin}[a*x]^3)/a - \text{ArcSin}[a*x]^4/(4a^2) + (x^2\text{ArcSin}[a*x]^4)/2}$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(a+b\*ArcSin[c\*x])^(n+1)/(b\*c\*Sqrt[d]\*(n+1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d+e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((f\_)\*(x\_))^(m\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m-1)\*Sqrt[d+e\*x^2]\*(a+b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m-1))/(c^2\*m), Int[((f\*x)^(m-2)\*(a+b\*ArcSin[c\*x])^n)/Sqrt[d+e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1-c^2\*x^2])/(c\*m\*Sqrt[d+e\*x^2]), Int[(f\*x)^(m-1)\*(a+b\*ArcSin[c\*x])^(n-1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d+e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rubi steps

$$\begin{aligned}
\int x \sin^{-1}(ax)^4 dx &= \frac{1}{2}x^2 \sin^{-1}(ax)^4 - (2a) \int \frac{x^2 \sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx \\
&= \frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} + \frac{1}{2}x^2 \sin^{-1}(ax)^4 - 3 \int x \sin^{-1}(ax)^2 dx - \frac{\int \frac{\sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx}{a} \\
&= -\frac{3}{2}x^2 \sin^{-1}(ax)^2 + \frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} - \frac{\sin^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^4 + (3a) \int \frac{x^2 \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx \\
&= -\frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} - \frac{3}{2}x^2 \sin^{-1}(ax)^2 + \frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} - \frac{\sin^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^4 \\
&= \frac{3x^2}{4} - \frac{3x\sqrt{1-a^2x^2} \sin^{-1}(ax)}{2a} + \frac{3 \sin^{-1}(ax)^2}{4a^2} - \frac{3}{2}x^2 \sin^{-1}(ax)^2 + \frac{x\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} - \frac{\sin^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^4
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 96, normalized size = 0.86

$$\frac{3a^2x^2 + (2a^2x^2 - 1) \sin^{-1}(ax)^4 + 4ax\sqrt{1-a^2x^2} \sin^{-1}(ax)^3 + (3 - 6a^2x^2) \sin^{-1}(ax)^2 - 6ax\sqrt{1-a^2x^2} \sin^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcSin[a\*x]^4,x]

[Out] (3\*a^2\*x^2 - 6\*a\*x\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x] + (3 - 6\*a^2\*x^2)\*ArcSin[a\*x]^2 + 4\*a\*x\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3 + (-1 + 2\*a^2\*x^2)\*ArcSin[a\*x]^4)/(4\*a^2)

**fricas [A]** time = 0.48, size = 82, normalized size = 0.74

$$\frac{(2a^2x^2 - 1) \arcsin(ax)^4 + 3a^2x^2 - 3(2a^2x^2 - 1) \arcsin(ax)^2 + 2(2ax \arcsin(ax)^3 - 3ax \arcsin(ax))\sqrt{-a^2x^2 + 1}}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^4,x, algorithm="fricas")

[Out] 1/4\*((2\*a^2\*x^2 - 1)\*arcsin(a\*x)^4 + 3\*a^2\*x^2 - 3\*(2\*a^2\*x^2 - 1)\*arcsin(a\*x)^2 + 2\*(2\*a\*x\*arcsin(a\*x)^3 - 3\*a\*x\*arcsin(a\*x))\*sqrt(-a^2\*x^2 + 1))/a^2

**giac [A]** time = 0.15, size = 127, normalized size = 1.14

$$\frac{\sqrt{-a^2x^2 + 1} x \arcsin(ax)^3}{a} + \frac{(a^2x^2 - 1) \arcsin(ax)^4}{2a^2} + \frac{\arcsin(ax)^4}{4a^2} - \frac{3\sqrt{-a^2x^2 + 1} x \arcsin(ax)}{2a} - \frac{3(a^2x^2 - 1) \arcsin(ax)^2}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^4,x, algorithm="giac")

[Out] sqrt(-a^2\*x^2 + 1)\*x\*arcsin(a\*x)^3/a + 1/2\*(a^2\*x^2 - 1)\*arcsin(a\*x)^4/a^2 + 1/4\*arcsin(a\*x)^4/a^2 - 3/2\*sqrt(-a^2\*x^2 + 1)\*x\*arcsin(a\*x)/a - 3/2\*(a^2\*x^2 - 1)\*arcsin(a\*x)^2/a^2 - 3/4\*arcsin(a\*x)^2/a^2 + 3/4\*(a^2\*x^2 - 1)/a^2 + 3/8/a^2

**maple [A]** time = 0.06, size = 117, normalized size = 1.05

$$\frac{(a^2x^2-1)\arcsin(ax)^4}{2} + \arcsin(ax)^3 \left( ax\sqrt{-a^2x^2+1} + \arcsin(ax) \right) - \frac{3(a^2x^2-1)\arcsin(ax)^2}{2} - \frac{3\arcsin(ax)(ax\sqrt{-a^2x^2+1} + \arcsin(ax))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsin(a*x)^4,x)`

[Out]  $\frac{1}{a^2} \left( \frac{1}{2} (a^2 x^2 - 1) \arcsin(ax)^4 + \arcsin(ax)^3 (ax \sqrt{-a^2 x^2 + 1})^{1/2} + \arcsin(ax) - \frac{3}{2} (a^2 x^2 - 1) \arcsin(ax)^2 - \frac{3}{2} \arcsin(ax) (ax \sqrt{-a^2 x^2 + 1})^{1/2} + \arcsin(ax) \right) + \frac{3}{4} \arcsin(ax)^2 + \frac{3}{4} a^2 x^2 - \frac{3}{4} \arcsin(ax)^4$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} x^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^4 + 2a \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^3}{a^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsin(a*x)^4,x, algorithm="maxima")`

[Out]  $\frac{1}{2} x^2 \arctan^2(ax, \sqrt{ax+1} \sqrt{-ax+1})^4 + 2a \int \sqrt{ax+1} \sqrt{-ax+1} x^2 \arctan^2(ax, \sqrt{ax+1} \sqrt{-ax+1})^3 / (a^2 x^2 - 1) dx$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \arcsin(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*asin(a*x)^4,x)`

[Out] `int(x*asin(a*x)^4, x)`

**sympy** [A] time = 1.90, size = 104, normalized size = 0.94

$$\begin{cases} \frac{x^2 \operatorname{asin}^4(ax)}{2} - \frac{3x^2 \operatorname{asin}^2(ax)}{2} + \frac{3x^2}{4} + \frac{x \sqrt{-a^2 x^2 + 1} \operatorname{asin}^3(ax)}{a} - \frac{3x \sqrt{-a^2 x^2 + 1} \operatorname{asin}(ax)}{2a} - \frac{\operatorname{asin}^4(ax)}{4a^2} + \frac{3 \operatorname{asin}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*asin(a*x)**4,x)`

[Out] `Piecewise((x**2*asin(a*x)**4/2 - 3*x**2*asin(a*x)**2/2 + 3*x**2/4 + x*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/a - 3*x*sqrt(-a**2*x**2 + 1)*asin(a*x)/(2*a) - asin(a*x)**4/(4*a**2) + 3*asin(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))`

### 3.37 $\int \sin^{-1}(ax)^4 dx$

**Optimal.** Leaf size=69

$$\frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} - \frac{24\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} + x \sin^{-1}(ax)^4 - 12x \sin^{-1}(ax)^2 + 24x$$

[Out] 24\*x-12\*x\*arcsin(a\*x)^2+x\*arcsin(a\*x)^4-24\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)/a+4\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)/a

**Rubi [A]** time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4619, 4677, 8}

$$\frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} - \frac{24\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} + x \sin^{-1}(ax)^4 - 12x \sin^{-1}(ax)^2 + 24x$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^4,x]

[Out] 24\*x - (24\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/a - 12\*x\*ArcSin[a\*x]^2 + (4\*sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/a + x\*ArcSin[a\*x]^4

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x]))^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \sin^{-1}(ax)^4 dx &= x \sin^{-1}(ax)^4 - (4a) \int \frac{x \sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx \\ &= \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} + x \sin^{-1}(ax)^4 - 12 \int \sin^{-1}(ax)^2 dx \\ &= -12x \sin^{-1}(ax)^2 + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} + x \sin^{-1}(ax)^4 + (24a) \int \frac{x \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx \\ &= -\frac{24\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} - 12x \sin^{-1}(ax)^2 + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} + x \sin^{-1}(ax)^4 + 24 \int 1 dx \\ &= 24x - \frac{24\sqrt{1-a^2x^2} \sin^{-1}(ax)}{a} - 12x \sin^{-1}(ax)^2 + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{a} + x \sin^{-1}(ax)^4 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 69, normalized size = 1.00

$$\frac{4\sqrt{1-a^2x^2}\sin^{-1}(ax)^3}{a} - \frac{24\sqrt{1-a^2x^2}\sin^{-1}(ax)}{a} + x\sin^{-1}(ax)^4 - 12x\sin^{-1}(ax)^2 + 24x$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^4,x]

[Out] 24\*x - (24\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x])/a - 12\*x\*ArcSin[a\*x]^2 + (4\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^3)/a + x\*ArcSin[a\*x]^4

**fricas [A]** time = 0.41, size = 55, normalized size = 0.80

$$\frac{ax \arcsin(ax)^4 - 12ax \arcsin(ax)^2 + 24ax + 4\sqrt{-a^2x^2 + 1}(\arcsin(ax)^3 - 6 \arcsin(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4,x, algorithm="fricas")

[Out] (a\*x\*arcsin(a\*x)^4 - 12\*a\*x\*arcsin(a\*x)^2 + 24\*a\*x + 4\*sqrt(-a^2\*x^2 + 1)\*(arcsin(a\*x)^3 - 6\*arcsin(a\*x)))/a

**giac [A]** time = 0.12, size = 65, normalized size = 0.94

$$x \arcsin(ax)^4 - 12x \arcsin(ax)^2 + \frac{4\sqrt{-a^2x^2 + 1} \arcsin(ax)^3}{a} + 24x - \frac{24\sqrt{-a^2x^2 + 1} \arcsin(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4,x, algorithm="giac")

[Out] x\*arcsin(a\*x)^4 - 12\*x\*arcsin(a\*x)^2 + 4\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)^3/a + 24\*x - 24\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)/a

**maple [A]** time = 0.04, size = 67, normalized size = 0.97

$$\frac{ax \arcsin(ax)^4 + 4 \arcsin(ax)^3 \sqrt{-a^2x^2 + 1} - 12ax \arcsin(ax)^2 + 24ax - 24 \arcsin(ax) \sqrt{-a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^4,x)

[Out] 1/a\*(a\*x\*arcsin(a\*x)^4+4\*arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)-12\*a\*x\*arcsin(a\*x)^2+24\*a\*x-24\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2))

**maxima [A]** time = 0.54, size = 75, normalized size = 1.09

$$x \arcsin(ax)^4 + \frac{4\sqrt{-a^2x^2 + 1} \arcsin(ax)^3}{a} - 12 \left( \frac{x \arcsin(ax)^2}{a} - \frac{2 \left( x - \frac{\sqrt{-a^2x^2 + 1} \arcsin(ax)}{a} \right)}{a} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4,x, algorithm="maxima")

[Out] x\*arcsin(a\*x)^4 + 4\*sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)^3/a - 12\*(x\*arcsin(a\*x)^2/a - 2\*(x - sqrt(-a^2\*x^2 + 1)\*arcsin(a\*x)/a)/a)\*a

**mupad [B]** time = 0.14, size = 48, normalized size = 0.70

$$x \left( \operatorname{asin}(ax)^4 - 12 \operatorname{asin}(ax)^2 + 24 \right) + \frac{4 \operatorname{asin}(ax) \sqrt{1 - a^2 x^2} \left( \operatorname{asin}(ax)^2 - 6 \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^4,x)`

[Out] `x*(asin(a*x)^4 - 12*asin(a*x)^2 + 24) + (4*asin(a*x)*(1 - a^2*x^2)^(1/2)*(asin(a*x)^2 - 6))/a`

**sympy [A]** time = 0.86, size = 65, normalized size = 0.94

$$\begin{cases} x \operatorname{asin}^4(ax) - 12x \operatorname{asin}^2(ax) + 24x + \frac{4\sqrt{-a^2x^2+1} \operatorname{asin}^3(ax)}{a} - \frac{24\sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)**4,x)`

[Out] `Piecewise((x*asin(a*x)**4 - 12*x*asin(a*x)**2 + 24*x + 4*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/a - 24*sqrt(-a**2*x**2 + 1)*asin(a*x)/a, Ne(a, 0)), (0, True))`

$$3.38 \quad \int \frac{\sin^{-1}(ax)^4}{x} dx$$

**Optimal.** Leaf size=113

$$-2i \sin^{-1}(ax)^3 \operatorname{Li}_2(e^{2i \sin^{-1}(ax)}) + 3 \sin^{-1}(ax)^2 \operatorname{Li}_3(e^{2i \sin^{-1}(ax)}) + 3i \sin^{-1}(ax) \operatorname{Li}_4(e^{2i \sin^{-1}(ax)}) - \frac{3}{2} \operatorname{Li}_5(e^{2i \sin^{-1}(ax)}) - \frac{1}{5}$$

[Out]  $-1/5 * I * \arcsin(a*x)^5 + \arcsin(a*x)^4 * \ln(1 - (I*a*x + (-a^2*x^2+1)^{(1/2)})^2) - 2*I*a$   
 $\operatorname{rcsin}(a*x)^3 * \operatorname{polylog}(2, (I*a*x + (-a^2*x^2+1)^{(1/2)})^2) + 3*\arcsin(a*x)^2 * \operatorname{polylo}$   
 $\operatorname{g}(3, (I*a*x + (-a^2*x^2+1)^{(1/2)})^2) + 3*I*\arcsin(a*x) * \operatorname{polylog}(4, (I*a*x + (-a^2*x^$   
 $2+1)^{(1/2)})^2) - 3/2 * \operatorname{polylog}(5, (I*a*x + (-a^2*x^2+1)^{(1/2)})^2)$

**Rubi [A]** time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4625, 3717, 2190, 2531, 6609, 2282, 6589}

$$-2i \sin^{-1}(ax)^3 \operatorname{PolyLog}(2, e^{2i \sin^{-1}(ax)}) + 3 \sin^{-1}(ax)^2 \operatorname{PolyLog}(3, e^{2i \sin^{-1}(ax)}) + 3i \sin^{-1}(ax) \operatorname{PolyLog}(4, e^{2i \sin^{-1}(ax)}) - \frac{3}{2} \operatorname{PolyLog}(5, e^{2i \sin^{-1}(ax)}) - \frac{1}{5}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcSin}[a*x]^4/x, x]$

[Out]  $(-I/5) * \operatorname{ArcSin}[a*x]^5 + \operatorname{ArcSin}[a*x]^4 * \operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcSin}[a*x])}] - (2*I)$   
 $* \operatorname{ArcSin}[a*x]^3 * \operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcSin}[a*x])}] + 3 * \operatorname{ArcSin}[a*x]^2 * \operatorname{PolyLog}[$   
 $3, E^{((2*I)*\operatorname{ArcSin}[a*x])}] + (3*I) * \operatorname{ArcSin}[a*x] * \operatorname{PolyLog}[4, E^{((2*I)*\operatorname{ArcSin}[a$   
 $x])}] - (3 * \operatorname{PolyLog}[5, E^{((2*I)*\operatorname{ArcSin}[a*x])}])/2$

#### Rule 2190

$\operatorname{Int}[\frac{((F_)^{((g_*) * ((e_*) + (f_*) * (x_)))})^{(n_*) * ((c_*) + (d_*) * (x_))^{(m_*)}}{((a_*) + (b_*) * ((F_)^{((g_*) * ((e_*) + (f_*) * (x_)))})^{(n_*)})}, x\_Symbol] :> \operatorname{Simp}$   
 $[\frac{(c + d*x)^m * \operatorname{Log}[1 + (b * (F^{(g * (e + f*x)))^n) / a]}{b * f * g * n * \operatorname{Log}[F]}, x] - \operatorname{Dist}$   
 $[\frac{(d * m)}{b * f * g * n * \operatorname{Log}[F]}, \operatorname{Int}[\frac{(c + d*x)^{(m-1)} * \operatorname{Log}[1 + (b * (F^{(g * (e + f*x)))^n) / a]}{a}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] :> \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x]$   
 $, \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_*) * ((a_*) * (v_)^{(n_)})^{(m_*)} /; \operatorname{FreeQ}$   
 $\{a, m, n\}, x\} \&\& \operatorname{IntegerQ}[m * n] \&\& \operatorname{!MatchQ}[u, E^{((c_*) * ((a_*) + (b_*) * x)) * (F_)[v_]} /; \operatorname{FreeQ}\{a, b, c\}, x\} \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_*) * ((F_)^{((c_*) * ((a_*) + (b_*) * (x_)))})^{(n_*)}] * ((f_*) + (g_*)$   
 $* (x_))^{(m_*)}, x\_Symbol] :> -\operatorname{Simp}[\frac{(f + g*x)^m * \operatorname{PolyLog}[2, -(e * (F^{(c * (a + b*x)))^n) / a]}{b * c * n * \operatorname{Log}[F]}, x] + \operatorname{Dist}[\frac{(g * m)}{b * c * n * \operatorname{Log}[F]}, \operatorname{Int}[\frac{(f + g*x)^{(m-1)} * \operatorname{PolyLog}[2, -(e * (F^{(c * (a + b*x)))^n) / a]}{a}, x], x] /; \operatorname{FreeQ}\{F, a, b, c, e, f, g, n\}, x\} \&\& \operatorname{GtQ}[m, 0]$

#### Rule 3717

$\operatorname{Int}[\frac{((c_*) + (d_*) * (x_))^{(m_*)} * \tan[(e_*) + \operatorname{Pi} * (k_*) + (f_*) * (x_)]}{(c + d*x)^{(m+1)} / (d * (m+1))}, x\_Symbol] :> \operatorname{Simp}[\frac{(I * (c + d*x)^{(m+1)}) / (d * (m+1))}{(1 + E^{(2 * I * k * \operatorname{Pi})} * E^{(2 * I * (e + f*x))}) / (1 + E^{(2 * I * k * \operatorname{Pi})} * E^{(2 * I * (e + f*x))})}, x]$   
 $/; \operatorname{FreeQ}\{c, d, e, f\}, x\} \&\& \operatorname{IntegerQ}[4 * k] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(ax)^4}{x} dx &= \text{Subst} \left( \int x^4 \cot(x) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{5}i \sin^{-1}(ax)^5 - 2i \text{Subst} \left( \int \frac{e^{2ix} x^4}{1 - e^{2ix}} dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{5}i \sin^{-1}(ax)^5 + \sin^{-1}(ax)^4 \log(1 - e^{2i \sin^{-1}(ax)}) - 4 \text{Subst} \left( \int x^3 \log(1 - e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{5}i \sin^{-1}(ax)^5 + \sin^{-1}(ax)^4 \log(1 - e^{2i \sin^{-1}(ax)}) - 2i \sin^{-1}(ax)^3 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + 6i \text{Subst} \left( \int x^2 \log(1 - e^{2ix}) dx, x, \sin^{-1}(ax) \right) \\ &= -\frac{1}{5}i \sin^{-1}(ax)^5 + \sin^{-1}(ax)^4 \log(1 - e^{2i \sin^{-1}(ax)}) - 2i \sin^{-1}(ax)^3 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + 3 \sin^{-1}(ax)^2 \text{Li}_3(e^{2i \sin^{-1}(ax)}) \\ &= -\frac{1}{5}i \sin^{-1}(ax)^5 + \sin^{-1}(ax)^4 \log(1 - e^{2i \sin^{-1}(ax)}) - 2i \sin^{-1}(ax)^3 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + 3 \sin^{-1}(ax)^2 \text{Li}_3(e^{2i \sin^{-1}(ax)}) \\ &= -\frac{1}{5}i \sin^{-1}(ax)^5 + \sin^{-1}(ax)^4 \log(1 - e^{2i \sin^{-1}(ax)}) - 2i \sin^{-1}(ax)^3 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + 3 \sin^{-1}(ax)^2 \text{Li}_3(e^{2i \sin^{-1}(ax)}) \\ &= -\frac{1}{5}i \sin^{-1}(ax)^5 + \sin^{-1}(ax)^4 \log(1 - e^{2i \sin^{-1}(ax)}) - 2i \sin^{-1}(ax)^3 \text{Li}_2(e^{2i \sin^{-1}(ax)}) + 3 \sin^{-1}(ax)^2 \text{Li}_3(e^{2i \sin^{-1}(ax)}) \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 113, normalized size = 1.00

$$2i \sin^{-1}(ax)^3 \text{Li}_2(e^{-2i \sin^{-1}(ax)}) + 3 \sin^{-1}(ax)^2 \text{Li}_3(e^{-2i \sin^{-1}(ax)}) - 3i \sin^{-1}(ax) \text{Li}_4(e^{-2i \sin^{-1}(ax)}) - \frac{3}{2} \text{Li}_5(e^{-2i \sin^{-1}(ax)}) + \frac{1}{5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSin[a*x]^4/x, x]
```

```
[Out] (I/5)*ArcSin[a*x]^5 + ArcSin[a*x]^4*Log[1 - E^((-2*I)*ArcSin[a*x])] + (2*I)*ArcSin[a*x]^3*PolyLog[2, E^((-2*I)*ArcSin[a*x])] + 3*ArcSin[a*x]^2*PolyLog[3, E^((-2*I)*ArcSin[a*x])] - (3*I)*ArcSin[a*x]*PolyLog[4, E^((-2*I)*ArcSin[a*x])] - (3*PolyLog[5, E^((-2*I)*ArcSin[a*x])])/2
```

**fricas [F]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^4}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(arcsin(a\*x)^4/x,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^4/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^4/x, x)

**maple** [A] time = 0.06, size = 287, normalized size = 2.54

$$-\frac{i \arcsin(ax)^5}{5} + \arcsin(ax)^4 \ln\left(1 + iax + \sqrt{-a^2x^2 + 1}\right) - 4i \arcsin(ax)^3 \operatorname{polylog}\left(2, -iax - \sqrt{-a^2x^2 + 1}\right) + 12$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^4/x,x)

[Out]  $-1/5*I*\arcsin(a*x)^5 + \arcsin(a*x)^4*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)}) - 4*I*\arcsin(a*x)^3*\operatorname{polylog}(2, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + 12*\arcsin(a*x)^2*\operatorname{polylog}(3, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + 24*I*\arcsin(a*x)*\operatorname{polylog}(4, -I*a*x - (-a^2*x^2+1)^{(1/2)}) - 24*\operatorname{polylog}(5, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + \arcsin(a*x)^4*\ln(1-I*a*x - (-a^2*x^2+1)^{(1/2)}) - 4*I*\arcsin(a*x)^3*\operatorname{polylog}(2, I*a*x + (-a^2*x^2+1)^{(1/2)}) + 12*\arcsin(a*x)^2*\operatorname{polylog}(3, I*a*x + (-a^2*x^2+1)^{(1/2)}) + 24*I*\arcsin(a*x)*\operatorname{polylog}(4, I*a*x + (-a^2*x^2+1)^{(1/2)}) - 24*\operatorname{polylog}(5, I*a*x + (-a^2*x^2+1)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x,x, algorithm="maxima")

[Out] integrate(arcsin(a\*x)^4/x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}(ax)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^4/x,x)

[Out] int(asin(a\*x)^4/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^4(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*4/x,x)

[Out] Integral(asin(a\*x)\*\*4/x, x)

$$3.39 \quad \int \frac{\sin^{-1}(ax)^4}{x^2} dx$$

**Optimal.** Leaf size=156

$$12ia \sin^{-1}(ax)^2 \text{Li}_2(-e^{i \sin^{-1}(ax)}) - 12ia \sin^{-1}(ax)^2 \text{Li}_2(e^{i \sin^{-1}(ax)}) - 24a \sin^{-1}(ax) \text{Li}_3(-e^{i \sin^{-1}(ax)}) + 24a \sin^{-1}(ax) \text{Li}_3(e^{i \sin^{-1}(ax)})$$

```
[Out] -arcsin(a*x)^4/x-8*a*arcsin(a*x)^3*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))+12*I*a
*a*arcsin(a*x)^2*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-12*I*a*arcsin(a*x)^2*po
lylog(2,I*a*x+(-a^2*x^2+1)^(1/2))-24*a*arcsin(a*x)*polylog(3,-I*a*x-(-a^2*x
^2+1)^(1/2))+24*a*arcsin(a*x)*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))-24*I*a*po
lylog(4,-I*a*x-(-a^2*x^2+1)^(1/2))+24*I*a*polylog(4,I*a*x+(-a^2*x^2+1)^(1/2
))
```

**Rubi [A]** time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4627, 4709, 4183, 2531, 6609, 2282, 6589}

$$12ia \sin^{-1}(ax)^2 \text{PolyLog}(2, -e^{i \sin^{-1}(ax)}) - 12ia \sin^{-1}(ax)^2 \text{PolyLog}(2, e^{i \sin^{-1}(ax)}) - 24a \sin^{-1}(ax) \text{PolyLog}(3, -e^{i \sin^{-1}(ax)}) + 24a \sin^{-1}(ax) \text{PolyLog}(3, e^{i \sin^{-1}(ax)})$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[a*x]^4/x^2,x]
```

```
[Out] -(ArcSin[a*x]^4/x) - 8*a*ArcSin[a*x]^3*ArcTanh[E^(I*ArcSin[a*x])] + (12*I)*
a*ArcSin[a*x]^2*PolyLog[2, -E^(I*ArcSin[a*x])] - (12*I)*a*ArcSin[a*x]^2*Pol
yLog[2, E^(I*ArcSin[a*x])] - 24*a*ArcSin[a*x]*PolyLog[3, -E^(I*ArcSin[a*x])
] + 24*a*ArcSin[a*x]*PolyLog[3, E^(I*ArcSin[a*x])] - (24*I)*a*PolyLog[4, -E
^(I*ArcSin[a*x])] + (24*I)*a*PolyLog[4, E^(I*ArcSin[a*x])]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

#### Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
```

\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4709

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\_.)\*(x\_)^(m\_)]/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Dist[1/(c^(m + 1)\*Sqrt[d]), Subst[Int[(a + b\*x)^n\*Sin[x]^m, x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
 \int \frac{\sin^{-1}(ax)^4}{x^2} dx &= -\frac{\sin^{-1}(ax)^4}{x} + (4a) \int \frac{\sin^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} dx \\
 &= -\frac{\sin^{-1}(ax)^4}{x} + (4a) \text{Subst}\left(\int x^3 \csc(x) dx, x, \sin^{-1}(ax)\right) \\
 &= -\frac{\sin^{-1}(ax)^4}{x} - 8a \sin^{-1}(ax)^3 \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) - (12a) \text{Subst}\left(\int x^2 \log(1 - e^{ix}) dx, x, \sin^{-1}(ax)\right) \\
 &= -\frac{\sin^{-1}(ax)^4}{x} - 8a \sin^{-1}(ax)^3 \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) + 12ia \sin^{-1}(ax)^2 \text{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) - 12ia \sin^{-1}(ax) \text{Li}_3\left(-e^{i \sin^{-1}(ax)}\right) \\
 &= -\frac{\sin^{-1}(ax)^4}{x} - 8a \sin^{-1}(ax)^3 \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) + 12ia \sin^{-1}(ax)^2 \text{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) - 12ia \sin^{-1}(ax) \text{Li}_3\left(-e^{i \sin^{-1}(ax)}\right) \\
 &= -\frac{\sin^{-1}(ax)^4}{x} - 8a \sin^{-1}(ax)^3 \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) + 12ia \sin^{-1}(ax)^2 \text{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) - 12ia \sin^{-1}(ax) \text{Li}_3\left(-e^{i \sin^{-1}(ax)}\right) \\
 &= -\frac{\sin^{-1}(ax)^4}{x} - 8a \sin^{-1}(ax)^3 \tanh^{-1}\left(e^{i \sin^{-1}(ax)}\right) + 12ia \sin^{-1}(ax)^2 \text{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) - 12ia \sin^{-1}(ax) \text{Li}_3\left(-e^{i \sin^{-1}(ax)}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 198, normalized size = 1.27

$$a \left( 12i \sin^{-1}(ax)^2 \text{Li}_2\left(e^{-i \sin^{-1}(ax)}\right) + 12i \sin^{-1}(ax)^2 \text{Li}_2\left(-e^{i \sin^{-1}(ax)}\right) + 24 \sin^{-1}(ax) \text{Li}_3\left(e^{-i \sin^{-1}(ax)}\right) - 24 \sin^{-1}(ax) \text{Li}_3\left(-e^{i \sin^{-1}(ax)}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^4/x^2,x]

[Out] a\*((-1/2\*I)\*Pi^4 + I\*ArcSin[a\*x]^4 - ArcSin[a\*x]^4/(a\*x) + 4\*ArcSin[a\*x]^3\*Log[1 - E^((-I)\*ArcSin[a\*x])] - 4\*ArcSin[a\*x]^3\*Log[1 + E^(I\*ArcSin[a\*x])]) + (12\*I)\*ArcSin[a\*x]^2\*PolyLog[2, E^((-I)\*ArcSin[a\*x])] + (12\*I)\*ArcSin[a\*x]^2\*PolyLog[2, -E^(I\*ArcSin[a\*x])] + 24\*ArcSin[a\*x]\*PolyLog[3, E^((-I)\*ArcSin[a\*x])] - 24\*ArcSin[a\*x]\*PolyLog[3, -E^(I\*ArcSin[a\*x])] - (24\*I)\*PolyLog[4, E^((-I)\*ArcSin[a\*x])] - (24\*I)\*PolyLog[4, -E^(I\*ArcSin[a\*x])])

**fricas** [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arcsin(ax)^4}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x^2,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^4/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x^2,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^4/x^2, x)

**maple** [A] time = 0.11, size = 241, normalized size = 1.54

$$-\frac{\arcsin(ax)^4}{x} - 4a \arcsin(ax)^3 \ln\left(1 + iax + \sqrt{-a^2x^2 + 1}\right) + 4a \arcsin(ax)^3 \ln\left(1 - iax - \sqrt{-a^2x^2 + 1}\right) - 24a \arcsin$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^4/x^2,x)

[Out]  $-\arcsin(a*x)^4/x - 4*a*\arcsin(a*x)^3*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)}) + 4*a*\arcsin(a*x)^3*\ln(1-I*a*x-(-a^2*x^2+1)^{(1/2)}) - 24*a*\arcsin(a*x)*\text{polylog}(3, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + 24*a*\arcsin(a*x)*\text{polylog}(3, I*a*x + (-a^2*x^2+1)^{(1/2)}) + 12*I*a*\arcsin(a*x)^2*\text{polylog}(2, -I*a*x - (-a^2*x^2+1)^{(1/2)}) - 12*I*a*\arcsin(a*x)^2*\text{polylog}(2, I*a*x + (-a^2*x^2+1)^{(1/2)}) - 24*I*a*\text{polylog}(4, -I*a*x - (-a^2*x^2+1)^{(1/2)}) + 24*I*a*\text{polylog}(4, I*a*x + (-a^2*x^2+1)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)^4 + 4ax \int \frac{\sqrt{-ax+1} \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^3}{\sqrt{ax+1}(ax-1)x} dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x^2,x, algorithm="maxima")

[Out]  $-(\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^4 + 4*a*x*\text{integrate}(\sqrt{a*x+1}*\sqrt{-a*x+1}*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^3/(a^2*x^3 - x), x))/x$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{asin}(ax)^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^4/x^2,x)

[Out] int(asin(a\*x)^4/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^4(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(a*x)**4/x**2,x)
```

```
[Out] Integral(asin(a*x)**4/x**2, x)
```

### 3.40 $\int \frac{\sin^{-1}(ax)^4}{x^3} dx$

**Optimal.** Leaf size=119

$$-6ia^2 \sin^{-1}(ax) \operatorname{Li}_2(e^{2i \sin^{-1}(ax)}) + 3a^2 \operatorname{Li}_3(e^{2i \sin^{-1}(ax)}) - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - 2ia^2 \sin^{-1}(ax)^3 + 6a^2 \sin^{-1}(ax)^2 \log$$

[Out]  $-2Ia^2 \arcsin(ax)^3 - 1/2 \arcsin(ax)^4/x^2 + 6a^2 \arcsin(ax)^2 \ln(1 - (Ia^2x + (-a^2x^2+1)^{(1/2)})^2) - 6Ia^2 \arcsin(ax) \operatorname{polylog}(2, (Ia^2x + (-a^2x^2+1)^{(1/2)})^2) + 3a^2 \operatorname{polylog}(3, (Ia^2x + (-a^2x^2+1)^{(1/2)})^2) - 2a^2 \arcsin(ax)^3 - (-a^2x^2+1)^{(1/2)}/x$

**Rubi [A]** time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$ , Rules used = {4627, 4681, 4625, 3717, 2190, 2531, 2282, 6589}

$$-6ia^2 \sin^{-1}(ax) \operatorname{PolyLog}(2, e^{2i \sin^{-1}(ax)}) + 3a^2 \operatorname{PolyLog}(3, e^{2i \sin^{-1}(ax)}) - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - 2ia^2 \sin^{-1}(ax)^3 + 6a^2 \sin^{-1}(ax)^2 \log$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^4/x^3,x]

[Out]  $(-2I)a^2 \operatorname{ArcSin}[a*x]^3 - (2a \operatorname{Sqrt}[1 - a^2x^2] \operatorname{ArcSin}[a*x]^3)/x - \operatorname{ArcSin}[a*x]^4/(2x^2) + 6a^2 \operatorname{ArcSin}[a*x]^2 \operatorname{Log}[1 - E^{((2I) \operatorname{ArcSin}[a*x])}] - (6I)a^2 \operatorname{ArcSin}[a*x] \operatorname{PolyLog}[2, E^{((2I) \operatorname{ArcSin}[a*x])}] + 3a^2 \operatorname{PolyLog}[3, E^{((2I) \operatorname{ArcSin}[a*x])}]$

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^(c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m-1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 3717

Int[(((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m+1))/(d\*(m+1)), x] - Dist[2\*I, Int[(c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))]/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*((d\_.)\*(x\_.))^m, x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4681

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*((f\_.)\*(x\_.))^m\*((d\_.) + (e\_.)\*(x\_)^2)^p, x\_Symbol] := Simp[((f\*x)^(m + 1)\*(d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*f\*(m + 1)), x] - Dist[(b\*c\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(f\*(m + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m + 1)\*(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && EqQ[m + 2\*p + 3, 0] && NeQ[m, -1]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^p]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
 \int \frac{\sin^{-1}(ax)^4}{x^3} dx &= -\frac{\sin^{-1}(ax)^4}{2x^2} + (2a) \int \frac{\sin^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}} dx \\
 &= -\frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - \frac{\sin^{-1}(ax)^4}{2x^2} + (6a^2) \int \frac{\sin^{-1}(ax)^2}{x} dx \\
 &= -\frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - \frac{\sin^{-1}(ax)^4}{2x^2} + (6a^2) \text{Subst} \left( \int x^2 \cot(x) dx, x, \sin^{-1}(ax) \right) \\
 &= -2ia^2 \sin^{-1}(ax)^3 - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - \frac{\sin^{-1}(ax)^4}{2x^2} - (12ia^2) \text{Subst} \left( \int \frac{e^{2ix}x^2}{1-e^{2ix}} dx, x, \sin^{-1}(ax) \right) \\
 &= -2ia^2 \sin^{-1}(ax)^3 - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - \frac{\sin^{-1}(ax)^4}{2x^2} + 6a^2 \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)}) \\
 &= -2ia^2 \sin^{-1}(ax)^3 - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - \frac{\sin^{-1}(ax)^4}{2x^2} + 6a^2 \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)}) \\
 &= -2ia^2 \sin^{-1}(ax)^3 - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - \frac{\sin^{-1}(ax)^4}{2x^2} + 6a^2 \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)}) \\
 &= -2ia^2 \sin^{-1}(ax)^3 - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{x} - \frac{\sin^{-1}(ax)^4}{2x^2} + 6a^2 \sin^{-1}(ax)^2 \log(1 - e^{2i \sin^{-1}(ax)})
 \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 124, normalized size = 1.04

$$-\frac{\sin^{-1}(ax)^4}{2x^2} + \frac{1}{4}a^2 \left( -\frac{8\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{ax} + 24i \sin^{-1}(ax) \text{Li}_2(e^{-2i \sin^{-1}(ax)}) + 12\text{Li}_3(e^{-2i \sin^{-1}(ax)}) + 8i \sin^{-1}(ax) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^4/x^3, x]

[Out]  $-1/2*\text{ArcSin}[a*x]^4/x^2 + (a^2*((-I)*\text{Pi}^3 + (8*I)*\text{ArcSin}[a*x]^3 - (8*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^3)/(a*x) + 24*\text{ArcSin}[a*x]^2*\text{Log}[1 - E^{((-2*I)*\text{ArcSin}[a*x])}] + (24*I)*\text{ArcSin}[a*x]*\text{PolyLog}[2, E^{((-2*I)*\text{ArcSin}[a*x])}] + 12*\text{PolyLog}[3, E^{((-2*I)*\text{ArcSin}[a*x])}])))/4$

**fricas** [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arcsin(ax)^4}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(a*x)^4/x^3,x, algorithm="fricas")`

[Out] `integral(arcsin(a*x)^4/x^3, x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^4}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(a*x)^4/x^3,x, algorithm="giac")`

[Out] `integrate(arcsin(a*x)^4/x^3, x)`

**maple** [A] time = 0.16, size = 227, normalized size = 1.91

$$-2ia^2 \arcsin(ax)^3 - \frac{2a \arcsin(ax)^3 \sqrt{-a^2x^2 + 1}}{x} - \frac{\arcsin(ax)^4}{2x^2} + 6a^2 \arcsin(ax)^2 \ln\left(1 + iax + \sqrt{-a^2x^2 + 1}\right) - 12ia^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(a*x)^4/x^3,x)`

[Out]  $-2*I*a^2*\arcsin(a*x)^3-2*a*\arcsin(a*x)^3*(-a^2*x^2+1)^{(1/2)}/x-1/2*\arcsin(a*x)^4/x^2+6*a^2*\arcsin(a*x)^2*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)})-12*I*a^2*\arcsin(a*x)*\text{polylog}(2,-I*a*x-(-a^2*x^2+1)^{(1/2)})+12*a^2*\text{polylog}(3,-I*a*x-(-a^2*x^2+1)^{(1/2)})+6*a^2*\arcsin(a*x)^2*\ln(1-I*a*x-(-a^2*x^2+1)^{(1/2)})-12*I*a^2*\arcsin(a*x)*\text{polylog}(2,I*a*x+(-a^2*x^2+1)^{(1/2)})+12*a^2*\text{polylog}(3,I*a*x+(-a^2*x^2+1)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^4 + \frac{1}{2}\left(\sqrt{ax+1}\sqrt{-ax+1}\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^3 + 8x \int \frac{7\sqrt{ax+1}\sqrt{-ax+1}}{2x^2}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(a*x)^4/x^3,x, algorithm="maxima")`

[Out]  $-1/2*(\arctan2(a*x, \text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1))^4 + 4*a*x^2*\text{integrate}(\text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1)*\arctan2(a*x, \text{sqrt}(a*x + 1)*\text{sqrt}(-a*x + 1))^3/(a^2*x^4 - x^2), x))/x^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{asin}(ax)^4}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(asin(a*x)^4/x^3,x)
```

```
[Out] int(asin(a*x)^4/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^4(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(a*x)**4/x**3,x)
```

```
[Out] Integral(asin(a*x)**4/x**3, x)
```

$$3.41 \quad \int \frac{\sin^{-1}(ax)^4}{x^4} dx$$

**Optimal.** Leaf size=276

$$2ia^3 \sin^{-1}(ax)^2 \text{Li}_2(-e^{i \sin^{-1}(ax)}) - 2ia^3 \sin^{-1}(ax)^2 \text{Li}_2(e^{i \sin^{-1}(ax)}) - 4a^3 \sin^{-1}(ax) \text{Li}_3(-e^{i \sin^{-1}(ax)}) + 4a^3 \sin^{-1}(ax) \text{Li}_3(e^{i \sin^{-1}(ax)})$$

```
[Out] -2*a^2*arcsin(a*x)^2/x-1/3*arcsin(a*x)^4/x^3-8*a^3*arcsin(a*x)*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))-4/3*a^3*arcsin(a*x)^3*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))+4*I*a^3*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))+2*I*a^3*arcsin(a*x)^2*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-4*I*a^3*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))-2*I*a^3*arcsin(a*x)^2*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))-4*a^3*arcsin(a*x)*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2))+4*a^3*arcsin(a*x)*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))-4*I*a^3*polylog(4,-I*a*x-(-a^2*x^2+1)^(1/2))+4*I*a^3*polylog(4,I*a*x+(-a^2*x^2+1)^(1/2))-2/3*a*arcsin(a*x)^3*(-a^2*x^2+1)^(1/2)/x^2
```

**Rubi [A]** time = 0.41, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {4627, 4701, 4709, 4183, 2531, 6609, 2282, 6589, 2279, 2391}

$$2ia^3 \sin^{-1}(ax)^2 \text{PolyLog}(2, -e^{i \sin^{-1}(ax)}) - 2ia^3 \sin^{-1}(ax)^2 \text{PolyLog}(2, e^{i \sin^{-1}(ax)}) - 4a^3 \sin^{-1}(ax) \text{PolyLog}(3, -e^{i \sin^{-1}(ax)}) + 4a^3 \sin^{-1}(ax) \text{PolyLog}(3, e^{i \sin^{-1}(ax)})$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[a*x]^4/x^4, x]
```

```
[Out] (-2*a^2*ArcSin[a*x]^2)/x - (2*a*Sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/(3*x^2) - ArcSin[a*x]^4/(3*x^3) - 8*a^3*ArcSin[a*x]*ArcTanh[E^(I*ArcSin[a*x])]/(3*x^2) - (4*a^3*ArcSin[a*x]^3*ArcTanh[E^(I*ArcSin[a*x])])/3 + (4*I)*a^3*PolyLog[2, -E^(I*ArcSin[a*x])]/(3*x^2) + (2*I)*a^3*ArcSin[a*x]^2*PolyLog[2, -E^(I*ArcSin[a*x])]/(3*x^2) - (4*I)*a^3*PolyLog[2, E^(I*ArcSin[a*x])]/(3*x^2) - (2*I)*a^3*ArcSin[a*x]^2*PolyLog[2, E^(I*ArcSin[a*x])]/(3*x^2) - 4*a^3*ArcSin[a*x]*PolyLog[3, -E^(I*ArcSin[a*x])]/(3*x^2) + 4*a^3*ArcSin[a*x]*PolyLog[3, E^(I*ArcSin[a*x])]/(3*x^2) - (4*I)*a^3*PolyLog[4, -E^(I*ArcSin[a*x])]/(3*x^2) + (4*I)*a^3*PolyLog[4, E^(I*ArcSin[a*x])]/(3*x^2)
```

**Rule 2279**

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

**Rule 2282**

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_.) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

**Rule 2391**

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

**Rule 2531**

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[(f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x]
```

)))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 4183

Int[csc[(e\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n]\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4701

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n]\*((f\_.)\*(x\_))^(m\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*f\*(m + 1)), x] + (Dist[(c^2\*(m + 2\*p + 3))/(f^2\*(m + 1)), Int[(f\*x)^(m + 2)\*(d + e\*x^2)^p\*(a + b\*ArcSin[c\*x])^n, x], x] - Dist[(b\*c\*n\*d\*IntPart[p]\*(d + e\*x^2)^FracPart[p])/(f\*(m + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(f\*x)^(m + 1)\*(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m]

#### Rule 4709

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n]\*((x\_)^(m\_))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Dist[1/(c^(m + 1)\*Sqrt[d]), Subst[Int[(a + b\*x)^n\*Sin[x]^m, x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(ax)^4}{x^4} dx &= -\frac{\sin^{-1}(ax)^4}{3x^3} + \frac{1}{3}(4a) \int \frac{\sin^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}} dx \\
&= -\frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{3x^2} - \frac{\sin^{-1}(ax)^4}{3x^3} + (2a^2) \int \frac{\sin^{-1}(ax)^2}{x^2} dx + \frac{1}{3} (2a^3) \int \frac{\sin^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} dx \\
&= -\frac{2a^2 \sin^{-1}(ax)^2}{x} - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{3x^2} - \frac{\sin^{-1}(ax)^4}{3x^3} + \frac{1}{3} (2a^3) \text{Subst} \left( \int x^3 \csc(x) dx, x, \right. \\
&= -\frac{2a^2 \sin^{-1}(ax)^2}{x} - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{3x^2} - \frac{\sin^{-1}(ax)^4}{3x^3} - \frac{4}{3} a^3 \sin^{-1}(ax)^3 \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{2a^2 \sin^{-1}(ax)^2}{x} - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{3x^2} - \frac{\sin^{-1}(ax)^4}{3x^3} - 8a^3 \sin^{-1}(ax) \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{2a^2 \sin^{-1}(ax)^2}{x} - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{3x^2} - \frac{\sin^{-1}(ax)^4}{3x^3} - 8a^3 \sin^{-1}(ax) \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{2a^2 \sin^{-1}(ax)^2}{x} - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{3x^2} - \frac{\sin^{-1}(ax)^4}{3x^3} - 8a^3 \sin^{-1}(ax) \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right) \\
&= -\frac{2a^2 \sin^{-1}(ax)^2}{x} - \frac{2a\sqrt{1-a^2x^2} \sin^{-1}(ax)^3}{3x^2} - \frac{\sin^{-1}(ax)^4}{3x^3} - 8a^3 \sin^{-1}(ax) \tanh^{-1} \left( e^{i \sin^{-1}(ax)} \right)
\end{aligned}$$

**Mathematica [A]** time = 3.97, size = 399, normalized size = 1.45

$$\frac{1}{24} a^3 \left( -\frac{8 \sin^4 \left( \frac{1}{2} \sin^{-1}(ax) \right) \sin^{-1}(ax)^4}{a^3 x^3} + 48i \sin^{-1}(ax)^2 \text{Li}_2 \left( e^{-i \sin^{-1}(ax)} \right) + 96 \sin^{-1}(ax) \text{Li}_3 \left( e^{-i \sin^{-1}(ax)} \right) - 96 \sin^{-1}(ax) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^4/x^4,x]

[Out] (a^3\*((-2\*I)\*Pi^4 + (4\*I)\*ArcSin[a\*x]^4 - 24\*ArcSin[a\*x]^2\*Cot[ArcSin[a\*x]/2] - 2\*ArcSin[a\*x]^4\*Cot[ArcSin[a\*x]/2] - 4\*ArcSin[a\*x]^3\*Csc[ArcSin[a\*x]/2]^2 - (a\*x\*ArcSin[a\*x]^4\*Csc[ArcSin[a\*x]/2]^4)/2 + 16\*ArcSin[a\*x]^3\*Log[1 - E^((-I)\*ArcSin[a\*x])] + 96\*ArcSin[a\*x]\*Log[1 - E^(I\*ArcSin[a\*x])] - 96\*ArcSin[a\*x]\*Log[1 + E^(I\*ArcSin[a\*x])] - 16\*ArcSin[a\*x]^3\*Log[1 + E^(I\*ArcSin[a\*x])]) + (48\*I)\*ArcSin[a\*x]^2\*PolyLog[2, E^((-I)\*ArcSin[a\*x])] + (48\*I)\*(2 + ArcSin[a\*x]^2)\*PolyLog[2, -E^(I\*ArcSin[a\*x])] - (96\*I)\*PolyLog[2, E^(I\*ArcSin[a\*x])] + 96\*ArcSin[a\*x]\*PolyLog[3, E^((-I)\*ArcSin[a\*x])] - 96\*ArcSin[a\*x]\*PolyLog[3, -E^(I\*ArcSin[a\*x])] - (96\*I)\*PolyLog[4, E^((-I)\*ArcSin[a\*x])] - (96\*I)\*PolyLog[4, -E^(I\*ArcSin[a\*x])] + 4\*ArcSin[a\*x]^3\*Sec[ArcSin[a\*x]/2]^2 - (8\*ArcSin[a\*x]^4\*Sin[ArcSin[a\*x]/2]^4)/(a^3\*x^3) - 24\*ArcSin[a\*x]^2\*Tan[ArcSin[a\*x]/2] - 2\*ArcSin[a\*x]^4\*Tan[ArcSin[a\*x]/2]))/24

**fricas [F]** time = 1.06, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\arcsin(ax)^4}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x^4,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^4/x^4, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^4}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x^4,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^4/x^4, x)

**maple** [A] time = 0.22, size = 409, normalized size = 1.48

$$\frac{2a \arcsin(ax)^3 \sqrt{-a^2x^2+1}}{3x^2} - \frac{2a^2 \arcsin(ax)^2}{x} - \frac{\arcsin(ax)^4}{3x^3} - \frac{2a^3 \arcsin(ax)^3 \ln\left(1+iax+\sqrt{-a^2x^2+1}\right)}{3} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^4/x^4,x)

[Out]  $-2/3*a*\arcsin(a*x)^3*(-a^2*x^2+1)^{(1/2)}/x^2-2*a^2*\arcsin(a*x)^2/x-1/3*\arcsin(a*x)^4/x^3-2/3*a^3*\arcsin(a*x)^3*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)})+2*I*a^3*\arcsin(a*x)^2*\operatorname{polylog}(2,-I*a*x-(-a^2*x^2+1)^{(1/2)})-4*a^3*\arcsin(a*x)*\operatorname{polylog}(3,-I*a*x-(-a^2*x^2+1)^{(1/2)})-4*I*a^3*\operatorname{polylog}(4,-I*a*x-(-a^2*x^2+1)^{(1/2)})+2/3*a^3*\arcsin(a*x)^3*\ln(1-I*a*x-(-a^2*x^2+1)^{(1/2)})-2*I*a^3*\arcsin(a*x)^2*\operatorname{polylog}(2,I*a*x+(-a^2*x^2+1)^{(1/2)})+4*a^3*\arcsin(a*x)*\operatorname{polylog}(3,I*a*x+(-a^2*x^2+1)^{(1/2)})+4*I*a^3*\operatorname{polylog}(4,I*a*x+(-a^2*x^2+1)^{(1/2)})-4*a^3*\arcsin(a*x)*\ln(1+I*a*x+(-a^2*x^2+1)^{(1/2)})+4*I*a^3*\operatorname{polylog}(2,-I*a*x-(-a^2*x^2+1)^{(1/2)})+4*a^3*\arcsin(a*x)*\ln(1-I*a*x-(-a^2*x^2+1)^{(1/2)})-4*I*a^3*\operatorname{polylog}(2,I*a*x+(-a^2*x^2+1)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4ax^3 \int \frac{\sqrt{ax+1}\sqrt{-ax+1} \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^3}{a^2x^5-x^3} dx + \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^4/x^4,x, algorithm="maxima")

[Out]  $-1/3*(12*a*x^3*\operatorname{integrate}(1/3*\sqrt{a*x+1}*\sqrt{-a*x+1}*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^3/(a^2*x^5-x^3), x) + \arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^4)/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asin}(ax)^4}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^4/x^4,x)

[Out] int(asin(a\*x)^4/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^4(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*4/x\*\*4,x)

[Out] Integral(asin(a\*x)\*\*4/x\*\*4, x)

$$3.42 \quad \int \frac{x^6}{\sin^{-1}(ax)} dx$$

Optimal. Leaf size=55

$$\frac{5\text{Ci}(\sin^{-1}(ax))}{64a^7} - \frac{9\text{Ci}(3\sin^{-1}(ax))}{64a^7} + \frac{5\text{Ci}(5\sin^{-1}(ax))}{64a^7} - \frac{\text{Ci}(7\sin^{-1}(ax))}{64a^7}$$

[Out] 5/64\*Ci(arcsin(a\*x))/a^7-9/64\*Ci(3\*arcsin(a\*x))/a^7+5/64\*Ci(5\*arcsin(a\*x))/a^7-1/64\*Ci(7\*arcsin(a\*x))/a^7

**Rubi [A]** time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4635, 4406, 3302}

$$\frac{5\text{CosIntegral}(\sin^{-1}(ax))}{64a^7} - \frac{9\text{CosIntegral}(3\sin^{-1}(ax))}{64a^7} + \frac{5\text{CosIntegral}(5\sin^{-1}(ax))}{64a^7} - \frac{\text{CosIntegral}(7\sin^{-1}(ax))}{64a^7}$$

Antiderivative was successfully verified.

[In] Int[x^6/ArcSin[a\*x],x]

[Out] (5\*CosIntegral[ArcSin[a\*x]])/(64\*a^7) - (9\*CosIntegral[3\*ArcSin[a\*x]])/(64\*a^7) + (5\*CosIntegral[5\*ArcSin[a\*x]])/(64\*a^7) - CosIntegral[7\*ArcSin[a\*x]]/(64\*a^7)

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n\*Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{x^6}{\sin^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^6(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^7} \\ &= \frac{\text{Subst}\left(\int \left(\frac{5\cos(x)}{64x} - \frac{9\cos(3x)}{64x} + \frac{5\cos(5x)}{64x} - \frac{\cos(7x)}{64x}\right) dx, x, \sin^{-1}(ax)\right)}{a^7} \\ &= -\frac{\text{Subst}\left(\int \frac{\cos(7x)}{x} dx, x, \sin^{-1}(ax)\right)}{64a^7} + \frac{5\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{64a^7} + \frac{5\text{Subst}\left(\int \frac{\cos(5x)}{x} dx, x, \sin^{-1}(ax)\right)}{64a^7} \\ &= \frac{5\text{Ci}(\sin^{-1}(ax))}{64a^7} - \frac{9\text{Ci}(3\sin^{-1}(ax))}{64a^7} + \frac{5\text{Ci}(5\sin^{-1}(ax))}{64a^7} - \frac{\text{Ci}(7\sin^{-1}(ax))}{64a^7} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.73

$$\frac{-5\text{Ci}(\sin^{-1}(ax)) + 9\text{Ci}(3\sin^{-1}(ax)) - 5\text{Ci}(5\sin^{-1}(ax)) + \text{Ci}(7\sin^{-1}(ax))}{64a^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/ArcSin[a\*x], x]

[Out] -1/64\*(-5\*CosIntegral[ArcSin[a\*x]] + 9\*CosIntegral[3\*ArcSin[a\*x]] - 5\*CosIntegral[5\*ArcSin[a\*x]] + CosIntegral[7\*ArcSin[a\*x]])/a^7

**fricas [F]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x), x, algorithm="fricas")

[Out] integral(x^6/arcsin(a\*x), x)

**giac [A]** time = 0.15, size = 47, normalized size = 0.85

$$-\frac{\text{Ci}(7\arcsin(ax))}{64a^7} + \frac{5\text{Ci}(5\arcsin(ax))}{64a^7} - \frac{9\text{Ci}(3\arcsin(ax))}{64a^7} + \frac{5\text{Ci}(\arcsin(ax))}{64a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x), x, algorithm="giac")

[Out] -1/64\*cos\_integral(7\*arcsin(a\*x))/a^7 + 5/64\*cos\_integral(5\*arcsin(a\*x))/a^7 - 9/64\*cos\_integral(3\*arcsin(a\*x))/a^7 + 5/64\*cos\_integral(arcsin(a\*x))/a^7

**maple [A]** time = 0.06, size = 40, normalized size = 0.73

$$\frac{\frac{5\text{Ci}(\arcsin(ax))}{64} - \frac{9\text{Ci}(3\arcsin(ax))}{64} + \frac{5\text{Ci}(5\arcsin(ax))}{64} - \frac{\text{Ci}(7\arcsin(ax))}{64}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/arcsin(a\*x), x)

[Out] 1/a^7\*(5/64\*Ci(arcsin(a\*x))-9/64\*Ci(3\*arcsin(a\*x))+5/64\*Ci(5\*arcsin(a\*x))-1/64\*Ci(7\*arcsin(a\*x)))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x), x, algorithm="maxima")

[Out] integrate(x^6/arcsin(a\*x), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/asin(a*x),x)
```

```
[Out] int(x^6/asin(a*x), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^6}{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/asin(a*x),x)
```

```
[Out] Integral(x**6/asin(a*x), x)
```



$$3.43 \quad \int \frac{x^5}{\sin^{-1}(ax)} dx$$

Optimal. Leaf size=43

$$\frac{5\text{Si}\left(2\sin^{-1}(ax)\right)}{32a^6} - \frac{\text{Si}\left(4\sin^{-1}(ax)\right)}{8a^6} + \frac{\text{Si}\left(6\sin^{-1}(ax)\right)}{32a^6}$$

[Out] 5/32\*Si(2\*arcsin(a\*x))/a^6-1/8\*Si(4\*arcsin(a\*x))/a^6+1/32\*Si(6\*arcsin(a\*x))/a^6

**Rubi [A]** time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4635, 4406, 3299}

$$\frac{5\text{Si}\left(2\sin^{-1}(ax)\right)}{32a^6} - \frac{\text{Si}\left(4\sin^{-1}(ax)\right)}{8a^6} + \frac{\text{Si}\left(6\sin^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

[In] Int[x^5/ArcSin[a\*x],x]

[Out] (5\*SinIntegral[2\*ArcSin[a\*x]])/(32\*a^6) - SinIntegral[4\*ArcSin[a\*x]]/(8\*a^6) + SinIntegral[6\*ArcSin[a\*x]]/(32\*a^6)

Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_.)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n \* Sin[x]^m \* Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{\sin^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^5(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^6} \\ &= \frac{\text{Subst}\left(\int \left(\frac{5\sin(2x)}{32x} - \frac{\sin(4x)}{8x} + \frac{\sin(6x)}{32x}\right) dx, x, \sin^{-1}(ax)\right)}{a^6} \\ &= \frac{\text{Subst}\left(\int \frac{\sin(6x)}{x} dx, x, \sin^{-1}(ax)\right)}{32a^6} - \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x} dx, x, \sin^{-1}(ax)\right)}{8a^6} + \frac{5\text{Subst}\left(\int \frac{\sin(2x)}{x} dx, x, \sin^{-1}(ax)\right)}{32a^6} \\ &= \frac{5\text{Si}\left(2\sin^{-1}(ax)\right)}{32a^6} - \frac{\text{Si}\left(4\sin^{-1}(ax)\right)}{8a^6} + \frac{\text{Si}\left(6\sin^{-1}(ax)\right)}{32a^6} \end{aligned}$$

**Mathematica** [A] time = 0.12, size = 33, normalized size = 0.77

$$\frac{5\text{Si}\left(2\sin^{-1}(ax)\right) - 4\text{Si}\left(4\sin^{-1}(ax)\right) + \text{Si}\left(6\sin^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/ArcSin[a\*x],x]

[Out] (5\*SinIntegral[2\*ArcSin[a\*x]] - 4\*SinIntegral[4\*ArcSin[a\*x]] + SinIntegral[6\*ArcSin[a\*x]])/(32\*a^6)

**fricas** [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x),x, algorithm="fricas")

[Out] integral(x^5/arcsin(a\*x), x)

**giac** [A] time = 0.14, size = 37, normalized size = 0.86

$$\frac{\text{Si}(6 \arcsin(ax))}{32 a^6} - \frac{\text{Si}(4 \arcsin(ax))}{8 a^6} + \frac{5 \text{Si}(2 \arcsin(ax))}{32 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x),x, algorithm="giac")

[Out] 1/32\*sin\_integral(6\*arcsin(a\*x))/a^6 - 1/8\*sin\_integral(4\*arcsin(a\*x))/a^6 + 5/32\*sin\_integral(2\*arcsin(a\*x))/a^6

**maple** [A] time = 0.06, size = 33, normalized size = 0.77

$$\frac{\frac{5\text{Si}(2\arcsin(ax))}{32} - \frac{\text{Si}(4\arcsin(ax))}{8} + \frac{\text{Si}(6\arcsin(ax))}{32}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/arcsin(a\*x),x)

[Out] 1/a^6\*(5/32\*Si(2\*arcsin(a\*x))-1/8\*Si(4\*arcsin(a\*x))+1/32\*Si(6\*arcsin(a\*x)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(x^5/arcsin(a\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/asin(a*x),x)
```

```
[Out] int(x^5/asin(a*x), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^5}{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/asin(a*x),x)
```

```
[Out] Integral(x**5/asin(a*x), x)
```

$$3.44 \quad \int \frac{x^4}{\sin^{-1}(ax)} dx$$

Optimal. Leaf size=41

$$\frac{\text{Ci}(\sin^{-1}(ax))}{8a^5} - \frac{3\text{Ci}(3\sin^{-1}(ax))}{16a^5} + \frac{\text{Ci}(5\sin^{-1}(ax))}{16a^5}$$

[Out] 1/8\*Ci(arcsin(a\*x))/a^5-3/16\*Ci(3\*arcsin(a\*x))/a^5+1/16\*Ci(5\*arcsin(a\*x))/a^5

**Rubi [A]** time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4635, 4406, 3302}

$$\frac{\text{CosIntegral}(\sin^{-1}(ax))}{8a^5} - \frac{3\text{CosIntegral}(3\sin^{-1}(ax))}{16a^5} + \frac{\text{CosIntegral}(5\sin^{-1}(ax))}{16a^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcSin[a\*x],x]

[Out] CosIntegral[ArcSin[a\*x]]/(8\*a^5) - (3\*CosIntegral[3\*ArcSin[a\*x]])/(16\*a^5) + CosIntegral[5\*ArcSin[a\*x]]/(16\*a^5)

Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n\*Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{\sin^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^4(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^5} \\ &= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{8x} - \frac{3\cos(3x)}{16x} + \frac{\cos(5x)}{16x}\right) dx, x, \sin^{-1}(ax)\right)}{a^5} \\ &= \frac{\text{Subst}\left(\int \frac{\cos(5x)}{x} dx, x, \sin^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{8a^5} - \frac{3\text{Subst}\left(\int \frac{\cos(3x)}{x} dx, x, \sin^{-1}(ax)\right)}{16a^5} \\ &= \frac{\text{Ci}(\sin^{-1}(ax))}{8a^5} - \frac{3\text{Ci}(3\sin^{-1}(ax))}{16a^5} + \frac{\text{Ci}(5\sin^{-1}(ax))}{16a^5} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 0.76

$$\frac{2\text{Ci}(\sin^{-1}(ax)) - 3\text{Ci}(3\sin^{-1}(ax)) + \text{Ci}(5\sin^{-1}(ax))}{16a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcSin[a\*x],x]

[Out] (2\*CosIntegral[ArcSin[a\*x]] - 3\*CosIntegral[3\*ArcSin[a\*x]] + CosIntegral[5\*ArcSin[a\*x]])/(16\*a^5)

**fricas [F]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x),x, algorithm="fricas")

[Out] integral(x^4/arcsin(a\*x), x)

**giac [A]** time = 0.13, size = 35, normalized size = 0.85

$$\frac{\text{Ci}(5 \arcsin(ax))}{16 a^5} - \frac{3 \text{Ci}(3 \arcsin(ax))}{16 a^5} + \frac{\text{Ci}(\arcsin(ax))}{8 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x),x, algorithm="giac")

[Out] 1/16\*cos\_integral(5\*arcsin(a\*x))/a^5 - 3/16\*cos\_integral(3\*arcsin(a\*x))/a^5 + 1/8\*cos\_integral(arcsin(a\*x))/a^5

**maple [A]** time = 0.03, size = 31, normalized size = 0.76

$$\frac{\frac{\text{Ci}(\arcsin(ax))}{8} - \frac{3\text{Ci}(3\arcsin(ax))}{16} + \frac{\text{Ci}(5\arcsin(ax))}{16}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arcsin(a\*x),x)

[Out] 1/a^5\*(1/8\*Ci(arcsin(a\*x))-3/16\*Ci(3\*arcsin(a\*x))+1/16\*Ci(5\*arcsin(a\*x)))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(x^4/arcsin(a\*x), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asin(a*x),x)
```

```
[Out] int(x^4/asin(a*x), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^4}{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asin(a*x),x)
```

```
[Out] Integral(x**4/asin(a*x), x)
```

$$3.45 \quad \int \frac{x^3}{\sin^{-1}(ax)} dx$$

Optimal. Leaf size=29

$$\frac{\text{Si}\left(2 \sin^{-1}(ax)\right)}{4a^4} - \frac{\text{Si}\left(4 \sin^{-1}(ax)\right)}{8a^4}$$

[Out] 1/4\*Si(2\*arcsin(a\*x))/a^4-1/8\*Si(4\*arcsin(a\*x))/a^4

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4635, 4406, 3299}

$$\frac{\text{Si}\left(2 \sin^{-1}(ax)\right)}{4a^4} - \frac{\text{Si}\left(4 \sin^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcSin[a\*x], x]

[Out] SinIntegral[2\*ArcSin[a\*x]]/(4\*a^4) - SinIntegral[4\*ArcSin[a\*x]]/(8\*a^4)

Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n\*Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sin^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^3(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^4} \\ &= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x} - \frac{\sin(4x)}{8x}\right) dx, x, \sin^{-1}(ax)\right)}{a^4} \\ &= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{x} dx, x, \sin^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x} dx, x, \sin^{-1}(ax)\right)}{4a^4} \\ &= \frac{\text{Si}\left(2 \sin^{-1}(ax)\right)}{4a^4} - \frac{\text{Si}\left(4 \sin^{-1}(ax)\right)}{8a^4} \end{aligned}$$

Mathematica [A] time = 0.08, size = 24, normalized size = 0.83

$$\frac{\text{Si}\left(4 \sin^{-1}(ax)\right) - 2\text{Si}\left(2 \sin^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcSin[a\*x],x]

[Out] -1/8\*(-2\*SinIntegral[2\*ArcSin[a\*x]] + SinIntegral[4\*ArcSin[a\*x]])/a^4

**fricas** [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x),x, algorithm="fricas")

[Out] integral(x^3/arcsin(a\*x), x)

**giac** [A] time = 0.15, size = 25, normalized size = 0.86

$$-\frac{\text{Si}(4 \arcsin(ax))}{8 a^4} + \frac{\text{Si}(2 \arcsin(ax))}{4 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x),x, algorithm="giac")

[Out] -1/8\*sin\_integral(4\*arcsin(a\*x))/a^4 + 1/4\*sin\_integral(2\*arcsin(a\*x))/a^4

**maple** [A] time = 0.03, size = 24, normalized size = 0.83

$$\frac{\frac{\text{Si}(2 \arcsin(ax))}{4} - \frac{\text{Si}(4 \arcsin(ax))}{8}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arcsin(a\*x),x)

[Out] 1/a^4\*(1/4\*Si(2\*arcsin(a\*x))-1/8\*Si(4\*arcsin(a\*x)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(x^3/arcsin(a\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/asin(a\*x),x)

[Out] int(x^3/asin(a\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\text{asin}(ax)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asin(a*x),x)
```

```
[Out] Integral(x**3/asin(a*x), x)
```

$$3.46 \quad \int \frac{x^2}{\sin^{-1}(ax)} dx$$

Optimal. Leaf size=27

$$\frac{\text{Ci}(\sin^{-1}(ax))}{4a^3} - \frac{\text{Ci}(3\sin^{-1}(ax))}{4a^3}$$

[Out] 1/4\*Ci(arcsin(a\*x))/a^3-1/4\*Ci(3\*arcsin(a\*x))/a^3

**Rubi [A]** time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4635, 4406, 3302}

$$\frac{\text{CosIntegral}(\sin^{-1}(ax))}{4a^3} - \frac{\text{CosIntegral}(3\sin^{-1}(ax))}{4a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcSin[a\*x],x]

[Out] CosIntegral[ArcSin[a\*x]]/(4\*a^3) - CosIntegral[3\*ArcSin[a\*x]]/(4\*a^3)

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n\*Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sin^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^3} \\ &= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{4x} - \frac{\cos(3x)}{4x}\right) dx, x, \sin^{-1}(ax)\right)}{a^3} \\ &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int \frac{\cos(3x)}{x} dx, x, \sin^{-1}(ax)\right)}{4a^3} \\ &= \frac{\text{Ci}(\sin^{-1}(ax))}{4a^3} - \frac{\text{Ci}(3\sin^{-1}(ax))}{4a^3} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 0.81

$$\frac{\text{Ci}(\sin^{-1}(ax)) - \text{Ci}(3\sin^{-1}(ax))}{4a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/ArcSin[a\*x],x]

[Out] (CosIntegral[ArcSin[a\*x]] - CosIntegral[3\*ArcSin[a\*x]])/(4\*a^3)

**fricas** [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x),x, algorithm="fricas")

[Out] integral(x^2/arcsin(a\*x), x)

**giac** [A] time = 0.16, size = 23, normalized size = 0.85

$$-\frac{\text{Ci}(3 \arcsin(ax))}{4 a^3} + \frac{\text{Ci}(\arcsin(ax))}{4 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x),x, algorithm="giac")

[Out] -1/4\*cos\_integral(3\*arcsin(a\*x))/a^3 + 1/4\*cos\_integral(arcsin(a\*x))/a^3

**maple** [A] time = 0.03, size = 22, normalized size = 0.81

$$\frac{\frac{\text{Ci}(\arcsin(ax))}{4} - \frac{\text{Ci}(3 \arcsin(ax))}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arcsin(a\*x),x)

[Out] 1/a^3\*(1/4\*Ci(arcsin(a\*x))-1/4\*Ci(3\*arcsin(a\*x)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(x^2/arcsin(a\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/asin(a\*x),x)

[Out] int(x^2/asin(a\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asin(a*x),x)
```

```
[Out] Integral(x**2/asin(a*x), x)
```

$$3.47 \quad \int \frac{x}{\sin^{-1}(ax)} dx$$

Optimal. Leaf size=14

$$\frac{\text{Si}(2 \sin^{-1}(ax))}{2a^2}$$

[Out] 1/2\*Si(2\*arcsin(a\*x))/a^2

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4635, 4406, 12, 3299}

$$\frac{\text{Si}(2 \sin^{-1}(ax))}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSin[a\*x], x]

[Out] SinIntegral[2\*ArcSin[a\*x]]/(2\*a^2)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n \* Sin[x]^m \* Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sin^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^2} \\ &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2x} dx, x, \sin^{-1}(ax)\right)}{a^2} \\ &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x} dx, x, \sin^{-1}(ax)\right)}{2a^2} \\ &= \frac{\text{Si}(2 \sin^{-1}(ax))}{2a^2} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 14, normalized size = 1.00

$$\frac{\text{Si}\left(2 \sin^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcSin[a\*x],x]

[Out] SinIntegral[2\*ArcSin[a\*x]]/(2\*a^2)

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x),x, algorithm="fricas")

[Out] integral(x/arcsin(a\*x), x)

**giac** [A] time = 0.15, size = 12, normalized size = 0.86

$$\frac{\text{Si}\left(2 \arcsin(ax)\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x),x, algorithm="giac")

[Out] 1/2\*sin\_integral(2\*arcsin(a\*x))/a^2

**maple** [A] time = 0.04, size = 13, normalized size = 0.93

$$\frac{\text{Si}\left(2 \arcsin(ax)\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arcsin(a\*x),x)

[Out] 1/2\*Si(2\*arcsin(a\*x))/a^2

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(x/arcsin(a\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/asin(a\*x),x)

[Out] int(x/asin(a\*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/asin(a\*x),x)

[Out] Integral(x/asin(a\*x), x)

$$3.48 \quad \int \frac{1}{\sin^{-1}(ax)} dx$$

**Optimal.** Leaf size=9

$$\frac{\text{Ci}(\sin^{-1}(ax))}{a}$$

[Out] Ci(arcsin(a\*x))/a

**Rubi [A]** time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4623, 3302}

$$\frac{\text{CosIntegral}(\sin^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(-1), x]

[Out] CosIntegral[ArcSin[a\*x]]/a

**Rule 3302**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

**Rule 4623**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{\sin^{-1}(ax)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a} \\ &= \frac{\text{Ci}(\sin^{-1}(ax))}{a} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 9, normalized size = 1.00

$$\frac{\text{Ci}(\sin^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^(-1), x]

[Out] CosIntegral[ArcSin[a\*x]]/a

**fricas [F]** time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/arcsin(a\*x),x, algorithm="fricas")

[Out] integral(1/arcsin(a\*x), x)

**giac** [A] time = 0.18, size = 9, normalized size = 1.00

$$\frac{\text{Ci}(\arcsin(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x),x, algorithm="giac")

[Out] cos\_integral(arcsin(a\*x))/a

**maple** [A] time = 0.02, size = 10, normalized size = 1.11

$$\frac{\text{Ci}(\arcsin(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arcsin(a\*x),x)

[Out] Ci(arcsin(a\*x))/a

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(1/arcsin(a\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/asin(a\*x),x)

[Out] int(1/asin(a\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\text{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/asin(a\*x),x)

[Out] Integral(1/asin(a\*x), x)

$$3.49 \quad \int \frac{1}{x \sin^{-1}(ax)} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x \sin^{-1}(ax)}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x), x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x \sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*ArcSin[a\*x]), x]

[Out] Defer[Int][1/(x\*ArcSin[a\*x]), x]

Rubi steps

$$\int \frac{1}{x \sin^{-1}(ax)} dx = \int \frac{1}{x \sin^{-1}(ax)} dx$$

**Mathematica [A]** time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*ArcSin[a\*x]), x]

[Out] Integrate[1/(x\*ArcSin[a\*x]), x]

**fricas [A]** time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x \arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x), x, algorithm="fricas")

[Out] integral(1/(x\*arcsin(a\*x)), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x), x, algorithm="giac")

[Out] integrate(1/(x\*arcsin(a\*x)), x)

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x),x)

[Out] int(1/x/arcsin(a\*x),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(1/(x\*arcsin(a\*x)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)),x)

[Out] int(1/(x\*asin(a\*x)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x),x)

[Out] Integral(1/(x\*asin(a\*x)), x)

$$3.50 \quad \int \frac{1}{x^2 \sin^{-1}(ax)} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sin^{-1}(ax)}, x\right)$$

[Out] Unintegrable(1/x^2/arcsin(a\*x), x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 \sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*ArcSin[a\*x]), x]

[Out] Defer[Int][1/(x^2\*ArcSin[a\*x]), x]

Rubi steps

$$\int \frac{1}{x^2 \sin^{-1}(ax)} dx = \int \frac{1}{x^2 \sin^{-1}(ax)} dx$$

Mathematica [A] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*ArcSin[a\*x]), x]

[Out] Integrate[1/(x^2\*ArcSin[a\*x]), x]

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x), x, algorithm="fricas")

[Out] integral(1/(x^2\*arcsin(a\*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x), x, algorithm="giac")

[Out] integrate(1/(x^2\*arcsin(a\*x)), x)

**maple** [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arcsin(a\*x),x)

[Out] int(1/x^2/arcsin(a\*x),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate(1/(x^2\*arcsin(a\*x)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*asin(a\*x)),x)

[Out] int(1/(x^2\*asin(a\*x)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/asin(a\*x),x)

[Out] Integral(1/(x\*\*2\*asin(a\*x)), x)

$$3.51 \quad \int \frac{x^6}{\sin^{-1}(ax)^2} dx$$

**Optimal.** Leaf size=83

$$-\frac{5\text{Si}(\sin^{-1}(ax))}{64a^7} + \frac{27\text{Si}(3\sin^{-1}(ax))}{64a^7} - \frac{25\text{Si}(5\sin^{-1}(ax))}{64a^7} + \frac{7\text{Si}(7\sin^{-1}(ax))}{64a^7} - \frac{x^6\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

[Out]  $-5/64*\text{Si}(\arcsin(a*x))/a^7+27/64*\text{Si}(3*\arcsin(a*x))/a^7-25/64*\text{Si}(5*\arcsin(a*x))/a^7+7/64*\text{Si}(7*\arcsin(a*x))/a^7-x^6*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)$

**Rubi [A]** time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4631, 3299}

$$-\frac{5\text{Si}(\sin^{-1}(ax))}{64a^7} + \frac{27\text{Si}(3\sin^{-1}(ax))}{64a^7} - \frac{25\text{Si}(5\sin^{-1}(ax))}{64a^7} + \frac{7\text{Si}(7\sin^{-1}(ax))}{64a^7} - \frac{x^6\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^6/\text{ArcSin}[a*x]^2, x]$

[Out]  $-(x^6*\text{Sqrt}[1 - a^2*x^2])/(a*\text{ArcSin}[a*x]) - (5*\text{SinIntegral}[\text{ArcSin}[a*x]])/(64*a^7) + (27*\text{SinIntegral}[3*\text{ArcSin}[a*x]])/(64*a^7) - (25*\text{SinIntegral}[5*\text{ArcSin}[a*x]])/(64*a^7) + (7*\text{SinIntegral}[7*\text{ArcSin}[a*x]])/(64*a^7)$

**Rule 3299**

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

**Rule 4631**

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(x^m*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^{(n+1)})/(b*c*(n+1)), x] - \text{Dist}[1/(b*c^{(m+1)}*(n+1)), \text{Subst}[\text{Int}[\text{ExpandTrigReduce}[(a + b*x)^{(n+1)}, \text{Sin}[x]^{(m-1)}*(m - (m+1)*\text{Sin}[x]^2), x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -2] \ \&\& \ \text{LtQ}[n, -1]$

**Rubi steps**

$$\begin{aligned} \int \frac{x^6}{\sin^{-1}(ax)^2} dx &= -\frac{x^6\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{5\sin(x)}{64x} + \frac{27\sin(3x)}{64x} - \frac{25\sin(5x)}{64x} + \frac{7\sin(7x)}{64x}\right) dx, x, \sin^{-1}(ax)\right)}{a^7} \\ &= -\frac{x^6\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} - \frac{5\text{Subst}\left(\int\frac{\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{64a^7} + \frac{7\text{Subst}\left(\int\frac{\sin(7x)}{x} dx, x, \sin^{-1}(ax)\right)}{64a^7} - \frac{25\text{Subst}\left(\int\frac{\sin(5x)}{x} dx, x, \sin^{-1}(ax)\right)}{64a^7} \\ &= -\frac{x^6\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} - \frac{5\text{Si}(\sin^{-1}(ax))}{64a^7} + \frac{27\text{Si}(3\sin^{-1}(ax))}{64a^7} - \frac{25\text{Si}(5\sin^{-1}(ax))}{64a^7} + \frac{7\text{Si}(7\sin^{-1}(ax))}{64a^7} \end{aligned}$$

**Mathematica [A]** time = 0.25, size = 86, normalized size = 1.04

$$\frac{64a^6x^6\sqrt{1-a^2x^2} + 5\sin^{-1}(ax)\text{Si}(\sin^{-1}(ax)) - 27\sin^{-1}(ax)\text{Si}(3\sin^{-1}(ax)) + 25\sin^{-1}(ax)\text{Si}(5\sin^{-1}(ax)) - 7\sin^{-1}(ax)\text{Si}(7\sin^{-1}(ax))}{64a^7\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/ArcSin[a\*x]^2,x]

[Out]  $-1/64*(64*a^6*x^6*\text{Sqrt}[1 - a^2*x^2] + 5*\text{ArcSin}[a*x]*\text{SinIntegral}[\text{ArcSin}[a*x]] - 27*\text{ArcSin}[a*x]*\text{SinIntegral}[3*\text{ArcSin}[a*x]] + 25*\text{ArcSin}[a*x]*\text{SinIntegral}[5*\text{ArcSin}[a*x]] - 7*\text{ArcSin}[a*x]*\text{SinIntegral}[7*\text{ArcSin}[a*x]])/(a^7*\text{ArcSin}[a*x])$

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(x^6/arcsin(a\*x)^2, x)

**giac** [B] time = 0.15, size = 161, normalized size = 1.94

$$\frac{(a^2x^2 - 1)^3 \sqrt{-a^2x^2 + 1}}{a^7 \arcsin(ax)} - \frac{3(a^2x^2 - 1)^2 \sqrt{-a^2x^2 + 1}}{a^7 \arcsin(ax)} + \frac{7 \text{Si}(7 \arcsin(ax))}{64 a^7} - \frac{25 \text{Si}(5 \arcsin(ax))}{64 a^7} + \frac{27 \text{Si}(3 \arcsin(ax))}{64 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x)^2,x, algorithm="giac")

[Out]  $-(a^2*x^2 - 1)^3*\text{sqrt}(-a^2*x^2 + 1)/(a^7*\text{arcsin}(a*x)) - 3*(a^2*x^2 - 1)^2*\text{sqrt}(-a^2*x^2 + 1)/(a^7*\text{arcsin}(a*x)) + 7/64*\text{sin\_integral}(7*\text{arcsin}(a*x))/a^7 - 25/64*\text{sin\_integral}(5*\text{arcsin}(a*x))/a^7 + 27/64*\text{sin\_integral}(3*\text{arcsin}(a*x))/a^7 - 5/64*\text{sin\_integral}(\text{arcsin}(a*x))/a^7 + 3*(-a^2*x^2 + 1)^(3/2)/(a^7*\text{arcsin}(a*x)) - \text{sqrt}(-a^2*x^2 + 1)/(a^7*\text{arcsin}(a*x))$

**maple** [A] time = 0.07, size = 105, normalized size = 1.27

$$\frac{-\frac{5\sqrt{-a^2x^2+1}}{64 \arcsin(ax)} - \frac{5 \text{Si}(\arcsin(ax))}{64} + \frac{9 \cos(3 \arcsin(ax))}{64 \arcsin(ax)} + \frac{27 \text{Si}(3 \arcsin(ax))}{64} - \frac{5 \cos(5 \arcsin(ax))}{64 \arcsin(ax)} - \frac{25 \text{Si}(5 \arcsin(ax))}{64} + \frac{\cos(7 \arcsin(ax))}{64 \arcsin(ax)}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/arcsin(a\*x)^2,x)

[Out]  $1/a^7*(-5/64/\text{arcsin}(a*x)*(-a^2*x^2+1)^(1/2)-5/64*\text{Si}(\text{arcsin}(a*x))+9/64/\text{arcsin}(a*x)*\cos(3*\text{arcsin}(a*x))+27/64*\text{Si}(3*\text{arcsin}(a*x))-5/64/\text{arcsin}(a*x)*\cos(5*\text{arcsin}(a*x))-25/64*\text{Si}(5*\text{arcsin}(a*x))+1/64/\text{arcsin}(a*x)*\cos(7*\text{arcsin}(a*x))+7/64*\text{Si}(7*\text{arcsin}(a*x)))$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\text{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/asin(a*x)^2,x)
```

```
[Out] int(x^6/asin(a*x)^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^6}{\operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/asin(a*x)**2,x)
```

```
[Out] Integral(x**6/asin(a*x)**2, x)
```



$$3.52 \quad \int \frac{x^5}{\sin^{-1}(ax)^2} dx$$

**Optimal.** Leaf size=71

$$\frac{5\text{Ci}\left(2\sin^{-1}(ax)\right)}{16a^6} - \frac{\text{Ci}\left(4\sin^{-1}(ax)\right)}{2a^6} + \frac{3\text{Ci}\left(6\sin^{-1}(ax)\right)}{16a^6} - \frac{x^5\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

[Out] 5/16\*Ci(2\*arcsin(a\*x))/a^6-1/2\*Ci(4\*arcsin(a\*x))/a^6+3/16\*Ci(6\*arcsin(a\*x))/a^6-x^5\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)

**Rubi [A]** time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4631, 3302}

$$\frac{5\text{CosIntegral}\left(2\sin^{-1}(ax)\right)}{16a^6} - \frac{\text{CosIntegral}\left(4\sin^{-1}(ax)\right)}{2a^6} + \frac{3\text{CosIntegral}\left(6\sin^{-1}(ax)\right)}{16a^6} - \frac{x^5\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^5/ArcSin[a\*x]^2,x]

[Out] -((x^5\*Sqrt[1 - a^2\*x^2])/(a\*ArcSin[a\*x])) + (5\*CosIntegral[2\*ArcSin[a\*x]])/(16\*a^6) - CosIntegral[4\*ArcSin[a\*x]]/(2\*a^6) + (3\*CosIntegral[6\*ArcSin[a\*x]])/(16\*a^6)

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{x^5}{\sin^{-1}(ax)^2} dx &= -\frac{x^5\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(\frac{5\cos(2x)}{16x} - \frac{\cos(4x)}{2x} + \frac{3\cos(6x)}{16x}\right) dx, x, \sin^{-1}(ax)\right)}{a^6} \\ &= -\frac{x^5\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{3\text{Subst}\left(\int\frac{\cos(6x)}{x} dx, x, \sin^{-1}(ax)\right)}{16a^6} + \frac{5\text{Subst}\left(\int\frac{\cos(2x)}{x} dx, x, \sin^{-1}(ax)\right)}{16a^6} \\ &= -\frac{x^5\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{5\text{Ci}\left(2\sin^{-1}(ax)\right)}{16a^6} - \frac{\text{Ci}\left(4\sin^{-1}(ax)\right)}{2a^6} + \frac{3\text{Ci}\left(6\sin^{-1}(ax)\right)}{16a^6} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 78, normalized size = 1.10

$$-\frac{10\sin^{-1}(ax)\text{Ci}\left(2\sin^{-1}(ax)\right) + 16\sin^{-1}(ax)\text{Ci}\left(4\sin^{-1}(ax)\right) - 6\sin^{-1}(ax)\text{Ci}\left(6\sin^{-1}(ax)\right) + 5\sin\left(2\sin^{-1}(ax)\right)}{32a^6\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/ArcSin[a\*x]^2,x]

[Out]  $-1/32*(-10*\text{ArcSin}[a*x]*\text{CosIntegral}[2*\text{ArcSin}[a*x]] + 16*\text{ArcSin}[a*x]*\text{CosIntegral}[4*\text{ArcSin}[a*x]] - 6*\text{ArcSin}[a*x]*\text{CosIntegral}[6*\text{ArcSin}[a*x]] + 5*\text{Sin}[2*\text{ArcSin}[a*x]] - 4*\text{Sin}[4*\text{ArcSin}[a*x]] + \text{Sin}[6*\text{ArcSin}[a*x]])/(a^6*\text{ArcSin}[a*x])$

**fricas** [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(x^5/arcsin(a\*x)^2, x)

**giac** [A] time = 0.14, size = 120, normalized size = 1.69

$$-\frac{(a^2x^2-1)^2\sqrt{-a^2x^2+1}x}{a^5\arcsin(ax)} + \frac{2(-a^2x^2+1)^{\frac{3}{2}}x}{a^5\arcsin(ax)} - \frac{\sqrt{-a^2x^2+1}x}{a^5\arcsin(ax)} + \frac{3\text{Ci}(6\arcsin(ax))}{16a^6} - \frac{\text{Ci}(4\arcsin(ax))}{2a^6} + \frac{5\text{Ci}(2\arcsin(ax))}{16a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x)^2,x, algorithm="giac")

[Out]  $-(a^2*x^2 - 1)^2*\text{sqrt}(-a^2*x^2 + 1)*x/(a^5*\text{arcsin}(a*x)) + 2*(-a^2*x^2 + 1)^{\frac{3}{2}}*x/(a^5*\text{arcsin}(a*x)) - \text{sqrt}(-a^2*x^2 + 1)*x/(a^5*\text{arcsin}(a*x)) + 3/16*\text{cos\_integral}(6*\text{arcsin}(a*x))/a^6 - 1/2*\text{cos\_integral}(4*\text{arcsin}(a*x))/a^6 + 5/16*\text{cos\_integral}(2*\text{arcsin}(a*x))/a^6$

**maple** [A] time = 0.06, size = 78, normalized size = 1.10

$$\frac{-\frac{5\sin(2\arcsin(ax))}{32\arcsin(ax)} + \frac{5\text{Ci}(2\arcsin(ax))}{16} + \frac{\sin(4\arcsin(ax))}{8\arcsin(ax)} - \frac{\text{Ci}(4\arcsin(ax))}{2} - \frac{\sin(6\arcsin(ax))}{32\arcsin(ax)} + \frac{3\text{Ci}(6\arcsin(ax))}{16}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/arcsin(a\*x)^2,x)

[Out]  $1/a^6*(-5/32/\arcsin(a*x)*\sin(2*\arcsin(a*x))+5/16*\text{Ci}(2*\arcsin(a*x))+1/8/\arcsin(a*x)*\sin(4*\arcsin(a*x))-1/2*\text{Ci}(4*\arcsin(a*x))-1/32/\arcsin(a*x)*\sin(6*\arcsin(a*x))+3/16*\text{Ci}(6*\arcsin(a*x)))$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\text{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/asin(a*x)^2,x)
```

```
[Out] int(x^5/asin(a*x)^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^5}{\operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/asin(a*x)**2,x)
```

```
[Out] Integral(x**5/asin(a*x)**2, x)
```

$$3.53 \quad \int \frac{x^4}{\sin^{-1}(ax)^2} dx$$

Optimal. Leaf size=69

$$-\frac{\text{Si}(\sin^{-1}(ax))}{8a^5} + \frac{9\text{Si}(3\sin^{-1}(ax))}{16a^5} - \frac{5\text{Si}(5\sin^{-1}(ax))}{16a^5} - \frac{x^4\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

[Out]  $-1/8*\text{Si}(\arcsin(a*x))/a^5+9/16*\text{Si}(3*\arcsin(a*x))/a^5-5/16*\text{Si}(5*\arcsin(a*x))/a^5-x^4*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4631, 3299}

$$-\frac{\text{Si}(\sin^{-1}(ax))}{8a^5} + \frac{9\text{Si}(3\sin^{-1}(ax))}{16a^5} - \frac{5\text{Si}(5\sin^{-1}(ax))}{16a^5} - \frac{x^4\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcSin[a\*x]^2,x]

[Out]  $-(x^4*\text{Sqrt}[1-a^2*x^2])/(a*\text{ArcSin}[a*x]) - \text{SinIntegral}[\text{ArcSin}[a*x]]/(8*a^5) + (9*\text{SinIntegral}[3*\text{ArcSin}[a*x]])/(16*a^5) - (5*\text{SinIntegral}[5*\text{ArcSin}[a*x]])/(16*a^5)$

Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{\sin^{-1}(ax)^2} dx &= -\frac{x^4\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\sin(x)}{8x} + \frac{9\sin(3x)}{16x} - \frac{5\sin(5x)}{16x}\right) dx, x, \sin^{-1}(ax)\right)}{a^5} \\ &= -\frac{x^4\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int\frac{\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{8a^5} - \frac{5\text{Subst}\left(\int\frac{\sin(5x)}{x} dx, x, \sin^{-1}(ax)\right)}{16a^5} + \frac{9\text{Subst}\left(\int\frac{\sin(3x)}{x} dx, x, \sin^{-1}(ax)\right)}{16a^5} \\ &= -\frac{x^4\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} - \frac{\text{Si}(\sin^{-1}(ax))}{8a^5} + \frac{9\text{Si}(3\sin^{-1}(ax))}{16a^5} - \frac{5\text{Si}(5\sin^{-1}(ax))}{16a^5} \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 61, normalized size = 0.88

$$\frac{16a^4x^4\sqrt{1-a^2x^2}}{\sin^{-1}(ax)} + \frac{2\text{Si}(\sin^{-1}(ax)) - 9\text{Si}(3\sin^{-1}(ax)) + 5\text{Si}(5\sin^{-1}(ax))}{16a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcSin[a\*x]^2,x]

[Out]  $-1/16*((16*a^4*x^4*\sqrt{1 - a^2*x^2})/\text{ArcSin}[a*x] + 2*\text{SinIntegral}[\text{ArcSin}[a*x]] - 9*\text{SinIntegral}[3*\text{ArcSin}[a*x]] + 5*\text{SinIntegral}[5*\text{ArcSin}[a*x]])/a^5$

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(x^4/arcsin(a\*x)^2, x)

**giac** [A] time = 0.16, size = 115, normalized size = 1.67

$$\frac{(a^2x^2 - 1)^2 \sqrt{-a^2x^2 + 1}}{a^5 \arcsin(ax)} - \frac{5 \text{Si}(5 \arcsin(ax))}{16 a^5} + \frac{9 \text{Si}(3 \arcsin(ax))}{16 a^5} - \frac{\text{Si}(\arcsin(ax))}{8 a^5} + \frac{2(-a^2x^2 + 1)^{\frac{3}{2}} \sqrt{-a^2x^2 + 1}}{a^5 \arcsin(ax)} - \frac{\sqrt{-a^2x^2 + 1}}{a^5 \arcsin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^2,x, algorithm="giac")

[Out]  $-(a^2*x^2 - 1)^2*\sqrt{-a^2*x^2 + 1}/(a^5*\arcsin(a*x)) - 5/16*\text{sin\_integral}(5*\arcsin(a*x))/a^5 + 9/16*\text{sin\_integral}(3*\arcsin(a*x))/a^5 - 1/8*\text{sin\_integral}(\arcsin(a*x))/a^5 + 2*(-a^2*x^2 + 1)^{(3/2)}/(a^5*\arcsin(a*x)) - \sqrt{-a^2*x^2 + 1}/(a^5*\arcsin(a*x))$

**maple** [A] time = 0.03, size = 81, normalized size = 1.17

$$\frac{\frac{\sqrt{-a^2x^2+1}}{8 \arcsin(ax)} - \frac{\text{Si}(\arcsin(ax))}{8} + \frac{3 \cos(3 \arcsin(ax))}{16 \arcsin(ax)} + \frac{9 \text{Si}(3 \arcsin(ax))}{16} - \frac{\cos(5 \arcsin(ax))}{16 \arcsin(ax)} - \frac{5 \text{Si}(5 \arcsin(ax))}{16}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arcsin(a\*x)^2,x)

[Out]  $1/a^5*(-1/8/\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}-1/8*\text{Si}(\arcsin(a*x))+3/16/\arcsin(a*x)*\cos(3*\arcsin(a*x))+9/16*\text{Si}(3*\arcsin(a*x))-1/16/\arcsin(a*x)*\cos(5*\arcsin(a*x))-5/16*\text{Si}(5*\arcsin(a*x)))$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\text{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/asin(a\*x)^2,x)

```
[Out] int(x^4/asin(a*x)^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^4}{\operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asin(a*x)**2,x)
```

```
[Out] Integral(x**4/asin(a*x)**2, x)
```

$$3.54 \quad \int \frac{x^3}{\sin^{-1}(ax)^2} dx$$

Optimal. Leaf size=57

$$\frac{\text{Ci}\left(2\sin^{-1}(ax)\right)}{2a^4} - \frac{\text{Ci}\left(4\sin^{-1}(ax)\right)}{2a^4} - \frac{x^3\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

[Out]  $1/2*\text{Ci}(2*\arcsin(a*x))/a^4 - 1/2*\text{Ci}(4*\arcsin(a*x))/a^4 - x^3*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)$

**Rubi [A]** time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4631, 3302}

$$\frac{\text{CosIntegral}\left(2\sin^{-1}(ax)\right)}{2a^4} - \frac{\text{CosIntegral}\left(4\sin^{-1}(ax)\right)}{2a^4} - \frac{x^3\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcSin[a\*x]^2,x]

[Out]  $-((x^3*\text{Sqrt}[1 - a^2*x^2])/(a*\text{ArcSin}[a*x])) + \text{CosIntegral}[2*\text{ArcSin}[a*x]]/(2*a^4) - \text{CosIntegral}[4*\text{ArcSin}[a*x]]/(2*a^4)$

Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*(x\_)^m, x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sin^{-1}(ax)^2} dx &= -\frac{x^3\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(\frac{\cos(2x)}{2x} - \frac{\cos(4x)}{2x}\right) dx, x, \sin^{-1}(ax)\right)}{a^4} \\ &= -\frac{x^3\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int\frac{\cos(2x)}{x} dx, x, \sin^{-1}(ax)\right)}{2a^4} - \frac{\text{Subst}\left(\int\frac{\cos(4x)}{x} dx, x, \sin^{-1}(ax)\right)}{2a^4} \\ &= -\frac{x^3\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{\text{Ci}\left(2\sin^{-1}(ax)\right)}{2a^4} - \frac{\text{Ci}\left(4\sin^{-1}(ax)\right)}{2a^4} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 56, normalized size = 0.98

$$\frac{4\sin^{-1}(ax)\text{Ci}\left(2\sin^{-1}(ax)\right) - 4\sin^{-1}(ax)\text{Ci}\left(4\sin^{-1}(ax)\right) - 2\sin\left(2\sin^{-1}(ax)\right) + \sin\left(4\sin^{-1}(ax)\right)}{8a^4\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcSin[a\*x]^2,x]

[Out] (4\*ArcSin[a\*x]\*CosIntegral[2\*ArcSin[a\*x]] - 4\*ArcSin[a\*x]\*CosIntegral[4\*ArcSin[a\*x]] - 2\*Sin[2\*ArcSin[a\*x]] + Sin[4\*ArcSin[a\*x]])/(8\*a^4\*ArcSin[a\*x])

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(x^3/arcsin(a\*x)^2, x)

**giac** [A] time = 0.21, size = 72, normalized size = 1.26

$$\frac{(-a^2x^2+1)^{\frac{3}{2}}x}{a^3\arcsin(ax)} - \frac{\sqrt{-a^2x^2+1}x}{a^3\arcsin(ax)} - \frac{\text{Ci}(4\arcsin(ax))}{2a^4} + \frac{\text{Ci}(2\arcsin(ax))}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^2,x, algorithm="giac")

[Out] (-a^2\*x^2 + 1)^(3/2)\*x/(a^3\*arcsin(a\*x)) - sqrt(-a^2\*x^2 + 1)\*x/(a^3\*arcsin(a\*x)) - 1/2\*cos\_integral(4\*arcsin(a\*x))/a^4 + 1/2\*cos\_integral(2\*arcsin(a\*x))/a^4

**maple** [A] time = 0.03, size = 54, normalized size = 0.95

$$\frac{-\frac{\sin(2\arcsin(ax))}{4\arcsin(ax)} + \frac{\text{Ci}(2\arcsin(ax))}{2} + \frac{\sin(4\arcsin(ax))}{8\arcsin(ax)} - \frac{\text{Ci}(4\arcsin(ax))}{2}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arcsin(a\*x)^2,x)

[Out] 1/a^4\*(-1/4/arcsin(a\*x)\*sin(2\*arcsin(a\*x))+1/2\*Ci(2\*arcsin(a\*x))+1/8/arcsin(a\*x)\*sin(4\*arcsin(a\*x))-1/2\*Ci(4\*arcsin(a\*x)))

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\text{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/asin(a\*x)^2,x)

[Out] int(x^3/asin(a\*x)^2, x)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sin^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/sin(a\*x)\*\*2,x)

[Out] Integral(x\*\*3/sin(a\*x)\*\*2, x)

$$3.55 \quad \int \frac{x^2}{\sin^{-1}(ax)^2} dx$$

Optimal. Leaf size=55

$$-\frac{\text{Si}(\sin^{-1}(ax))}{4a^3} + \frac{3\text{Si}(3\sin^{-1}(ax))}{4a^3} - \frac{x^2\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

[Out]  $-1/4*\text{Si}(\arcsin(a*x))/a^3+3/4*\text{Si}(3*\arcsin(a*x))/a^3-x^2*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)$

**Rubi [A]** time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4631, 3299}

$$-\frac{\text{Si}(\sin^{-1}(ax))}{4a^3} + \frac{3\text{Si}(3\sin^{-1}(ax))}{4a^3} - \frac{x^2\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcSin[a\*x]^2,x]

[Out]  $-((x^2*\text{Sqrt}[1 - a^2*x^2])/(a*\text{ArcSin}[a*x])) - \text{SinIntegral}[\text{ArcSin}[a*x]]/(4*a^3) + (3*\text{SinIntegral}[3*\text{ArcSin}[a*x]])/(4*a^3)$

Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sin^{-1}(ax)^2} dx &= -\frac{x^2\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\sin(x)}{4x} + \frac{3\sin(3x)}{4x}\right) dx, x, \sin^{-1}(ax)\right)}{a^3} \\ &= -\frac{x^2\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int\frac{\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{4a^3} + \frac{3\text{Subst}\left(\int\frac{\sin(3x)}{x} dx, x, \sin^{-1}(ax)\right)}{4a^3} \\ &= -\frac{x^2\sqrt{1-a^2x^2}}{a\sin^{-1}(ax)} - \frac{\text{Si}(\sin^{-1}(ax))}{4a^3} + \frac{3\text{Si}(3\sin^{-1}(ax))}{4a^3} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 50, normalized size = 0.91

$$-\frac{4a^2x^2\sqrt{1-a^2x^2}}{\sin^{-1}(ax)} + \frac{\text{Si}(\sin^{-1}(ax)) - 3\text{Si}(3\sin^{-1}(ax))}{4a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/ArcSin[a\*x]^2,x]

[Out]  $-1/4*((4*a^2*x^2*\text{Sqrt}[1 - a^2*x^2])/ArcSin[a*x] + \text{SinIntegral}[ArcSin[a*x]] - 3*\text{SinIntegral}[3*ArcSin[a*x]])/a^3$

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(x^2/arcsin(a\*x)^2, x)

**giac** [A] time = 0.21, size = 68, normalized size = 1.24

$$\frac{3 \text{Si}(3 \arcsin(ax))}{4 a^3} - \frac{\text{Si}(\arcsin(ax))}{4 a^3} + \frac{(-a^2 x^2 + 1)^{\frac{3}{2}}}{a^3 \arcsin(ax)} - \frac{\sqrt{-a^2 x^2 + 1}}{a^3 \arcsin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^2,x, algorithm="giac")

[Out]  $3/4*\text{sin\_integral}(3*\arcsin(a*x))/a^3 - 1/4*\text{sin\_integral}(\arcsin(a*x))/a^3 + (-a^2*x^2 + 1)^{(3/2)/(a^3*\arcsin(a*x))} - \text{sqrt}(-a^2*x^2 + 1)/(a^3*\arcsin(a*x))$

**maple** [A] time = 0.03, size = 57, normalized size = 1.04

$$\frac{-\frac{\sqrt{-a^2 x^2 + 1}}{4 \arcsin(ax)} - \frac{\text{Si}(\arcsin(ax))}{4} + \frac{\cos(3 \arcsin(ax))}{4 \arcsin(ax)} + \frac{3 \text{Si}(3 \arcsin(ax))}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arcsin(a\*x)^2,x)

[Out]  $1/a^3*(-1/4/\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}-1/4*\text{Si}(\arcsin(a*x))+1/4/\arcsin(a*x)*\cos(3*\arcsin(a*x))+3/4*\text{Si}(3*\arcsin(a*x)))$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\text{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/asin(a\*x)^2,x)

[Out] int(x^2/asin(a\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sin^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/sin(a\*x)\*\*2,x)

[Out] Integral(x\*\*2/sin(a\*x)\*\*2, x)

$$3.56 \quad \int \frac{x}{\sin^{-1}(ax)^2} dx$$

**Optimal.** Leaf size=38

$$\frac{\text{Ci}(2 \sin^{-1}(ax))}{a^2} - \frac{x\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)}$$

[Out] Ci(2\*arcsin(a\*x))/a^2-x\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4631, 3302}

$$\frac{\text{CosIntegral}(2 \sin^{-1}(ax))}{a^2} - \frac{x\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSin[a\*x]^2,x]

[Out] -((x\*sqrt[1 - a^2\*x^2])/(a\*ArcSin[a\*x])) + CosIntegral[2\*ArcSin[a\*x]]/a^2

**Rule 3302**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

**Rule 4631**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^m, x\_Symbol] :> Simp[(x^m\*sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{x}{\sin^{-1}(ax)^2} dx &= -\frac{x\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^2} \\ &= -\frac{x\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)} + \frac{\text{Ci}(2 \sin^{-1}(ax))}{a^2} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 32, normalized size = 0.84

$$\frac{\text{Ci}(2 \sin^{-1}(ax))}{a^2} - \frac{\sin(2 \sin^{-1}(ax))}{2a^2 \sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcSin[a\*x]^2,x]

[Out] CosIntegral[2\*ArcSin[a\*x]]/a^2 - Sin[2\*ArcSin[a\*x]]/(2\*a^2\*ArcSin[a\*x])

**fricas [F]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(x/arcsin(a\*x)^2, x)

**giac** [A] time = 0.26, size = 36, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+1}x}{a\arcsin(ax)} + \frac{\text{Ci}(2\arcsin(ax))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^2,x, algorithm="giac")

[Out] -sqrt(-a^2\*x^2+1)\*x/(a\*arcsin(a\*x)) + cos\_integral(2\*arcsin(a\*x))/a^2

**maple** [A] time = 0.04, size = 28, normalized size = 0.74

$$\frac{-\frac{\sin(2\arcsin(ax))}{2\arcsin(ax)} + \text{Ci}(2\arcsin(ax))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arcsin(a\*x)^2,x)

[Out] 1/a^2\*(-1/2/arcsin(a\*x)\*sin(2\*arcsin(a\*x))+Ci(2\*arcsin(a\*x)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\sqrt{ax+1}\sqrt{-ax+1}x + \left( \int \frac{2\sqrt{-ax+1}a^2x^2}{\sqrt{ax+1}ax\arctan(ax,\sqrt{ax+1}\sqrt{-ax+1})-\sqrt{ax+1}\arctan(ax,\sqrt{ax+1}\sqrt{-ax+1})} dx + \int -\frac{1}{\sqrt{ax+1}ax\arctan(ax,\sqrt{ax+1}\sqrt{-ax+1})} dx \right)}{a\arctan(ax,\sqrt{ax+1}\sqrt{-ax+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] (a\*arctan2(a\*x, sqrt(a\*x+1)\*sqrt(-a\*x+1))\*integrate((2\*a^2\*x^2-1)\*sqrt(a\*x+1)\*sqrt(-a\*x+1)/((a^3\*x^2-a)\*arctan2(a\*x, sqrt(a\*x+1)\*sqrt(-a\*x+1))), x) - sqrt(a\*x+1)\*sqrt(-a\*x+1)\*x/(a\*arctan2(a\*x, sqrt(a\*x+1)\*sqrt(-a\*x+1))))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\text{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/asin(a\*x)^2,x)

[Out] int(x/asin(a\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\text{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/asin(a\*x)\*\*2,x)

[Out] Integral(x/asin(a\*x)\*\*2, x)

$$3.57 \quad \int \frac{1}{\sin^{-1}(ax)^2} dx$$

**Optimal.** Leaf size=36

$$-\frac{\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)} - \frac{\text{Si}(\sin^{-1}(ax))}{a}$$

[Out] -Si(arcsin(a\*x))/a-(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)

**Rubi [A]** time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4621, 4723, 3299}

$$-\frac{\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)} - \frac{\text{Si}(\sin^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(-2),x]

[Out] -(Sqrt[1 - a^2\*x^2]/(a\*ArcSin[a\*x])) - SinIntegral[ArcSin[a\*x]]/a

**Rule 3299**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

**Rule 4621**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

**Rule 4723**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^m\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{1}{\sin^{-1}(ax)^2} dx &= -\frac{\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)} - a \int \frac{x}{\sqrt{1-a^2x^2} \sin^{-1}(ax)} dx \\ &= -\frac{\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a} \\ &= -\frac{\sqrt{1-a^2x^2}}{a \sin^{-1}(ax)} - \frac{\text{Si}(\sin^{-1}(ax))}{a} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 32, normalized size = 0.89

$$-\frac{\frac{\sqrt{1-a^2x^2}}{\sin^{-1}(ax)} + \text{Si}(\sin^{-1}(ax))}{a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^(-2), x]

[Out] -((Sqrt[1 - a^2\*x^2]/ArcSin[a\*x] + SinIntegral[ArcSin[a\*x]])/a)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^(-2), x)

giac [A] time = 0.23, size = 34, normalized size = 0.94

$$-\frac{\text{Si}(\arcsin(ax))}{a} - \frac{\sqrt{-a^2x^2+1}}{a \arcsin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^2,x, algorithm="giac")

[Out] -sin\_integral(arcsin(a\*x))/a - sqrt(-a^2\*x^2 + 1)/(a\*arcsin(a\*x))

maple [A] time = 0.02, size = 33, normalized size = 0.92

$$\frac{-\frac{\sqrt{-a^2x^2+1}}{\arcsin(ax)} - \text{Si}(\arcsin(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arcsin(a\*x)^2,x)

[Out] 1/a\*(-1/arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)-Si(arcsin(a\*x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right) \int \frac{\sqrt{-ax+1} x}{\sqrt{ax+1} (ax-1) \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)} dx - \sqrt{ax+1} \sqrt{-ax+1}}{a \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] (a^2\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))\*integrate(sqrt(a\*x + 1)\*sqrt(-a\*x + 1)\*x/((a^2\*x^2 - 1)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))), x) - sqrt(a\*x + 1)\*sqrt(-a\*x + 1)/(a\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\text{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/asin(a\*x)^2,x)



```
[Out] int(1/asin(a*x)^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{\operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asin(a*x)**2,x)
```

```
[Out] Integral(asin(a*x)**(-2), x)
```

$$3.58 \quad \int \frac{1}{x \sin^{-1}(ax)^2} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x \sin^{-1}(ax)^2}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x)^2,x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x \sin^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*ArcSin[a\*x]^2), x]

[Out] Defer[Int][1/(x\*ArcSin[a\*x]^2), x]

Rubi steps

$$\int \frac{1}{x \sin^{-1}(ax)^2} dx = \int \frac{1}{x \sin^{-1}(ax)^2} dx$$

**Mathematica [A]** time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sin^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*ArcSin[a\*x]^2), x]

[Out] Integrate[1/(x\*ArcSin[a\*x]^2), x]

**fricas [A]** time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x \arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(1/(x\*arcsin(a\*x)^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^2,x, algorithm="giac")

[Out] integrate(1/(x\*arcsin(a\*x)^2), x)

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x)^2,x)

[Out] int(1/x/arcsin(a\*x)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right) \int \frac{\sqrt{-ax+1}}{\sqrt{ax+1} ax^3 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1}) - \sqrt{ax+1} x^2 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx - \sqrt{ax+1}}{ax \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] (a\*x\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))\*integrate(sqrt(a\*x + 1)\*sqrt(-a\*x + 1)/((a^3\*x^4 - a\*x^2)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))), x) - sqrt(a\*x + 1)\*sqrt(-a\*x + 1)/(a\*x\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)^2),x)

[Out] int(1/(x\*asin(a\*x)^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x)\*\*2,x)

[Out] Integral(1/(x\*asin(a\*x)\*\*2), x)

$$3.59 \quad \int \frac{1}{x^2 \sin^{-1}(ax)^2} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sin^{-1}(ax)^2}, x\right)$$

[Out] Unintegrable(1/x^2/arcsin(a\*x)^2,x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 \sin^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*ArcSin[a\*x]^2), x]

[Out] Defer[Int][1/(x^2\*ArcSin[a\*x]^2), x]

Rubi steps

$$\int \frac{1}{x^2 \sin^{-1}(ax)^2} dx = \int \frac{1}{x^2 \sin^{-1}(ax)^2} dx$$

Mathematica [A] time = 9.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sin^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*ArcSin[a\*x]^2), x]

[Out] Integrate[1/(x^2\*ArcSin[a\*x]^2), x]

fricas [A] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral(1/(x^2\*arcsin(a\*x)^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^2,x, algorithm="giac")

[Out] integrate(1/(x^2\*arcsin(a\*x)^2), x)

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arcsin(a\*x)^2,x)

[Out] int(1/x^2/arcsin(a\*x)^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*asin(a\*x)^2),x)

[Out] int(1/(x^2\*asin(a\*x)^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/asin(a\*x)\*\*2,x)

[Out] Integral(1/(x\*\*2\*asin(a\*x)\*\*2), x)

$$3.60 \quad \int \frac{x^4}{\sin^{-1}(ax)^3} dx$$

**Optimal.** Leaf size=98

$$-\frac{\text{Ci}(\sin^{-1}(ax))}{16a^5} + \frac{27\text{Ci}(3\sin^{-1}(ax))}{32a^5} - \frac{25\text{Ci}(5\sin^{-1}(ax))}{32a^5} - \frac{2x^3}{a^2\sin^{-1}(ax)} - \frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} + \frac{5x^5}{2\sin^{-1}(ax)}$$

[Out]  $-2*x^3/a^2/\arcsin(a*x)+5/2*x^5/\arcsin(a*x)-1/16*\text{Ci}(\arcsin(a*x))/a^5+27/32*\text{Ci}(3*\arcsin(a*x))/a^5-25/32*\text{Ci}(5*\arcsin(a*x))/a^5-1/2*x^4*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^2$

**Rubi [A]** time = 0.34, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4633, 4719, 4635, 4406, 3302}

$$-\frac{\text{CosIntegral}(\sin^{-1}(ax))}{16a^5} + \frac{27\text{CosIntegral}(3\sin^{-1}(ax))}{32a^5} - \frac{25\text{CosIntegral}(5\sin^{-1}(ax))}{32a^5} - \frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{2x^3}{a^2\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4/\text{ArcSin}[a*x]^3, x]$

[Out]  $-(x^4*\text{Sqrt}[1 - a^2*x^2])/(2*a*\text{ArcSin}[a*x]^2) - (2*x^3)/(a^2*\text{ArcSin}[a*x]) + (5*x^5)/(2*\text{ArcSin}[a*x]) - \text{CosIntegral}[\text{ArcSin}[a*x]]/(16*a^5) + (27*\text{CosIntegral}[3*\text{ArcSin}[a*x]])/(32*a^5) - (25*\text{CosIntegral}[5*\text{ArcSin}[a*x]])/(32*a^5)$

Rule 3302

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]]^n*\text{Cos}[a + b*x]^p, x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 4633

$\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(x^m*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^{(n+1)})/(b*c*(n+1)), x] + (\text{Dist}[(c*(m+1))/(b*(n+1)), \text{Int}[(x^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n+1)})/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Dist}[m/(b*c*(n+1)), \text{Int}[(x^{(m-1)}*(a + b*\text{ArcSin}[c*x])^{(n+1)})/\text{Sqrt}[1 - c^2*x^2], x], x]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{LtQ}[n, -2]$

Rule 4635

$\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sin}[x]^m*\text{Cos}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, n\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 4719

$\text{Int}[(((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)*((f_.)*(x_.))^{(m_.)}), x\_Symbol] \rightarrow \text{Simp}[(f*x)^m*(a + b*\text{ArcSin}[c*x])^{(n+1)})/(b*c*\text{Sqrt}[d]*(n+1)), x] - \text{Dist}[(f*m)/(b*c*\text{Sqrt}[d]*(n+1)), \text{Int}[(f*x)^{(m-1)}*(a + b*\text{ArcSin}[c*x])^{(n+1)}], x]$

1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] & & EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{\sin^{-1}(ax)^3} dx &= -\frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} + \frac{2\int \frac{x^3}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx}{a} - \frac{1}{2}(5a) \int \frac{x^5}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx \\ &= -\frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{2x^3}{a^2\sin^{-1}(ax)} + \frac{5x^5}{2\sin^{-1}(ax)} - \frac{25}{2} \int \frac{x^4}{\sin^{-1}(ax)} dx + \frac{6\int \frac{x^2}{\sin^{-1}(ax)} dx}{a^2} \\ &= -\frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{2x^3}{a^2\sin^{-1}(ax)} + \frac{5x^5}{2\sin^{-1}(ax)} + \frac{6\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^5} \\ &= -\frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{2x^3}{a^2\sin^{-1}(ax)} + \frac{5x^5}{2\sin^{-1}(ax)} + \frac{6\text{Subst}\left(\int \left(\frac{\cos(x)}{4x} - \frac{\cos(3x)}{4x}\right) dx, x, \sin^{-1}(ax)\right)}{a^5} \\ &= -\frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{2x^3}{a^2\sin^{-1}(ax)} + \frac{5x^5}{2\sin^{-1}(ax)} - \frac{25\text{Subst}\left(\int \frac{\cos(5x)}{x} dx, x, \sin^{-1}(ax)\right)}{32a^5} + \frac{3\text{Subst}\left(\int \frac{\cos(7x)}{x} dx, x, \sin^{-1}(ax)\right)}{32a^5} \\ &= -\frac{x^4\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{2x^3}{a^2\sin^{-1}(ax)} + \frac{5x^5}{2\sin^{-1}(ax)} - \frac{\text{Ci}\left(\sin^{-1}(ax)\right)}{16a^5} + \frac{27\text{Ci}\left(3\sin^{-1}(ax)\right)}{32a^5} - \frac{25\text{Ci}\left(5\sin^{-1}(ax)\right)}{32a^5} + \frac{3\text{Ci}\left(7\sin^{-1}(ax)\right)}{32a^5} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 103, normalized size = 1.05

$$\frac{-80a^5x^5\sin^{-1}(ax) + 64a^3x^3\sin^{-1}(ax) + 16a^4x^4\sqrt{1-a^2x^2} + 2\sin^{-1}(ax)^2\text{Ci}\left(\sin^{-1}(ax)\right) - 27\sin^{-1}(ax)^2\text{Ci}\left(3\sin^{-1}(ax)\right) + 25\sin^{-1}(ax)^2\text{Ci}\left(5\sin^{-1}(ax)\right) - 3\sin^{-1}(ax)^2\text{Ci}\left(7\sin^{-1}(ax)\right)}{32a^5\sin^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcSin[a\*x]^3,x]

[Out] -1/32\*(16\*a^4\*x^4\*Sqrt[1 - a^2\*x^2] + 64\*a^3\*x^3\*ArcSin[a\*x] - 80\*a^5\*x^5\*ArcSin[a\*x] + 2\*ArcSin[a\*x]^2\*CosIntegral[ArcSin[a\*x]] - 27\*ArcSin[a\*x]^2\*CosIntegral[3\*ArcSin[a\*x]] + 25\*ArcSin[a\*x]^2\*CosIntegral[5\*ArcSin[a\*x]])/(a^5\*ArcSin[a\*x]^2)

**fricas [F]** time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\arcsin(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^3,x, algorithm="fricas")

[Out] integral(x^4/arcsin(a\*x)^3, x)

**giac [A]** time = 0.28, size = 170, normalized size = 1.73

$$\frac{5(a^2x^2-1)^2x}{2a^4\arcsin(ax)} + \frac{3(a^2x^2-1)x}{a^4\arcsin(ax)} + \frac{x}{2a^4\arcsin(ax)} - \frac{25\text{Ci}(5\arcsin(ax))}{32a^5} + \frac{27\text{Ci}(3\arcsin(ax))}{32a^5} - \frac{\text{Ci}(\arcsin(ax))}{16a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^3,x, algorithm="giac")

[Out]  $5/2*(a^2*x^2 - 1)^2*x/(a^4*\arcsin(ax)) + 3*(a^2*x^2 - 1)*x/(a^4*\arcsin(ax)) + 1/2*x/(a^4*\arcsin(ax)) - 25/32*\cos\_integral(5*\arcsin(ax))/a^5 + 27/32*\cos\_integral(3*\arcsin(ax))/a^5 - 1/16*\cos\_integral(\arcsin(ax))/a^5 - 1/2*(a^2*x^2 - 1)^2*\sqrt{-a^2*x^2 + 1}/(a^5*\arcsin(ax)^2) + (-a^2*x^2 + 1)^{3/2}/(a^5*\arcsin(ax)^2) - 1/2*\sqrt{-a^2*x^2 + 1}/(a^5*\arcsin(ax)^2)$

**maple** [A] time = 0.06, size = 121, normalized size = 1.23

$$\frac{-\frac{\sqrt{-a^2x^2+1}}{16\arcsin(ax)^2} + \frac{ax}{16\arcsin(ax)} - \frac{\text{Ci}(\arcsin(ax))}{16} + \frac{3\cos(3\arcsin(ax))}{32\arcsin(ax)^2} - \frac{9\sin(3\arcsin(ax))}{32\arcsin(ax)} + \frac{27\text{Ci}(3\arcsin(ax))}{32} - \frac{\cos(5\arcsin(ax))}{32\arcsin(ax)^2} + \frac{5\sin(5\arcsin(ax))}{32\arcsin(ax)}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/arcsin(a*x)^3,x)`

[Out]  $1/a^5*(-1/16/\arcsin(ax)^2*(-a^2*x^2+1)^{(1/2)}+1/16*a*x/\arcsin(ax)-1/16*\text{Ci}(\arcsin(ax))+3/32/\arcsin(ax)^2*\cos(3*\arcsin(ax))-9/32/\arcsin(ax)*\sin(3*\arcsin(ax))+27/32*\text{Ci}(3*\arcsin(ax))-1/32/\arcsin(ax)^2*\cos(5*\arcsin(ax))+5/32/\arcsin(ax)*\sin(5*\arcsin(ax))-25/32*\text{Ci}(5*\arcsin(ax)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}ax^4 + \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2 \int \frac{(25a^2x^2-12)x^2}{\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx - (5a^2x^5 - 4x^3) \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})}{2a^2 \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arcsin(a*x)^3,x, algorithm="maxima")`

[Out]  $-1/2*(\sqrt{ax+1}\sqrt{-ax+1}*ax^4 + \arctan2(ax, \sqrt{ax+1}\sqrt{-ax+1}))^2*\integrate((25*a^2*x^4 - 12*x^2)/\arctan2(ax, \sqrt{ax+1}\sqrt{-ax+1}), x) - (5*a^2*x^5 - 4*x^3)*\arctan2(ax, \sqrt{ax+1}\sqrt{-ax+1})/(a^2*\arctan2(ax, \sqrt{ax+1}\sqrt{-ax+1})^2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\text{asin}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/asin(a*x)^3,x)`

[Out] `int(x^4/asin(a*x)^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\text{asin}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/asin(a*x)**3,x)`

[Out] `Integral(x**4/asin(a*x)**3, x)`



### 3.61 $\int \frac{x^3}{\sin^{-1}(ax)^3} dx$

**Optimal.** Leaf size=83

$$-\frac{\text{Si}(2 \sin^{-1}(ax))}{2a^4} + \frac{\text{Si}(4 \sin^{-1}(ax))}{a^4} - \frac{3x^2}{2a^2 \sin^{-1}(ax)} - \frac{x^3 \sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} + \frac{2x^4}{\sin^{-1}(ax)}$$

[Out]  $-3/2*x^2/a^2/\arcsin(a*x)+2*x^4/\arcsin(a*x)-1/2*Si(2*\arcsin(a*x))/a^4+Si(4*\arcsin(a*x))/a^4-1/2*x^3*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^2$

**Rubi [A]** time = 0.30, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4633, 4719, 4635, 4406, 3299, 12}

$$-\frac{\text{Si}(2 \sin^{-1}(ax))}{2a^4} + \frac{\text{Si}(4 \sin^{-1}(ax))}{a^4} - \frac{x^3 \sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} - \frac{3x^2}{2a^2 \sin^{-1}(ax)} + \frac{2x^4}{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcSin[a\*x]^3,x]

[Out]  $-(x^3*\text{Sqrt}[1 - a^2*x^2])/(2*a*\text{ArcSin}[a*x]^2) - (3*x^2)/(2*a^2*\text{ArcSin}[a*x]) + (2*x^4)/\text{ArcSin}[a*x] - \text{SinIntegral}[2*\text{ArcSin}[a*x]]/(2*a^4) + \text{SinIntegral}[4*\text{ArcSin}[a*x]]/a^4$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n \* Sin[x]^m \* Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rule 4719

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^n)*((f_.)*(x_.))^m)/Sqrt[(d_
+ (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^m*(a + b*ArcSin[c*x])^(n + 1))/(b
*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m -
1)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sin^{-1}(ax)^3} dx &= -\frac{x^3\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} + \frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx}{2a} - (2a) \int \frac{x^4}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{3x^2}{2a^2\sin^{-1}(ax)} + \frac{2x^4}{\sin^{-1}(ax)} - 8 \int \frac{x^3}{\sin^{-1}(ax)} dx + \frac{3\int \frac{x}{\sin^{-1}(ax)} dx}{a^2} \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{3x^2}{2a^2\sin^{-1}(ax)} + \frac{2x^4}{\sin^{-1}(ax)} + \frac{3\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^4} - \frac{8\text{Subst}\left(\int \frac{x}{\sin^{-1}(ax)} dx, x, \sin^{-1}(ax)\right)}{a^2} \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{3x^2}{2a^2\sin^{-1}(ax)} + \frac{2x^4}{\sin^{-1}(ax)} + \frac{3\text{Subst}\left(\int \frac{\sin(2x)}{2x} dx, x, \sin^{-1}(ax)\right)}{a^4} - \frac{8\text{Subst}\left(\int \frac{x}{\sin^{-1}(ax)} dx, x, \sin^{-1}(ax)\right)}{a^2} \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{3x^2}{2a^2\sin^{-1}(ax)} + \frac{2x^4}{\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^4} + \frac{3\text{Subst}\left(\int \frac{x}{\sin^{-1}(ax)} dx, x, \sin^{-1}(ax)\right)}{a^2} \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{3x^2}{2a^2\sin^{-1}(ax)} + \frac{2x^4}{\sin^{-1}(ax)} - \frac{\text{Si}\left(2\sin^{-1}(ax)\right)}{2a^4} + \frac{\text{Si}\left(4\sin^{-1}(ax)\right)}{a^4}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 73, normalized size = 0.88

$$\frac{\frac{a^2x^2\left((4a^2x^2-3)\sin^{-1}(ax)-ax\sqrt{1-a^2x^2}\right)}{\sin^{-1}(ax)^2} - \text{Si}\left(2\sin^{-1}(ax)\right) + 2\text{Si}\left(4\sin^{-1}(ax)\right)}{2a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcSin[a\*x]^3,x]

[Out] ((a^2\*x^2\*(-(a\*x\*Sqrt[1 - a^2\*x^2]) + (-3 + 4\*a^2\*x^2)\*ArcSin[a\*x]))/ArcSin[a\*x]^2 - SinIntegral[2\*ArcSin[a\*x]] + 2\*SinIntegral[4\*ArcSin[a\*x]])/(2\*a^4)

**fricas [F]** time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\arcsin(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^3,x, algorithm="fricas")

[Out] integral(x^3/arcsin(a\*x)^3, x)

**giac [A]** time = 0.27, size = 125, normalized size = 1.51

$$\frac{(-a^2x^2 + 1)^{\frac{3}{2}}x}{2a^3\arcsin(ax)^2} + \frac{2(a^2x^2 - 1)^2}{a^4\arcsin(ax)} + \frac{\text{Si}(4\arcsin(ax))}{a^4} - \frac{\text{Si}(2\arcsin(ax))}{2a^4} - \frac{\sqrt{-a^2x^2 + 1}x}{2a^3\arcsin(ax)^2} + \frac{5(a^2x^2 - 1)}{2a^4\arcsin(ax)} + \frac{1}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^3,x, algorithm="giac")

[Out]  $\frac{1}{2}(-a^2x^2 + 1)^{3/2}x/(a^3\arcsin(ax)^2) + 2(a^2x^2 - 1)^2/(a^4\arcsin(ax)) + \sin\_integral(4\arcsin(ax))/a^4 - 1/2\sin\_integral(2\arcsin(ax))/a^4 - 1/2\sqrt{-a^2x^2 + 1}x/(a^3\arcsin(ax)^2) + 5/2(a^2x^2 - 1)/(a^4\arcsin(ax)) + 1/2/(a^4\arcsin(ax))$

**maple [A]** time = 0.05, size = 82, normalized size = 0.99

$$\frac{\frac{\sin(2 \arcsin(ax))}{8 \arcsin(ax)^2} - \frac{\cos(2 \arcsin(ax))}{4 \arcsin(ax)} - \frac{\text{Si}(2 \arcsin(ax))}{2} + \frac{\sin(4 \arcsin(ax))}{16 \arcsin(ax)^2} + \frac{\cos(4 \arcsin(ax))}{4 \arcsin(ax)} + \text{Si}(4 \arcsin(ax))}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arcsin(a\*x)^3,x)

[Out]  $\frac{1}{a^4}(-1/8/\arcsin(ax)^2*\sin(2*\arcsin(ax))-1/4/\arcsin(ax)*\cos(2*\arcsin(ax))-1/2*\text{Si}(2*\arcsin(ax))+1/16/\arcsin(ax)^2*\sin(4*\arcsin(ax))+1/4/\arcsin(ax)*\cos(4*\arcsin(ax))+\text{Si}(4*\arcsin(ax)))$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}ax^3 + 2 \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2 \int \frac{(8a^2x^2-3)x}{\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx - (4a^2x^4 - 3x^2) \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})}{2a^2 \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^3,x, algorithm="maxima")

[Out]  $-1/2*(\sqrt{ax+1}*\sqrt{-ax+1}*a*x^3 + 2*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})^2*\int((8*a^2*x^3 - 3*x)/\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1}), x) - (4*a^2*x^4 - 3*x^2)*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1}))/a^2*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})^2$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\arcsin(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/asin(a\*x)^3,x)

[Out] int(x^3/asin(a\*x)^3, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\arcsin^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/asin(a\*x)\*\*3,x)

[Out] Integral(x\*\*3/asin(a\*x)\*\*3, x)

$$3.62 \quad \int \frac{x^2}{\sin^{-1}(ax)^3} dx$$

Optimal. Leaf size=82

$$-\frac{\text{Ci}(\sin^{-1}(ax))}{8a^3} + \frac{9\text{Ci}(3\sin^{-1}(ax))}{8a^3} - \frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{x}{a^2\sin^{-1}(ax)} + \frac{3x^3}{2\sin^{-1}(ax)}$$

[Out]  $-x/a^2/\arcsin(ax)+3/2*x^3/\arcsin(ax)-1/8*\text{Ci}(\arcsin(ax))/a^3+9/8*\text{Ci}(3*\arcsin(ax))/a^3-1/2*x^2*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(ax)^2$

**Rubi [A]** time = 0.25, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4633, 4719, 4635, 4406, 3302, 4623}

$$-\frac{\text{CosIntegral}(\sin^{-1}(ax))}{8a^3} + \frac{9\text{CosIntegral}(3\sin^{-1}(ax))}{8a^3} - \frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{x}{a^2\sin^{-1}(ax)} + \frac{3x^3}{2\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcSin[a\*x]^3,x]

[Out]  $-(x^2*\text{Sqrt}[1-a^2*x^2])/(2*a*\text{ArcSin}[a*x]^2) - x/(a^2*\text{ArcSin}[a*x]) + (3*x^3)/(2*\text{ArcSin}[a*x]) - \text{CosIntegral}[\text{ArcSin}[a*x]]/(8*a^3) + (9*\text{CosIntegral}[3*\text{ArcSin}[a*x]])/(8*a^3)$

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n\*cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*sin[x]^m\*cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

## Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\_)\*((f\_.)\*(x\_.))^m\_)/Sqrt[(d\_. + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] & & EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

## Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sin^{-1}(ax)^3} dx &= -\frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} + \frac{\int \frac{x}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx}{a} - \frac{1}{2}(3a) \int \frac{x^3}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx \\ &= -\frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{x}{a^2\sin^{-1}(ax)} + \frac{3x^3}{2\sin^{-1}(ax)} - \frac{9}{2} \int \frac{x^2}{\sin^{-1}(ax)} dx + \frac{\int \frac{1}{\sin^{-1}(ax)} dx}{a^2} \\ &= -\frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{x}{a^2\sin^{-1}(ax)} + \frac{3x^3}{2\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^3} - \frac{9 \text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{2a^3} \\ &= -\frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{x}{a^2\sin^{-1}(ax)} + \frac{3x^3}{2\sin^{-1}(ax)} + \frac{\text{Ci}\left(\sin^{-1}(ax)\right)}{a^3} - \frac{9 \text{Subst}\left(\int \left(\frac{\cos(x)}{4x} - \frac{\cos(3x)}{4x}\right) dx, x, \sin^{-1}(ax)\right)}{2a^3} \\ &= -\frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{x}{a^2\sin^{-1}(ax)} + \frac{3x^3}{2\sin^{-1}(ax)} + \frac{\text{Ci}\left(\sin^{-1}(ax)\right)}{a^3} - \frac{9 \text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{8a^3} \\ &= -\frac{x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{x}{a^2\sin^{-1}(ax)} + \frac{3x^3}{2\sin^{-1}(ax)} - \frac{\text{Ci}\left(\sin^{-1}(ax)\right)}{8a^3} + \frac{9\text{Ci}\left(3\sin^{-1}(ax)\right)}{8a^3} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 68, normalized size = 0.83

$$\frac{4ax\left((3a^2x^2-2)\sin^{-1}(ax)-ax\sqrt{1-a^2x^2}\right)}{\sin^{-1}(ax)^2} - \frac{\text{Ci}\left(\sin^{-1}(ax)\right) + 9\text{Ci}\left(3\sin^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/ArcSin[a\*x]^3,x]

[Out] ((4\*a\*x\*(-(a\*x\*Sqrt[1 - a^2\*x^2]) + (-2 + 3\*a^2\*x^2)\*ArcSin[a\*x]))/ArcSin[a\*x]^2 - CosIntegral[ArcSin[a\*x]] + 9\*CosIntegral[3\*ArcSin[a\*x]])/(8\*a^3)

**fricas [F]** time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\arcsin(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^3,x, algorithm="fricas")

[Out] integral(x^2/arcsin(a\*x)^3, x)

**giac [A]** time = 0.21, size = 102, normalized size = 1.24

$$\frac{3(a^2x^2-1)x}{2a^2\arcsin(ax)} + \frac{x}{2a^2\arcsin(ax)} + \frac{9\text{Ci}(3\arcsin(ax))}{8a^3} - \frac{\text{Ci}(\arcsin(ax))}{8a^3} + \frac{(-a^2x^2+1)^{\frac{3}{2}}}{2a^3\arcsin(ax)^2} - \frac{\sqrt{-a^2x^2+1}}{2a^3\arcsin(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^3,x, algorithm="giac")

[Out]  $\frac{3}{2}*(a^2*x^2 - 1)*x/(a^2*\arcsin(a*x)) + \frac{1}{2}*x/(a^2*\arcsin(a*x)) + \frac{9}{8}*\cos\_integral(3*\arcsin(a*x))/a^3 - \frac{1}{8}*\cos\_integral(\arcsin(a*x))/a^3 + \frac{1}{2}*(-a^2*x^2 + 1)^{(3/2)}/(a^3*\arcsin(a*x)^2) - \frac{1}{2}*sqrt(-a^2*x^2 + 1)/(a^3*\arcsin(a*x)^2)$

**maple** [A] time = 0.03, size = 82, normalized size = 1.00

$$\frac{-\frac{\sqrt{-a^2x^2+1}}{8\arcsin(ax)^2} + \frac{ax}{8\arcsin(ax)} - \frac{\text{Ci}(\arcsin(ax))}{8} + \frac{\cos(3\arcsin(ax))}{8\arcsin(ax)^2} - \frac{3\sin(3\arcsin(ax))}{8\arcsin(ax)} + \frac{9\text{Ci}(3\arcsin(ax))}{8}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arcsin(a\*x)^3,x)

[Out]  $\frac{1}{a^3}*(-\frac{1}{8}/\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}+1/8*a*x/\arcsin(a*x)-1/8*\text{Ci}(\arcsin(a*x))+1/8/\arcsin(a*x)^2*\cos(3*\arcsin(a*x))-3/8/\arcsin(a*x)*\sin(3*\arcsin(a*x))+9/8*\text{Ci}(3*\arcsin(a*x)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}ax^2 + \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2 \int \frac{9a^2x^2-2}{\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx - (3a^2x^3 - 2x) \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})}{2a^2 \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^3,x, algorithm="maxima")

[Out]  $-1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^2 + \arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)))^2*integrate((9*a^2*x^2 - 2)/\arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)), x) - (3*a^2*x^3 - 2*x)*\arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))/ (a^2*\arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\text{asin}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/asin(a\*x)^3,x)

[Out] int(x^2/asin(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{asin}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/asin(a\*x)\*\*3,x)

[Out] Integral(x\*\*2/asin(a\*x)\*\*3, x)

### 3.63 $\int \frac{x}{\sin^{-1}(ax)^3} dx$

**Optimal.** Leaf size=64

$$-\frac{\text{Si}(2 \sin^{-1}(ax))}{a^2} - \frac{x\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} - \frac{1}{2a^2 \sin^{-1}(ax)} + \frac{x^2}{\sin^{-1}(ax)}$$

[Out]  $-1/2/a^2/\arcsin(a*x)+x^2/\arcsin(a*x)-\text{Si}(2*\arcsin(a*x))/a^2-1/2*x*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^2$

**Rubi [A]** time = 0.17, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$ , Rules used = {4633, 4719, 4635, 4406, 12, 3299, 4641}

$$-\frac{\text{Si}(2 \sin^{-1}(ax))}{a^2} - \frac{x\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} - \frac{1}{2a^2 \sin^{-1}(ax)} + \frac{x^2}{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSin[a\*x]^3,x]

[Out]  $-(x*\text{Sqrt}[1 - a^2*x^2])/(2*a*\text{ArcSin}[a*x]^2) - 1/(2*a^2*\text{ArcSin}[a*x]) + x^2/\text{ArcSin}[a*x] - \text{SinIntegral}[2*\text{ArcSin}[a*x]]/a^2$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n \* Sin[x]^m \* Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.) / Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])^(n + 1) / (b\*c\*Sqrt[d]\*(n + 1)), x] /; Fre

$eQ[\{a, b, c, d, e, n\}, x] \ \&\& \ EqQ[c^2*d + e, 0] \ \&\& \ GtQ[d, 0] \ \&\& \ NeQ[n, -1]$

### Rule 4719

$\text{Int}[\left(\left(\left(a_{\cdot}\right) + \text{ArcSin}\left[\left(c_{\cdot}\right)*\left(x_{\cdot}\right)\right]*\left(b_{\cdot}\right)\right)^{\left(n_{\cdot}\right)}*\left(\left(f_{\cdot}\right)*\left(x_{\cdot}\right)\right)^{\left(m_{\cdot}\right)}\right)/\text{Sqrt}\left[\left(d_{\cdot}\right) + \left(e_{\cdot}\right)*\left(x_{\cdot}\right)^2\right], x\_Symbol] \ :> \ \text{Simp}\left[\left(\left(f*x\right)^m*\left(a + b*\text{ArcSin}\left[c*x\right]\right)^{\left(n + 1\right)}\right)/\left(b*c*\text{Sqrt}\left[d\right]*\left(n + 1\right)\right), x] - \text{Dist}\left[\left(f*m\right)/\left(b*c*\text{Sqrt}\left[d\right]*\left(n + 1\right)\right), \text{Int}\left[\left(f*x\right)^{\left(m - 1\right)}*\left(a + b*\text{ArcSin}\left[c*x\right]\right)^{\left(n + 1\right)}, x\right], x\right] \ ; \ \text{FreeQ}\left[\{a, b, c, d, e, f, m\}, x\right] \ \&\& \ \text{EqQ}\left[c^2*d + e, 0\right] \ \&\& \ \text{LtQ}\left[n, -1\right] \ \&\& \ \text{GtQ}\left[d, 0\right]$

### Rubi steps

$$\begin{aligned} \int \frac{x}{\sin^{-1}(ax)^3} dx &= -\frac{x\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx}{2a} - a \int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^2} dx \\ &= -\frac{x\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{1}{2a^2\sin^{-1}(ax)} + \frac{x^2}{\sin^{-1}(ax)} - 2 \int \frac{x}{\sin^{-1}(ax)} dx \\ &= -\frac{x\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{1}{2a^2\sin^{-1}(ax)} + \frac{x^2}{\sin^{-1}(ax)} - \frac{2 \text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^2} \\ &= -\frac{x\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{1}{2a^2\sin^{-1}(ax)} + \frac{x^2}{\sin^{-1}(ax)} - \frac{2 \text{Subst}\left(\int \frac{\sin(2x)}{2x} dx, x, \sin^{-1}(ax)\right)}{a^2} \\ &= -\frac{x\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{1}{2a^2\sin^{-1}(ax)} + \frac{x^2}{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x} dx, x, \sin^{-1}(ax)\right)}{a^2} \\ &= -\frac{x\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)^2} - \frac{1}{2a^2\sin^{-1}(ax)} + \frac{x^2}{\sin^{-1}(ax)} - \frac{\text{Si}\left(2\sin^{-1}(ax)\right)}{a^2} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 61, normalized size = 0.95

$$-\frac{ax\sqrt{1-a^2x^2} + (1-2a^2x^2)\sin^{-1}(ax) + 2\sin^{-1}(ax)^2\text{Si}\left(2\sin^{-1}(ax)\right)}{2a^2\sin^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcSin[a\*x]^3,x]

[Out]  $-1/2*(a*x*\text{Sqrt}[1 - a^2*x^2] + (1 - 2*a^2*x^2)*\text{ArcSin}[a*x] + 2*\text{ArcSin}[a*x]^2*\text{SinIntegral}[2*\text{ArcSin}[a*x]])/(a^2*\text{ArcSin}[a*x]^2)$

**fricas [F]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\arcsin(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^3,x, algorithm="fricas")

[Out] integral(x/arcsin(a\*x)^3, x)

**giac [A]** time = 0.21, size = 67, normalized size = 1.05

$$-\frac{\text{Si}\left(2\arcsin(ax)\right)}{a^2} - \frac{\sqrt{-a^2x^2+1}x}{2a\arcsin(ax)^2} + \frac{a^2x^2-1}{a^2\arcsin(ax)} + \frac{1}{2a^2\arcsin(ax)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^3,x, algorithm="giac")

[Out]  $-\frac{\sin(2\arcsin(ax))}{4\arcsin(ax)^2} - \frac{\cos(2\arcsin(ax))}{2\arcsin(ax)} - \frac{\text{Si}(2\arcsin(ax))}{a^2} - \frac{1}{2}\sqrt{-a^2x^2+1}x/(a\arcsin(ax)^2) + (a^2x^2-1)/(a^2\arcsin(ax)) + 1/2/(a^2\arcsin(ax))$

**maple** [A] time = 0.04, size = 45, normalized size = 0.70

$$\frac{-\frac{\sin(2\arcsin(ax))}{4\arcsin(ax)^2} - \frac{\cos(2\arcsin(ax))}{2\arcsin(ax)} - \text{Si}(2\arcsin(ax))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arcsin(a\*x)^3,x)

[Out]  $1/a^2*(-1/4/\arcsin(a*x)^2*\sin(2*\arcsin(a*x))-1/2/\arcsin(a*x)*\cos(2*\arcsin(a*x))-Si(2*\arcsin(a*x)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4a^2 \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2 \int \frac{x}{\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx + \sqrt{ax+1}\sqrt{-ax+1}ax - (2a^2x^2-1)\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})}{2a^2 \arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^3,x, algorithm="maxima")

[Out]  $-1/2*(4*a^2*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^2*\integrate(x/\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1}), x) + \sqrt{a*x+1}*\sqrt{-a*x+1}*a*x - (2*a^2*x^2-1)*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1}))/a^2*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\text{asin}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/asin(a\*x)^3,x)

[Out] int(x/asin(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\text{asin}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/asin(a\*x)\*\*3,x)

[Out] Integral(x/asin(a\*x)\*\*3, x)

$$3.64 \quad \int \frac{1}{\sin^{-1}(ax)^3} dx$$

**Optimal.** Leaf size=51

$$-\frac{\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} - \frac{\text{Ci}(\sin^{-1}(ax))}{2a} + \frac{x}{2 \sin^{-1}(ax)}$$

[Out] 1/2\*x/arcsin(a\*x)-1/2\*Ci(arcsin(a\*x))/a-1/2\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)^2

**Rubi [A]** time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4621, 4719, 4623, 3302}

$$-\frac{\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} - \frac{\text{CosIntegral}(\sin^{-1}(ax))}{2a} + \frac{x}{2 \sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(-3), x]

[Out] -Sqrt[1 - a^2\*x^2]/(2\*a\*ArcSin[a\*x]^2) + x/(2\*ArcSin[a\*x]) - CosIntegral[ArcSin[a\*x]]/(2\*a)

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n)\*((f\_.)\*(x\_))^(m\_)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sin^{-1}(ax)^3} dx &= -\frac{\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} - \frac{1}{2}a \int \frac{x}{\sqrt{1-a^2x^2} \sin^{-1}(ax)^2} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} + \frac{x}{2 \sin^{-1}(ax)} - \frac{1}{2} \int \frac{1}{\sin^{-1}(ax)} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} + \frac{x}{2 \sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \sin^{-1}(ax)\right)}{2a} \\
&= -\frac{\sqrt{1-a^2x^2}}{2a \sin^{-1}(ax)^2} + \frac{x}{2 \sin^{-1}(ax)} - \frac{\text{Ci}(\sin^{-1}(ax))}{2a}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 48, normalized size = 0.94

$$\frac{\sqrt{1-a^2x^2} + \sin^{-1}(ax)^2 \text{Ci}(\sin^{-1}(ax)) - ax \sin^{-1}(ax)}{2a \sin^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^(-3), x]

[Out] -1/2\*(Sqrt[1 - a^2\*x^2] - a\*x\*ArcSin[a\*x] + ArcSin[a\*x]^2\*CosIntegral[ArcSin[a\*x]])/(a\*ArcSin[a\*x]^2)

**fricas [F]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\arcsin(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^3, x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^(-3), x)

**giac [A]** time = 0.20, size = 43, normalized size = 0.84

$$\frac{x}{2 \arcsin(ax)} - \frac{\text{Ci}(\arcsin(ax))}{2a} - \frac{\sqrt{-a^2x^2 + 1}}{2a \arcsin(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^3, x, algorithm="giac")

[Out] 1/2\*x/arcsin(a\*x) - 1/2\*cos\_integral(arcsin(a\*x))/a - 1/2\*sqrt(-a^2\*x^2 + 1)/(a\*arcsin(a\*x)^2)

**maple [A]** time = 0.03, size = 43, normalized size = 0.84

$$\frac{-\frac{\sqrt{-a^2x^2+1}}{2 \arcsin(ax)^2} + \frac{ax}{2 \arcsin(ax)} - \frac{\text{Ci}(\arcsin(ax))}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arcsin(a\*x)^3, x)

[Out] 1/a\*(-1/2/arcsin(a\*x)^2\*(-a^2\*x^2+1)^(1/2)+1/2\*a\*x/arcsin(a\*x)-1/2\*Ci(arcsin(a\*x)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2 \int \frac{1}{\arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx - ax \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1}) + \sqrt{ax+1} \sqrt{-ax+1}}{2 a \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^3,x, algorithm="maxima")

[Out] -1/2\*(a\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2\*integrate(1/arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)), x) - a\*x\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)) + sqrt(a\*x + 1)\*sqrt(-a\*x + 1))/(a\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asin}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/asin(a\*x)^3,x)

[Out] int(1/asin(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asin}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/asin(a\*x)\*\*3,x)

[Out] Integral(asin(a\*x)\*\*(-3), x)

$$3.65 \quad \int \frac{1}{x \sin^{-1}(ax)^3} dx$$

**Optimal.** Leaf size=13

$$\text{Int}\left(\frac{1}{x \sin^{-1}(ax)^3}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x)^3, x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x \sin^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

[In] Int [1/(x\*ArcSin[a\*x]^3), x]

[Out] Defer[Int] [1/(x\*ArcSin[a\*x]^3), x]

Rubi steps

$$\int \frac{1}{x \sin^{-1}(ax)^3} dx = \int \frac{1}{x \sin^{-1}(ax)^3} dx$$

**Mathematica [A]** time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sin^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*ArcSin[a\*x]^3), x]

[Out] Integrate[1/(x\*ArcSin[a\*x]^3), x]

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x \arcsin(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^3, x, algorithm="fricas")

[Out] integral(1/(x\*arcsin(a\*x)^3), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^3, x, algorithm="giac")

[Out] integrate(1/(x\*arcsin(a\*x)^3), x)

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x)^3,x)

[Out] int(1/x/arcsin(a\*x)^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2x^2 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2 \int \frac{1}{x^3 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx - \sqrt{ax+1} \sqrt{-ax+1} ax + \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}{2a^2x^2 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^3,x, algorithm="maxima")

[Out] 1/2\*(2\*x^2\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2\*integrate(1/(x^3\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))), x) - sqrt(a\*x + 1)\*sqrt(-a\*x + 1)\*a\*x + arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)))/(a^2\*x^2\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{asin}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)^3),x)

[Out] int(1/(x\*asin(a\*x)^3), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asin}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x)\*\*3,x)

[Out] Integral(1/(x\*asin(a\*x)\*\*3), x)

$$3.66 \quad \int \frac{1}{x^2 \sin^{-1}(ax)^3} dx$$

**Optimal.** Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sin^{-1}(ax)^3}, x\right)$$

[Out] Unintegrable(1/x^2/arcsin(a\*x)^3, x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 \sin^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

[In] Int [1/(x^2\*ArcSin[a\*x]^3), x]

[Out] Defer[Int] [1/(x^2\*ArcSin[a\*x]^3), x]

Rubi steps

$$\int \frac{1}{x^2 \sin^{-1}(ax)^3} dx = \int \frac{1}{x^2 \sin^{-1}(ax)^3} dx$$

**Mathematica [A]** time = 6.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sin^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*ArcSin[a\*x]^3), x]

[Out] Integrate[1/(x^2\*ArcSin[a\*x]^3), x]

**fricas [A]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \arcsin(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^3, x, algorithm="fricas")

[Out] integral(1/(x^2\*arcsin(a\*x)^3), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^3, x, algorithm="giac")

[Out] integrate(1/(x^2\*arcsin(a\*x)^3), x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arcsin(a\*x)^3,x)

[Out] int(1/x^2/arcsin(a\*x)^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2 \int \frac{a^2 x^2 - 6}{x^4 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx + \sqrt{ax+1} \sqrt{-ax+1} ax + (a^2 x^2 - 2) \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}{2 a^2 x^3 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^3,x, algorithm="maxima")

[Out] -1/2\*(x^3\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2\*integrate((a^2\*x^2 - 6)/(x^4\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))), x) + sqrt(a\*x + 1)\*sqrt(-a\*x + 1)\*a\*x + (a^2\*x^2 - 2)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)))/(a^2\*x^3\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asin}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*asin(a\*x)^3),x)

[Out] int(1/(x^2\*asin(a\*x)^3), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asin}^3(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/asin(a\*x)\*\*3,x)

[Out] Integral(1/(x\*\*2\*asin(a\*x)\*\*3), x)



$$3.67 \quad \int \frac{x^4}{\sin^{-1}(ax)^4} dx$$

**Optimal.** Leaf size=158

$$\frac{\text{Si}(\sin^{-1}(ax))}{48a^5} - \frac{27\text{Si}(3\sin^{-1}(ax))}{32a^5} + \frac{125\text{Si}(5\sin^{-1}(ax))}{96a^5} - \frac{2x^3}{3a^2\sin^{-1}(ax)^2} + \frac{25x^4\sqrt{1-a^2x^2}}{6a\sin^{-1}(ax)} - \frac{x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{2x^2}{a^3}$$

[Out]  $-2/3*x^3/a^2/\arcsin(a*x)^2+5/6*x^5/\arcsin(a*x)^2+1/48*\text{Si}(\arcsin(a*x))/a^5-27/32*\text{Si}(3*\arcsin(a*x))/a^5+125/96*\text{Si}(5*\arcsin(a*x))/a^5-1/3*x^4*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^3-2*x^2*(-a^2*x^2+1)^{(1/2)}/a^3/\arcsin(a*x)+25/6*x^4*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)$

**Rubi [A]** time = 0.31, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4633, 4719, 4631, 3299}

$$\frac{\text{Si}(\sin^{-1}(ax))}{48a^5} - \frac{27\text{Si}(3\sin^{-1}(ax))}{32a^5} + \frac{125\text{Si}(5\sin^{-1}(ax))}{96a^5} + \frac{25x^4\sqrt{1-a^2x^2}}{6a\sin^{-1}(ax)} - \frac{x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{2x^3}{3a^2\sin^{-1}(ax)^2} - \frac{2x^2}{a^3}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcSin[a\*x]^4, x]

[Out]  $-(x^4*\text{Sqrt}[1 - a^2*x^2])/(3*a*\text{ArcSin}[a*x]^3) - (2*x^3)/(3*a^2*\text{ArcSin}[a*x]^2) + (5*x^5)/(6*\text{ArcSin}[a*x]^2) - (2*x^2*\text{Sqrt}[1 - a^2*x^2])/(a^3*\text{ArcSin}[a*x]) + (25*x^4*\text{Sqrt}[1 - a^2*x^2])/(6*a*\text{ArcSin}[a*x]) + \text{SinIntegral}[\text{ArcSin}[a*x]]/(48*a^5) - (27*\text{SinIntegral}[3*\text{ArcSin}[a*x]])/(32*a^5) + (125*\text{SinIntegral}[5*\text{ArcSin}[a*x]])/(96*a^5)$

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(x\_)^m\_., x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(x\_)^m\_., x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4719

Int((((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_)\*((f\_.)\*(x\_))^m\_./Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{\sin^{-1}(ax)^4} dx &= -\frac{x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} + \frac{4\int \frac{x^3}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx}{3a} - \frac{1}{3}(5a) \int \frac{x^5}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx \\
 &= -\frac{x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{2x^3}{3a^2\sin^{-1}(ax)^2} + \frac{5x^5}{6\sin^{-1}(ax)^2} - \frac{25}{6} \int \frac{x^4}{\sin^{-1}(ax)^2} dx + \frac{2\int \frac{x^2}{\sin^{-1}(ax)^2} dx}{a^2} \\
 &= -\frac{x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{2x^3}{3a^2\sin^{-1}(ax)^2} + \frac{5x^5}{6\sin^{-1}(ax)^2} - \frac{2x^2\sqrt{1-a^2x^2}}{a^3\sin^{-1}(ax)} + \frac{25x^4\sqrt{1-a^2x^2}}{6a\sin^{-1}(ax)} + \frac{2\text{Subst}}{a^2} \\
 &= -\frac{x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{2x^3}{3a^2\sin^{-1}(ax)^2} + \frac{5x^5}{6\sin^{-1}(ax)^2} - \frac{2x^2\sqrt{1-a^2x^2}}{a^3\sin^{-1}(ax)} + \frac{25x^4\sqrt{1-a^2x^2}}{6a\sin^{-1}(ax)} - \frac{\text{Subst}}{a^2} \\
 &= -\frac{x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{2x^3}{3a^2\sin^{-1}(ax)^2} + \frac{5x^5}{6\sin^{-1}(ax)^2} - \frac{2x^2\sqrt{1-a^2x^2}}{a^3\sin^{-1}(ax)} + \frac{25x^4\sqrt{1-a^2x^2}}{6a\sin^{-1}(ax)} + \frac{\text{Si}(\sin^{-1}(ax))}{4a^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.29, size = 159, normalized size = 1.01

$$\frac{80a^5x^5\sin^{-1}(ax) - 64a^3x^3\sin^{-1}(ax) - 192a^2x^2\sqrt{1-a^2x^2}\sin^{-1}(ax)^2 - 32a^4x^4\sqrt{1-a^2x^2} + 400a^4x^4\sqrt{1-a^2x^2}\sin^{-1}(ax)}{96a^5\sin^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcSin[a\*x]^4,x]

[Out] (-32\*a^4\*x^4\*Sqrt[1 - a^2\*x^2] - 64\*a^3\*x^3\*ArcSin[a\*x] + 80\*a^5\*x^5\*ArcSin[a\*x] - 192\*a^2\*x^2\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2 + 400\*a^4\*x^4\*Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2 + 2\*ArcSin[a\*x]^3\*SinIntegral[ArcSin[a\*x]] - 81\*ArcSin[a\*x]^3\*SinIntegral[3\*ArcSin[a\*x]] + 125\*ArcSin[a\*x]^3\*SinIntegral[5\*ArcSin[a\*x]])/(96\*a^5\*ArcSin[a\*x]^3)

**fricas [F]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\arcsin(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^4,x, algorithm="fricas")

[Out] integral(x^4/arcsin(a\*x)^4, x)

**giac [A]** time = 0.28, size = 250, normalized size = 1.58

$$\frac{5(a^2x^2-1)^2x}{6a^4\arcsin(ax)^2} + \frac{25(a^2x^2-1)^2\sqrt{-a^2x^2+1}}{6a^5\arcsin(ax)} + \frac{(a^2x^2-1)x}{a^4\arcsin(ax)^2} + \frac{125\text{Si}(5\arcsin(ax))}{96a^5} - \frac{27\text{Si}(3\arcsin(ax))}{32a^5} + \frac{\text{Si}(\arcsin(ax))}{4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^4,x, algorithm="giac")

[Out] 5/6\*(a^2\*x^2 - 1)^2\*x/(a^4\*arcsin(a\*x)^2) + 25/6\*(a^2\*x^2 - 1)^2\*sqrt(-a^2\*x^2 + 1)/(a^5\*arcsin(a\*x)) + (a^2\*x^2 - 1)\*x/(a^4\*arcsin(a\*x)^2) + 125/96\*Si(5\*arcsin(a\*x))/a^5 - 27/32\*Si(3\*arcsin(a\*x))/a^5 + 1/48\*Si(arcsin(a\*x))/a^5 - 19/3\*(-a^2\*x^2 + 1)^(3/2)/(a^5\*arcsin(a\*x))

$x)) + 1/6*x/(a^4*\arcsin(a*x)^2) - 1/3*(a^2*x^2 - 1)^2*\sqrt{-a^2*x^2 + 1}/(a^5*\arcsin(a*x)^3) + 13/6*\sqrt{-a^2*x^2 + 1}/(a^5*\arcsin(a*x)) + 2/3*(-a^2*x^2 + 1)^{(3/2)}/(a^5*\arcsin(a*x)^3) - 1/3*\sqrt{-a^2*x^2 + 1}/(a^5*\arcsin(a*x)^3)$

**maple [A]** time = 0.06, size = 171, normalized size = 1.08

$$\frac{-\frac{\sqrt{-a^2x^2+1}}{24 \arcsin(ax)^3} + \frac{ax}{48 \arcsin(ax)^2} + \frac{\sqrt{-a^2x^2+1}}{48 \arcsin(ax)} + \frac{\text{Si}(\arcsin(ax))}{48} + \frac{\cos(3 \arcsin(ax))}{16 \arcsin(ax)^3} - \frac{3 \sin(3 \arcsin(ax))}{32 \arcsin(ax)^2} - \frac{9 \cos(3 \arcsin(ax))}{32 \arcsin(ax)} - \frac{27 \sin(3 \arcsin(ax))}{32 \arcsin(ax)^3}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arcsin(a\*x)^4,x)

[Out] 1/a^5\*(-1/24/arcsin(a\*x)^3\*(-a^2\*x^2+1)^(1/2)+1/48\*a\*x/arcsin(a\*x)^2+1/48/arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2)+1/48\*Si(arcsin(a\*x))+1/16/arcsin(a\*x)^3\*cos(3\*arcsin(a\*x))-3/32/arcsin(a\*x)^2\*sin(3\*arcsin(a\*x))-9/32/arcsin(a\*x)\*cos(3\*arcsin(a\*x))-27/32\*Si(3\*arcsin(a\*x))-1/48/arcsin(a\*x)^3\*cos(5\*arcsin(a\*x))+5/96/arcsin(a\*x)^2\*sin(5\*arcsin(a\*x))+25/96/arcsin(a\*x)\*cos(5\*arcsin(a\*x))+125/96\*Si(5\*arcsin(a\*x)))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^3 \int \frac{(125a^4x^5 - 136a^2x^3 + 24x)\sqrt{ax+1}\sqrt{-ax+1}}{(a^5x^2 - a^3) \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx + (2a^2x^4 - (25a^2x^4 - 12x^2) \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1}))}{6a^3 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^4,x, algorithm="maxima")

[Out] -1/6\*(6\*a^3\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^3\*integrate(1/6\*(125\*a^4\*x^5 - 136\*a^2\*x^3 + 24\*x)\*sqrt(a\*x + 1)\*sqrt(-a\*x + 1)/((a^5\*x^2 - a^3)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))), x) + (2\*a^2\*x^4 - (25\*a^2\*x^4 - 12\*x^2)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2)\*sqrt(a\*x + 1)\*sqrt(-a\*x + 1) - (5\*a^3\*x^5 - 4\*a\*x^3)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))/((a^3\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^3)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\arcsin(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/asin(a\*x)^4,x)

[Out] int(x^4/asin(a\*x)^4, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\arcsin^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/asin(a\*x)\*\*4,x)

[Out] Integral(x\*\*4/asin(a\*x)\*\*4, x)

$$3.68 \quad \int \frac{x^3}{\sin^{-1}(ax)^4} dx$$

**Optimal.** Leaf size=144

$$-\frac{\text{Ci}(2 \sin^{-1}(ax))}{3a^4} + \frac{4\text{Ci}(4 \sin^{-1}(ax))}{3a^4} - \frac{x^2}{2a^2 \sin^{-1}(ax)^2} + \frac{8x^3 \sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)} - \frac{x^3 \sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} - \frac{x \sqrt{1-a^2x^2}}{a^3 \sin^{-1}(ax)} + \frac{2x^4}{3 \sin^{-1}(ax)^2}$$

[Out]  $-1/2*x^2/a^2/\arcsin(a*x)^2+2/3*x^4/\arcsin(a*x)^2-1/3*\text{Ci}(2*\arcsin(a*x))/a^4+4/3*\text{Ci}(4*\arcsin(a*x))/a^4-1/3*x^3*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^3-x*(-a^2*x^2+1)^{(1/2)}/a^3/\arcsin(a*x)+8/3*x^3*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)$

**Rubi [A]** time = 0.28, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4633, 4719, 4631, 3302}

$$-\frac{\text{CosIntegral}(2 \sin^{-1}(ax))}{3a^4} + \frac{4\text{CosIntegral}(4 \sin^{-1}(ax))}{3a^4} + \frac{8x^3 \sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)} - \frac{x^3 \sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} - \frac{x^2}{2a^2 \sin^{-1}(ax)^2} - \frac{x \sqrt{1-a^2x^2}}{a^3 \sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcSin[a\*x]^4,x]

[Out]  $-(x^3*\text{Sqrt}[1-a^2*x^2])/(3*a*\text{ArcSin}[a*x]^3) - x^2/(2*a^2*\text{ArcSin}[a*x]^2) + (2*x^4)/(3*\text{ArcSin}[a*x]^2) - (x*\text{Sqrt}[1-a^2*x^2])/(a^3*\text{ArcSin}[a*x]) + (8*x^3*\text{Sqrt}[1-a^2*x^2])/(3*a*\text{ArcSin}[a*x]) - \text{CosIntegral}[2*\text{ArcSin}[a*x]]/(3*a^4) + (4*\text{CosIntegral}[4*\text{ArcSin}[a*x]])/(3*a^4)$

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*((f\_.)\*(x\_))^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sin^{-1}(ax)^4} dx &= -\frac{x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} + \frac{\int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx}{a} - \frac{1}{3}(4a) \int \frac{x^4}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x^2}{2a^2\sin^{-1}(ax)^2} + \frac{2x^4}{3\sin^{-1}(ax)^2} - \frac{8}{3} \int \frac{x^3}{\sin^{-1}(ax)^2} dx + \frac{\int \frac{x}{\sin^{-1}(ax)^2} dx}{a^2} \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x^2}{2a^2\sin^{-1}(ax)^2} + \frac{2x^4}{3\sin^{-1}(ax)^2} - \frac{x\sqrt{1-a^2x^2}}{a^3\sin^{-1}(ax)} + \frac{8x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)} + \frac{\text{Subst}}{\sin^{-1}(ax)} \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x^2}{2a^2\sin^{-1}(ax)^2} + \frac{2x^4}{3\sin^{-1}(ax)^2} - \frac{x\sqrt{1-a^2x^2}}{a^3\sin^{-1}(ax)} + \frac{8x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)} + \frac{\text{Ci}(2\sin^{-1}(ax))}{\sin^{-1}(ax)} \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x^2}{2a^2\sin^{-1}(ax)^2} + \frac{2x^4}{3\sin^{-1}(ax)^2} - \frac{x\sqrt{1-a^2x^2}}{a^3\sin^{-1}(ax)} + \frac{8x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)} - \frac{\text{Ci}(2\sin^{-1}(ax))}{\sin^{-1}(ax)}
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 107, normalized size = 0.74

$$\frac{ax(-2a^2x^2\sqrt{1-a^2x^2}+ax(4a^2x^2-3)\sin^{-1}(ax)+2\sqrt{1-a^2x^2}(8a^2x^2-3)\sin^{-1}(ax)^2)}{\sin^{-1}(ax)^3} - 2\text{Ci}(2\sin^{-1}(ax)) + 8\text{Ci}(4\sin^{-1}(ax))}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcSin[a\*x]^4,x]

[Out] ((a\*x\*(-2\*a^2\*x^2\*Sqrt[1 - a^2\*x^2] + a\*x\*(-3 + 4\*a^2\*x^2)\*ArcSin[a\*x] + 2\*Sqrt[1 - a^2\*x^2]\*(-3 + 8\*a^2\*x^2)\*ArcSin[a\*x]^2))/ArcSin[a\*x]^3 - 2\*CosIntegral[2\*ArcSin[a\*x]] + 8\*CosIntegral[4\*ArcSin[a\*x]])/(6\*a^4)

**fricas [F]** time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\arcsin(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^4,x, algorithm="fricas")

[Out] integral(x^3/arcsin(a\*x)^4, x)

**giac [A]** time = 0.23, size = 174, normalized size = 1.21

$$-\frac{8(-a^2x^2+1)^{\frac{3}{2}}x}{3a^3\arcsin(ax)} + \frac{5\sqrt{-a^2x^2+1}x}{3a^3\arcsin(ax)} + \frac{4\text{Ci}(4\arcsin(ax))}{3a^4} - \frac{\text{Ci}(2\arcsin(ax))}{3a^4} + \frac{(-a^2x^2+1)^{\frac{3}{2}}x}{3a^3\arcsin(ax)^3} + \frac{2(a^2x^2-1)}{3a^4\arcsin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^4,x, algorithm="giac")

[Out] -8/3\*(-a^2\*x^2 + 1)^(3/2)\*x/(a^3\*arcsin(a\*x)) + 5/3\*sqrt(-a^2\*x^2 + 1)\*x/(a^3\*arcsin(a\*x)) + 4/3\*cos\_integral(4\*arcsin(a\*x))/a^4 - 1/3\*cos\_integral(2\*arcsin(a\*x))/a^4 + 1/3\*(-a^2\*x^2 + 1)^(3/2)\*x/(a^3\*arcsin(a\*x)^3) + 2/3\*(a^2\*x^2 - 1)/(a^4\*arcsin(a\*x)^2) - 1/3\*sqrt(-a^2\*x^2 + 1)\*x/(a^3\*arcsin(a\*x)^3) + 5/6\*(a^2\*x^2 - 1)/(a^4\*arcsin(a\*x)^2) + 1/6/(a^4\*arcsin(a\*x)^2)

**maple [A]** time = 0.06, size = 114, normalized size = 0.79

$$\frac{-\frac{\sin(2\arcsin(ax))}{12\arcsin(ax)^3} - \frac{\cos(2\arcsin(ax))}{12\arcsin(ax)^2} + \frac{\sin(2\arcsin(ax))}{6\arcsin(ax)} - \frac{\text{Ci}(2\arcsin(ax))}{3} + \frac{\sin(4\arcsin(ax))}{24\arcsin(ax)^3} + \frac{\cos(4\arcsin(ax))}{12\arcsin(ax)^2} - \frac{\sin(4\arcsin(ax))}{3\arcsin(ax)}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/arcsin(a*x)^4,x)`

[Out]  $1/a^4*(-1/12/\arcsin(ax)^3*\sin(2*\arcsin(ax))-1/12/\arcsin(ax)^2*\cos(2*\arcsin(ax))+1/6/\arcsin(ax)*\sin(2*\arcsin(ax))-1/3*Ci(2*\arcsin(ax))+1/24/\arcsin(ax)^3*\sin(4*\arcsin(ax))+1/12/\arcsin(ax)^2*\cos(4*\arcsin(ax))-1/3/\arcsin(ax)*\sin(4*\arcsin(ax))+4/3*Ci(4*\arcsin(ax)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2a^3 \arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)^3 \int \frac{(32a^4x^4 - 30a^2x^2 + 3)\sqrt{ax+1}\sqrt{-ax+1}}{(a^5x^2 - a^3)\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx + 2\left(a^2x^3 - (8a^2x^3 - 3x)\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})\right)$$

---


$$6a^3 \arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arcsin(a*x)^4,x, algorithm="maxima")`

[Out]  $-1/6*(6*a^3*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})^3*\integrate(1/3*(32*a^4*x^4 - 30*a^2*x^2 + 3)*\sqrt{ax+1}*\sqrt{-ax+1}/((a^5*x^2 - a^3)*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})), x) + 2*(a^2*x^3 - (8*a^2*x^3 - 3*x)*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})^2)*\sqrt{ax+1}*\sqrt{-ax+1} - (4*a^3*x^4 - 3*a*x^2)*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1}))/a^3*\arctan2(ax, \sqrt{ax+1}*\sqrt{-ax+1})^3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{asin}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/asin(a*x)^4,x)`

[Out] `int(x^3/asin(a*x)^4, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asin}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/asin(a*x)**4,x)`

[Out] `Integral(x**3/asin(a*x)**4, x)`

$$3.69 \quad \int \frac{x^2}{\sin^{-1}(ax)^4} dx$$

**Optimal.** Leaf size=141

$$\frac{\operatorname{Si}(\sin^{-1}(ax))}{24a^3} - \frac{9\operatorname{Si}(3\sin^{-1}(ax))}{8a^3} + \frac{3x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)} - \frac{x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x}{3a^2\sin^{-1}(ax)^2} - \frac{\sqrt{1-a^2x^2}}{3a^3\sin^{-1}(ax)} + \frac{x^3}{2\sin^{-1}(ax)^2}$$

[Out] -1/3\*x/a^2/arcsin(a\*x)^2+1/2\*x^3/arcsin(a\*x)^2+1/24\*Si(arcsin(a\*x))/a^3-9/8\*Si(3\*arcsin(a\*x))/a^3-1/3\*x^2\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)^3-1/3\*(-a^2\*x^2+1)^(1/2)/a^3/arcsin(a\*x)+3/2\*x^2\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)

**Rubi [A]** time = 0.30, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4633, 4719, 4631, 3299, 4621, 4723}

$$\frac{\operatorname{Si}(\sin^{-1}(ax))}{24a^3} - \frac{9\operatorname{Si}(3\sin^{-1}(ax))}{8a^3} + \frac{3x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)} - \frac{x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{\sqrt{1-a^2x^2}}{3a^3\sin^{-1}(ax)} - \frac{x}{3a^2\sin^{-1}(ax)^2} + \frac{x^3}{2\sin^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcSin[a\*x]^4,x]

[Out] -(x^2\*Sqrt[1 - a^2\*x^2])/(3\*a\*ArcSin[a\*x]^3) - x/(3\*a^2\*ArcSin[a\*x]^2) + x^3/(2\*ArcSin[a\*x]^2) - Sqrt[1 - a^2\*x^2]/(3\*a^3\*ArcSin[a\*x]) + (3\*x^2\*Sqrt[1 - a^2\*x^2])/(2\*a\*ArcSin[a\*x]) + SinIntegral[ArcSin[a\*x]]/(24\*a^3) - (9\*SinIntegral[3\*ArcSin[a\*x]])/(8\*a^3)

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_], x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(x\_)^m\_], x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(x\_)^m\_], x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4719

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(f\_.\*(x\_)^m\_)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b

```
*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] & & EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

**Rule 4723**

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x]^(2*p + 1), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\sin^{-1}(ax)^4} dx = -\frac{x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} + \frac{2\int \frac{x}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx}{3a} - a\int \frac{x^3}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx$$

$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x}{3a^2\sin^{-1}(ax)^2} + \frac{x^3}{2\sin^{-1}(ax)^2} - \frac{3}{2}\int \frac{x^2}{\sin^{-1}(ax)^2} dx + \frac{\int \frac{1}{\sin^{-1}(ax)^2} dx}{3a^2}$$

$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x}{3a^2\sin^{-1}(ax)^2} + \frac{x^3}{2\sin^{-1}(ax)^2} - \frac{\sqrt{1-a^2x^2}}{3a^3\sin^{-1}(ax)} + \frac{3x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)} - \frac{3\text{Subst}\left(\frac{1}{\sin^{-1}(ax)^2}, \sin^{-1}(ax)\right)}{3a^2}$$

$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x}{3a^2\sin^{-1}(ax)^2} + \frac{x^3}{2\sin^{-1}(ax)^2} - \frac{\sqrt{1-a^2x^2}}{3a^3\sin^{-1}(ax)} + \frac{3x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)} - \frac{\text{Subst}\left(\frac{1}{\sin^{-1}(ax)^2}, \sin^{-1}(ax)\right)}{3a^2}$$

$$= -\frac{x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{x}{3a^2\sin^{-1}(ax)^2} + \frac{x^3}{2\sin^{-1}(ax)^2} - \frac{\sqrt{1-a^2x^2}}{3a^3\sin^{-1}(ax)} + \frac{3x^2\sqrt{1-a^2x^2}}{2a\sin^{-1}(ax)} + \frac{\text{Si}\left(\sin^{-1}(ax)\right)}{24a^3}$$

**Mathematica [A]** time = 0.23, size = 102, normalized size = 0.72

$$\frac{-\frac{8a^2x^2\sqrt{1-a^2x^2}}{\sin^{-1}(ax)^3} + \frac{4ax(3a^2x^2-2)}{\sin^{-1}(ax)^2} + \frac{4\sqrt{1-a^2x^2}(9a^2x^2-2)}{\sin^{-1}(ax)} + \text{Si}\left(\sin^{-1}(ax)\right) - 27\text{Si}\left(3\sin^{-1}(ax)\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSin[a*x]^4,x]
[Out] ((-8*a^2*x^2*Sqrt[1 - a^2*x^2])/ArcSin[a*x]^3 + (4*a*x*(-2 + 3*a^2*x^2))/ArcSin[a*x]^2 + (4*Sqrt[1 - a^2*x^2]*(-2 + 9*a^2*x^2))/ArcSin[a*x] + SinIntegral[ArcSin[a*x]] - 27*SinIntegral[3*ArcSin[a*x]])/(24*a^3)
```

**fricas [F]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\arcsin(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsin(a*x)^4,x, algorithm="fricas")
[Out] integral(x^2/arcsin(a*x)^4, x)
```

**giac [A]** time = 0.21, size = 148, normalized size = 1.05

$$\frac{(a^2x^2 - 1)x}{2a^2\arcsin(ax)^2} - \frac{9\text{Si}(3\arcsin(ax))}{8a^3} + \frac{\text{Si}(\arcsin(ax))}{24a^3} - \frac{3(-a^2x^2 + 1)^{\frac{3}{2}}}{2a^3\arcsin(ax)} + \frac{x}{6a^2\arcsin(ax)^2} + \frac{7\sqrt{-a^2x^2 + 1}}{6a^3\arcsin(ax)} + \frac{\text{Si}\left(\sin^{-1}(ax)\right)}{24a^3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^4,x, algorithm="giac")

[Out]  $\frac{1}{2}(a^2x^2 - 1)x/(a^2\arcsin(ax)^2) - \frac{9}{8}\sin\_integral(3\arcsin(ax))/a^3 + \frac{1}{24}\sin\_integral(\arcsin(ax))/a^3 - \frac{3}{2}(-a^2x^2 + 1)^{(3/2)}/(a^3\arcsin(ax)) + \frac{1}{6}x/(a^2\arcsin(ax)^2) + \frac{7}{6}\sqrt{-a^2x^2 + 1}/(a^3\arcsin(ax)) + \frac{1}{3}(-a^2x^2 + 1)^{(3/2)}/(a^3\arcsin(ax)^3) - \frac{1}{3}\sqrt{-a^2x^2 + 1}/(a^3\arcsin(ax)^3)$

**maple [A]** time = 0.04, size = 117, normalized size = 0.83

$$\frac{-\frac{\sqrt{-a^2x^2+1}}{12\arcsin(ax)^3} + \frac{ax}{24\arcsin(ax)^2} + \frac{\sqrt{-a^2x^2+1}}{24\arcsin(ax)} + \frac{\text{Si}(\arcsin(ax))}{24} + \frac{\cos(3\arcsin(ax))}{12\arcsin(ax)^3} - \frac{\sin(3\arcsin(ax))}{8\arcsin(ax)^2} - \frac{3\cos(3\arcsin(ax))}{8\arcsin(ax)} - \frac{9\text{Si}(3\arcsin(ax))}{8\arcsin(ax)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arcsin(a\*x)^4,x)

[Out]  $\frac{1}{a^3}(-\frac{1}{12}\arcsin(ax)^3(-a^2x^2+1)^{(1/2)} + \frac{1}{24}ax/\arcsin(ax)^2 + \frac{1}{24}a\arcsin(ax)*(-a^2x^2+1)^{(1/2)} + \frac{1}{24}\text{Si}(\arcsin(ax)) + \frac{1}{12}\arcsin(ax)^3\cos(3\arcsin(ax)) - \frac{1}{8}\arcsin(ax)^2\sin(3\arcsin(ax)) - \frac{3}{8}\arcsin(ax)\cos(3\arcsin(ax)) - \frac{9}{8}\text{Si}(3\arcsin(ax)))$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)^3 \int \frac{(27a^2x^3-20x)\sqrt{ax+1}\sqrt{-ax+1}}{(a^3x^2-a)\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx + \left(2a^2x^2 - (9a^2x^2 - 2)\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})\right)}{6a^3 \arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^4,x, algorithm="maxima")

[Out]  $-\frac{1}{6}(6a^3\arctan^2(ax, \sqrt{ax+1}\sqrt{-ax+1})^3 \int \frac{(27a^2x^3-20x)\sqrt{ax+1}\sqrt{-ax+1}}{(a^3x^2-a)\arctan^2(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx + (2a^2x^2 - (9a^2x^2 - 2)\arctan^2(ax, \sqrt{ax+1}\sqrt{-ax+1}))\sqrt{ax+1}\sqrt{-ax+1} - (3a^3x^3 - 2a^2x)\arctan^2(ax, \sqrt{ax+1}\sqrt{-ax+1}))/a^3\arctan^2(ax, \sqrt{ax+1}\sqrt{-ax+1})^3$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\text{asin}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/asin(a\*x)^4,x)

[Out] int(x^2/asin(a\*x)^4, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{asin}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/asin(a\*x)\*\*4,x)

[Out] Integral(x\*\*2/asin(a\*x)\*\*4, x)

### 3.70 $\int \frac{x}{\sin^{-1}(ax)^4} dx$

**Optimal.** Leaf size=97

$$-\frac{2\text{Ci}(2\sin^{-1}(ax))}{3a^2} + \frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)} - \frac{x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{1}{6a^2\sin^{-1}(ax)^2} + \frac{x^2}{3\sin^{-1}(ax)^2}$$

[Out]  $-1/6/a^2/\arcsin(ax)^2+1/3*x^2/\arcsin(ax)^2-2/3*Ci(2*\arcsin(ax))/a^2-1/3*x*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(ax)^3+2/3*x*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(ax)$

**Rubi [A]** time = 0.16, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4633, 4719, 4631, 3302, 4641}

$$-\frac{2\text{CosIntegral}(2\sin^{-1}(ax))}{3a^2} + \frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)} - \frac{x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{1}{6a^2\sin^{-1}(ax)^2} + \frac{x^2}{3\sin^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSin[a\*x]^4,x]

[Out]  $-(x*\text{Sqrt}[1-a^2*x^2])/(3*a*\text{ArcSin}[a*x]^3) - 1/(6*a^2*\text{ArcSin}[a*x]^2) + x^2/(3*\text{ArcSin}[a*x]^2) + (2*x*\text{Sqrt}[1-a^2*x^2])/(3*a*\text{ArcSin}[a*x]) - (2*\text{CosIntegral}[2*\text{ArcSin}[a*x]])/(3*a^2)$

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4719

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*((f\_.)\*(x\_)^(m\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &

& EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

### Rubi steps

$$\begin{aligned} \int \frac{x}{\sin^{-1}(ax)^4} dx &= -\frac{x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx}{3a} - \frac{1}{3}(2a) \int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^3} dx \\ &= -\frac{x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{1}{6a^2\sin^{-1}(ax)^2} + \frac{x^2}{3\sin^{-1}(ax)^2} - \frac{2}{3} \int \frac{x}{\sin^{-1}(ax)^2} dx \\ &= -\frac{x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{1}{6a^2\sin^{-1}(ax)^2} + \frac{x^2}{3\sin^{-1}(ax)^2} + \frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)} - \frac{2 \operatorname{Subst}\left(\int \frac{\cos(2x)}{x} dx, x\right)}{3a^2} \\ &= -\frac{x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^3} - \frac{1}{6a^2\sin^{-1}(ax)^2} + \frac{x^2}{3\sin^{-1}(ax)^2} + \frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)} - \frac{2\operatorname{Ci}\left(2\sin^{-1}(ax)\right)}{3a^2} \end{aligned}$$

**Mathematica** [A] time = 0.12, size = 86, normalized size = 0.89

$$\frac{-2ax\sqrt{1-a^2x^2} + 4ax\sqrt{1-a^2x^2}\sin^{-1}(ax)^2 + (2a^2x^2 - 1)\sin^{-1}(ax) - 4\sin^{-1}(ax)^3\operatorname{Ci}\left(2\sin^{-1}(ax)\right)}{6a^2\sin^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcSin[a\*x]^4, x]

[Out]  $(-2*a*x*\sqrt{1 - a^2*x^2} + (-1 + 2*a^2*x^2)*\operatorname{ArcSin}[a*x] + 4*a*x*\sqrt{1 - a^2*x^2}*\operatorname{ArcSin}[a*x]^2 - 4*\operatorname{ArcSin}[a*x]^3*\operatorname{CosIntegral}[2*\operatorname{ArcSin}[a*x]])/(6*a^2*\operatorname{ArcSin}[a*x]^3)$

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x}{\operatorname{arcsin}(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^4, x, algorithm="fricas")

[Out] integral(x/arcsin(a\*x)^4, x)

**giac** [A] time = 0.38, size = 92, normalized size = 0.95

$$\frac{2\sqrt{-a^2x^2+1}x}{3a\operatorname{arcsin}(ax)} - \frac{2\operatorname{Ci}(2\operatorname{arcsin}(ax))}{3a^2} - \frac{\sqrt{-a^2x^2+1}x}{3a\operatorname{arcsin}(ax)^3} + \frac{a^2x^2-1}{3a^2\operatorname{arcsin}(ax)^2} + \frac{1}{6a^2\operatorname{arcsin}(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^4, x, algorithm="giac")

[Out]  $2/3*\sqrt{-a^2*x^2 + 1}*x/(a*\operatorname{arcsin}(a*x)) - 2/3*\operatorname{cos\_integral}(2*\operatorname{arcsin}(a*x))/a^2 - 1/3*\sqrt{-a^2*x^2 + 1}*x/(a*\operatorname{arcsin}(a*x)^3) + 1/3*(a^2*x^2 - 1)/(a^2*a*\operatorname{rccsin}(a*x)^2) + 1/6/(a^2*\operatorname{arcsin}(a*x)^2)$

**maple** [A] time = 0.04, size = 60, normalized size = 0.62

$$\frac{-\frac{\sin(2\operatorname{arcsin}(ax))}{6\operatorname{arcsin}(ax)^3} - \frac{\cos(2\operatorname{arcsin}(ax))}{6\operatorname{arcsin}(ax)^2} + \frac{\sin(2\operatorname{arcsin}(ax))}{3\operatorname{arcsin}(ax)} - \frac{2\operatorname{Ci}(2\operatorname{arcsin}(ax))}{3}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/arcsin(a*x)^4,x)`

[Out]  $1/a^2*(-1/6/\arcsin(a*x)^3*\sin(2*\arcsin(a*x))-1/6/\arcsin(a*x)^2*\cos(2*\arcsin(a*x))+1/3/\arcsin(a*x)*\sin(2*\arcsin(a*x))-2/3*Ci(2*\arcsin(a*x)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4a^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^3 \int \frac{(2a^2x^2-1)\sqrt{ax+1} \sqrt{-ax+1}}{(a^3x^2-a) \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx - 2 \left(2ax \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right) - 2a^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^3\right)}{6a^2 \arctan\left(ax, \sqrt{ax+1} \sqrt{-ax+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/arcsin(a*x)^4,x, algorithm="maxima")`

[Out]  $-1/6*(6*a^2*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^3*\integrate(2/3*(2*a^2*x^2-1)*\sqrt{a*x+1}*\sqrt{-a*x+1}/((a^3*x^2-a)*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})), x) - 2*(2*a*x*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^2 - a*x)*\sqrt{a*x+1}*\sqrt{-a*x+1} - (2*a^2*x^2-1)*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1}))/a^2*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asin}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/asin(a*x)^4,x)`

[Out] `int(x/asin(a*x)^4, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asin}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/asin(a*x)**4,x)`

[Out] `Integral(x/asin(a*x)**4, x)`

$$3.71 \quad \int \frac{1}{\sin^{-1}(ax)^4} dx$$

**Optimal.** Leaf size=78

$$\frac{\sqrt{1-a^2x^2}}{6a \sin^{-1}(ax)} - \frac{\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} + \frac{\text{Si}(\sin^{-1}(ax))}{6a} + \frac{x}{6 \sin^{-1}(ax)^2}$$

[Out] 1/6\*x/arcsin(a\*x)^2+1/6\*Si(arcsin(a\*x))/a-1/3\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)^3+1/6\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)

**Rubi [A]** time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4621, 4719, 4723, 3299}

$$\frac{\sqrt{1-a^2x^2}}{6a \sin^{-1}(ax)} - \frac{\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} + \frac{\text{Si}(\sin^{-1}(ax))}{6a} + \frac{x}{6 \sin^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(-4), x]

[Out] -Sqrt[1 - a^2\*x^2]/(3\*a\*ArcSin[a\*x]^3) + x/(6\*ArcSin[a\*x]^2) + Sqrt[1 - a^2\*x^2]/(6\*a\*ArcSin[a\*x]) + SinIntegral[ArcSin[a\*x]]/(6\*a)

Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n)\*((f\_.)\*(x\_))^(m\_)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^(m\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sin^{-1}(ax)^4} dx &= -\frac{\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} - \frac{1}{3}a \int \frac{x}{\sqrt{1-a^2x^2} \sin^{-1}(ax)^3} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} + \frac{x}{6 \sin^{-1}(ax)^2} - \frac{1}{6} \int \frac{1}{\sin^{-1}(ax)^2} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} + \frac{x}{6 \sin^{-1}(ax)^2} + \frac{\sqrt{1-a^2x^2}}{6a \sin^{-1}(ax)} + \frac{1}{6}a \int \frac{x}{\sqrt{1-a^2x^2} \sin^{-1}(ax)} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} + \frac{x}{6 \sin^{-1}(ax)^2} + \frac{\sqrt{1-a^2x^2}}{6a \sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, \sin^{-1}(ax)\right)}{6a} \\
&= -\frac{\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^3} + \frac{x}{6 \sin^{-1}(ax)^2} + \frac{\sqrt{1-a^2x^2}}{6a \sin^{-1}(ax)} + \frac{\text{Si}(\sin^{-1}(ax))}{6a}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 70, normalized size = 0.90

$$\frac{-2\sqrt{1-a^2x^2} + \sqrt{1-a^2x^2} \sin^{-1}(ax)^2 + \sin^{-1}(ax)^3 \text{Si}(\sin^{-1}(ax)) + ax \sin^{-1}(ax)}{6a \sin^{-1}(ax)^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^(-4), x]

[Out] (-2\*Sqrt[1 - a^2\*x^2] + a\*x\*ArcSin[a\*x] + Sqrt[1 - a^2\*x^2]\*ArcSin[a\*x]^2 + ArcSin[a\*x]^3\*SinIntegral[ArcSin[a\*x]])/(6\*a\*ArcSin[a\*x]^3)

**fricas [F]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\arcsin(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^4, x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^(-4), x)

**giac [A]** time = 0.25, size = 66, normalized size = 0.85

$$\frac{\text{Si}(\arcsin(ax))}{6a} + \frac{x}{6 \arcsin(ax)^2} + \frac{\sqrt{-a^2x^2+1}}{6a \arcsin(ax)} - \frac{\sqrt{-a^2x^2+1}}{3a \arcsin(ax)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^4, x, algorithm="giac")

[Out] 1/6\*sin\_integral(arcsin(a\*x))/a + 1/6\*x/arcsin(a\*x)^2 + 1/6\*sqrt(-a^2\*x^2 + 1)/(a\*arcsin(a\*x)) - 1/3\*sqrt(-a^2\*x^2 + 1)/(a\*arcsin(a\*x)^3)

**maple [A]** time = 0.03, size = 63, normalized size = 0.81

$$\frac{-\frac{\sqrt{-a^2x^2+1}}{3 \arcsin(ax)^3} + \frac{ax}{6 \arcsin(ax)^2} + \frac{\sqrt{-a^2x^2+1}}{6 \arcsin(ax)} + \frac{\text{Si}(\arcsin(ax))}{6}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arcsin(a\*x)^4, x)

[Out]  $1/a*(-1/3/\arcsin(ax)^3*(-a^2*x^2+1)^{(1/2)}+1/6*a*x/\arcsin(ax)^2+1/6/\arcsin(ax)*(-a^2*x^2+1)^{(1/2)}+1/6*Si(\arcsin(ax)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^3 \int \frac{\sqrt{ax+1} \sqrt{-ax+1} x}{(a^2 x^2 - 1) \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx - ax \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}{6 a \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^4,x, algorithm="maxima")

[Out]  $-1/6*(6*a^2*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1}))^3*\integrate(1/6*\sqrt{a*x+1}*\sqrt{-a*x+1}*x/((a^2*x^2-1)*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})), x) - a*x*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1}) - \sqrt{a*x+1}*\sqrt{-a*x+1}*(\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^2 - 2)/(a*\arctan2(a*x, \sqrt{a*x+1}*\sqrt{-a*x+1})^3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asin}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/asin(a\*x)^4,x)

[Out] int(1/asin(a\*x)^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asin}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/asin(a\*x)\*\*4,x)

[Out] Integral(asin(a\*x)\*\*(-4), x)

$$3.72 \quad \int \frac{1}{x \sin^{-1}(ax)^4} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x \sin^{-1}(ax)^4}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x)^4,x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x \sin^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*ArcSin[a\*x]^4),x]

[Out] Defer[Int][1/(x\*ArcSin[a\*x]^4), x]

Rubi steps

$$\int \frac{1}{x \sin^{-1}(ax)^4} dx = \int \frac{1}{x \sin^{-1}(ax)^4} dx$$

Mathematica [A] time = 2.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sin^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*ArcSin[a\*x]^4),x]

[Out] Integrate[1/(x\*ArcSin[a\*x]^4), x]

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x \arcsin(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^4,x, algorithm="fricas")

[Out] integral(1/(x\*arcsin(a\*x)^4), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^4,x, algorithm="giac")

[Out] integrate(1/(x\*arcsin(a\*x)^4), x)



**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x)^4,x)

[Out] int(1/x/arcsin(a\*x)^4,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2a^3x^3 \arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)^3 \int \frac{(2a^2x^2-3)\sqrt{ax+1}\sqrt{-ax+1}}{(a^5x^6-a^3x^4)\arctan(ax, \sqrt{ax+1}\sqrt{-ax+1})} dx - ax \arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)^3}{6a^3x^3 \arctan\left(ax, \sqrt{ax+1}\sqrt{-ax+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^4,x, algorithm="maxima")

[Out] -1/6\*(6\*a^3\*x^3\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^3\*integrate(1/3\*(2\*a^2\*x^2 - 3)\*sqrt(a\*x + 1)\*sqrt(-a\*x + 1)/((a^5\*x^6 - a^3\*x^4)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))), x) - a\*x\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)) + 2\*(a^2\*x^2 + arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2)\*sqrt(a\*x + 1)\*sqrt(-a\*x + 1)/(a^3\*x^3\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^3)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{asin}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)^4),x)

[Out] int(1/(x\*asin(a\*x)^4), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asin}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x)\*\*4,x)

[Out] Integral(1/(x\*asin(a\*x)\*\*4), x)

$$3.73 \quad \int \frac{1}{x^2 \sin^{-1}(ax)^4} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sin^{-1}(ax)^4}, x\right)$$

[Out] Unintegrable(1/x^2/arcsin(a\*x)^4,x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 \sin^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*ArcSin[a\*x]^4),x]

[Out] Defer[Int][1/(x^2\*ArcSin[a\*x]^4), x]

Rubi steps

$$\int \frac{1}{x^2 \sin^{-1}(ax)^4} dx = \int \frac{1}{x^2 \sin^{-1}(ax)^4} dx$$

Mathematica [A] time = 17.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sin^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*ArcSin[a\*x]^4),x]

[Out] Integrate[1/(x^2\*ArcSin[a\*x]^4), x]

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \arcsin(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^4,x, algorithm="fricas")

[Out] integral(1/(x^2\*arcsin(a\*x)^4), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^4,x, algorithm="giac")

[Out] integrate(1/(x^2\*arcsin(a\*x)^4), x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \arcsin(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arcsin(a\*x)^4,x)

[Out] int(1/x^2/arcsin(a\*x)^4,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 x^4 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})^3 \int \frac{(a^4 x^4 - 20 a^2 x^2 + 24) \sqrt{ax+1} \sqrt{-ax+1}}{(a^5 x^7 - a^3 x^5) \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})} dx - (2 a^2 x^2 - (a^2 x^2 - 6) \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1}))}{6 a^3 x^4 \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^4,x, algorithm="maxima")

[Out] 1/6\*(6\*a^3\*x^4\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^3\*integrate(1/6\*(a^4\*x^4 - 20\*a^2\*x^2 + 24)\*sqrt(a\*x + 1)\*sqrt(-a\*x + 1)/((a^5\*x^7 - a^3\*x^5)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))), x) - (2\*a^2\*x^2 - (a^2\*x^2 - 6)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^2)\*sqrt(a\*x + 1)\*sqrt(-a\*x + 1) - (a^3\*x^3 - 2\*a\*x)\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1)))/(a^3\*x^4\*arctan2(a\*x, sqrt(a\*x + 1)\*sqrt(-a\*x + 1))^3)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asin}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*asin(a\*x)^4),x)

[Out] int(1/(x^2\*asin(a\*x)^4), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asin}^4(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/asin(a\*x)\*\*4,x)

[Out] Integral(1/(x\*\*2\*asin(a\*x)\*\*4), x)

### 3.74 $\int x^4 \sqrt{\sin^{-1}(ax)} dx$

**Optimal.** Leaf size=121

$$-\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5} + \frac{\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{\frac{\pi}{10}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{80a^5} + \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)}$$

[Out]  $-1/800*\text{FresnelS}(10^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*10^{(1/2)}*\text{Pi}^{(1/2)}/a^5+1/96*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^5-1/16*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^5+1/5*x^5*\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {4629, 4723, 3312, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5} + \frac{\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{\frac{\pi}{10}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{80a^5} + \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*\text{Sqrt}[\text{ArcSin}[a*x]], x]$

[Out]  $(x^5*\text{Sqrt}[\text{ArcSin}[a*x]])/5 - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(8*a^5) + (\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(16*a^5) - (\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(80*a^5)$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\sin[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x], x] /; \text{FreeQ}\{c, d, e, f, x\} \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

#### Rule 3312

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \sin[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x \&\& \text{IGtQ}[n, 1] \&\& (!\text{RationalQ}[m] \mid\mid (\text{GeQ}[m, -1] \&\& \text{LtQ}[m, 1]))$

#### Rule 3351

$\text{Int}[\sin[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f, x\}$

#### Rule 4629

$\text{Int}[(a_. + \text{ArcSin}[c_.*(x_.)]*(b_.))^{(n_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*(a + b*\text{ArcSin}[c*x])^n)/(m+1), x] - \text{Dist}[(b*c*n)/(m+1), \text{Int}[(x^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c\}, x \&\& \text{IGtQ}[m, 0] \&\& \text{GtQ}[n, 0]$

#### Rule 4723

$\text{Int}[(a_. + \text{ArcSin}[c_.*(x_.)]*(b_.))^{(n_.)}*(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[d^p/c^{(m+1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\sin[x]^m*\cos[x]^{(2*p+1)}, x], x, \text{ArcSin}[c*x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[2*p] \&\& \text{GtQ}[p, -1] \&\& \text{IGtQ}[m, 0] \&\& (\text{IntegerQ}[p] \mid\mid \text{GtQ}[d, 0])$

Rubi steps

$$\begin{aligned}
\int x^4 \sqrt{\sin^{-1}(ax)} dx &= \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)} - \frac{1}{10} a \int \frac{x^5}{\sqrt{1-a^2 x^2} \sqrt{\sin^{-1}(ax)}} dx \\
&= \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin^5(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{10a^5} \\
&= \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{5 \sin(x)}{8\sqrt{x}} - \frac{5 \sin(3x)}{16\sqrt{x}} + \frac{\sin(5x)}{16\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{10a^5} \\
&= \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin(5x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{160a^5} + \frac{\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{32a^5} \\
&= \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \sin(5x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{80a^5} + \frac{\text{Subst}\left(\int \sin(3x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{16a^5} \\
&= \frac{1}{5} x^5 \sqrt{\sin^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5} + \frac{\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{\frac{\pi}{10}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{80a^5}
\end{aligned}$$

**Mathematica [C]** time = 0.10, size = 204, normalized size = 1.69

$$i\sqrt{\sin^{-1}(ax)} \left(-150\sqrt{i\sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, -i\sin^{-1}(ax)\right) + 150\sqrt{-i\sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, i\sin^{-1}(ax)\right) + 25\sqrt{3}\sqrt{i\sin^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*Sqrt[ArcSin[a\*x]], x]

[Out] ((I/2400)\*Sqrt[ArcSin[a\*x]]\*(-150\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, (-I)\*ArcSin[a\*x]] + 150\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, I\*ArcSin[a\*x]] + 25\*Sqrt[3]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, (-3\*I)\*ArcSin[a\*x]] - 25\*Sqrt[3]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (3\*I)\*ArcSin[a\*x]] - 3\*Sqrt[5]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, (-5\*I)\*ArcSin[a\*x]] + 3\*Sqrt[5]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (5\*I)\*ArcSin[a\*x]]))/(a^5\*Sqrt[ArcSin[a\*x]^2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.30, size = 247, normalized size = 2.04

$$-\frac{(i-1)\sqrt{10}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{10}\sqrt{\arcsin(ax)}\right)}{3200a^5} + \frac{(i+1)\sqrt{10}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{10}\sqrt{\arcsin(ax)}\right)}{3200a^5} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] -(1/3200\*I - 1/3200)\*sqrt(10)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(10)\*sqrt(arcsin(a\*x)))/a^5 + (1/3200\*I + 1/3200)\*sqrt(10)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(10)\*sqrt(arcsin(a\*x)))/a^5 + ...

```
rt(10)*sqrt(arcsin(a*x))/a^5 + (1/384*I - 1/384)*sqrt(6)*sqrt(pi)*erf((1/2
*I - 1/2)*sqrt(6)*sqrt(arcsin(a*x)))/a^5 - (1/384*I + 1/384)*sqrt(6)*sqrt(p
i)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arcsin(a*x)))/a^5 - (1/64*I - 1/64)*sqrt
(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arcsin(a*x)))/a^5 + (1/64*I + 1
/64)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arcsin(a*x)))/a^5 - 1
/160*I*sqrt(arcsin(a*x))*e^(5*I*arcsin(a*x))/a^5 + 1/32*I*sqrt(arcsin(a*x))
*e^(3*I*arcsin(a*x))/a^5 - 1/16*I*sqrt(arcsin(a*x))*e^(I*arcsin(a*x))/a^5 +
1/16*I*sqrt(arcsin(a*x))*e^(-I*arcsin(a*x))/a^5 - 1/32*I*sqrt(arcsin(a*x))
*e^(-3*I*arcsin(a*x))/a^5 + 1/160*I*sqrt(arcsin(a*x))*e^(-5*I*arcsin(a*x))/
a^5
```

**maple [A]** time = 0.13, size = 143, normalized size = 1.18

$$\frac{3\sqrt{5}\sqrt{2}\sqrt{\arcsin(ax)}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{5}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - 25\sqrt{3}\sqrt{2}\sqrt{\arcsin(ax)}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + 150\sqrt{2}\sqrt{\arcsin(ax)}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)}{240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsin(a*x)^(1/2),x)
```

```
[Out] -1/2400/a^5/arcsin(a*x)^(1/2)*(3*5^(1/2)*2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)
*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)*arcsin(a*x)^(1/2))-25*3^(1/2)*2^(1/2)*ar
csin(a*x)^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcsin(a*x)^(1/2
))+150*2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arcsin(
a*x)^(1/2))-300*a*x*arcsin(a*x)+150*arcsin(a*x)*sin(3*arcsin(a*x))-30*arcsi
n(a*x)*sin(5*arcsin(a*x))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsin(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asin(a*x)^(1/2),x)
```

```
[Out] int(x^4*asin(a*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asin(a*x)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(asin(a*x)), x)
```

### 3.75 $\int x^3 \sqrt{\sin^{-1}(ax)} dx$

**Optimal.** Leaf size=95

$$-\frac{\sqrt{\frac{\pi}{2}} C\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{64a^4} + \frac{\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4} - \frac{3\sqrt{\sin^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4 \sqrt{\sin^{-1}(ax)}$$

[Out]  $-1/128*\text{FresnelC}(2*2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(ax)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^4+1/16*\text{FresnelC}(2*\arcsin(ax)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^4-3/32*\arcsin(ax)^{(1/2)}/a^4+1/4*x^4*\arcsin(ax)^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {4629, 4723, 3312, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{64a^4} + \frac{\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4} - \frac{3\sqrt{\sin^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4 \sqrt{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] `Int[x^3*Sqrt[ArcSin[a*x]],x]`

[Out]  $(-3*\text{Sqrt}[\text{ArcSin}[a*x]])/(32*a^4) + (x^4*\text{Sqrt}[\text{ArcSin}[a*x]])/4 - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(64*a^4) + (\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/ \text{Sqrt}[\text{Pi}]])/(16*a^4)$

#### Rule 3304

`Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

#### Rule 3312

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

#### Rule 3352

`Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]`

#### Rule 4629

`Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_), x_Symbol] := Simp[(x^(m + 1)*(a + b*ArcSin[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[(x^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]`

#### Rule 4723

`Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x]^(2*p + 1), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])`

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{\sin^{-1}(ax)} dx &= \frac{1}{4} x^4 \sqrt{\sin^{-1}(ax)} - \frac{1}{8} a \int \frac{x^4}{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}} dx \\
&= \frac{1}{4} x^4 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin^4(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^4} \\
&= \frac{1}{4} x^4 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{8a^4} \\
&= -\frac{3\sqrt{\sin^{-1}(ax)}}{32a^4} + \frac{1}{4} x^4 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{64a^4} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{16a^4} \\
&= -\frac{3\sqrt{\sin^{-1}(ax)}}{32a^4} + \frac{1}{4} x^4 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \cos(4x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{32a^4} + \frac{\text{Subst}\left(\int \cos(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{16a^4} \\
&= -\frac{3\sqrt{\sin^{-1}(ax)}}{32a^4} + \frac{1}{4} x^4 \sqrt{\sin^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} C\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{64a^4} + \frac{\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{16a^4}
\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 138, normalized size = 1.45

$$\frac{\sqrt{\sin^{-1}(ax)} \left( -4\sqrt{2} \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, -2i \sin^{-1}(ax)\right) - 4\sqrt{2} \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, 2i \sin^{-1}(ax)\right) + \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, -2i \sin^{-1}(ax)\right) + \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, 2i \sin^{-1}(ax)\right) \right)}{128a^4 \sqrt{\sin^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*Sqrt[ArcSin[a\*x]],x]

[Out] (Sqrt[ArcSin[a\*x]]\*(-4\*Sqrt[2]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, (-2\*I)\*ArcSin[a\*x]] - 4\*Sqrt[2]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (2\*I)\*ArcSin[a\*x]] + Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, (-4\*I)\*ArcSin[a\*x]] + Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (4\*I)\*ArcSin[a\*x]]))/(128\*a^4\*Sqrt[ArcSin[a\*x]^2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.53, size = 153, normalized size = 1.61

$$\frac{(i+1) \sqrt{2} \sqrt{\pi} \operatorname{erf}\left((i-1) \sqrt{2} \sqrt{\arcsin(ax)}\right)}{512 a^4} - \frac{(i-1) \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(- (i+1) \sqrt{2} \sqrt{\arcsin(ax)}\right)}{512 a^4} - \frac{(i+1) \sqrt{\pi} \operatorname{erf}\left(i \sqrt{\arcsin(ax)}\right)}{64 a^4} + \frac{(i-1) \sqrt{\pi} \operatorname{erf}\left(-i \sqrt{\arcsin(ax)}\right)}{64 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(1/2),x, algorithm="giac")

[Out] (1/512\*I + 1/512)\*sqrt(2)\*sqrt(pi)\*erf((I - 1)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^4 - (1/512\*I - 1/512)\*sqrt(2)\*sqrt(pi)\*erf(-(I + 1)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^4 - (1/64\*I + 1/64)\*sqrt(pi)\*erf((I - 1)\*sqrt(arcsin(a\*x)))/a^4 + (1/64\*I - 1/64)\*sqrt(pi)\*erf(-(I + 1)\*sqrt(arcsin(a\*x)))/a^4



$$\frac{1}{64}I - \frac{1}{64}\sqrt{\pi}\operatorname{erf}(-I + 1)\sqrt{\arcsin(ax)}/a^4 + \frac{1}{64}\sqrt{\arcsin(ax)}e^{(4I\arcsin(ax))}/a^4 - \frac{1}{16}\sqrt{\arcsin(ax)}e^{(2I\arcsin(ax))}/a^4 - \frac{1}{16}\sqrt{\arcsin(ax)}e^{(-2I\arcsin(ax))}/a^4 + \frac{1}{64}\sqrt{\arcsin(ax)}e^{(-4I\arcsin(ax))}/a^4$$

**maple** [A] time = 0.08, size = 90, normalized size = 0.95

$$\frac{\operatorname{FresnelC}\left(\frac{2\sqrt{2}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\arcsin(ax)}\sqrt{\pi} + 16\arcsin(ax)\cos(2\arcsin(ax)) - 8\operatorname{FresnelC}\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)}{128a^4\sqrt{\arcsin(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arcsin(a\*x)^(1/2),x)

[Out]  $-1/128/a^4/\arcsin(a*x)^{(1/2)}*(\operatorname{FresnelC}(2*2^{(1/2)}/\pi^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*\pi^{(1/2)}+16*\arcsin(a*x)*\cos(2*\arcsin(a*x))-8*\operatorname{FresnelC}(2*\arcsin(a*x)^{(1/2)}/\pi^{(1/2)})*\arcsin(a*x)^{(1/2)}*\pi^{(1/2)}-4*\arcsin(a*x)*\cos(4*\arcsin(a*x)))$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*asin(a\*x)^(1/2),x)

[Out] int(x^3\*asin(a\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*asin(a\*x)\*\*(1/2),x)

[Out] Integral(x\*\*3\*sqrt(asin(a\*x)), x)

### 3.76 $\int x^2 \sqrt{\sin^{-1}(ax)} dx$

**Optimal.** Leaf size=86

$$-\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{12a^3} + \frac{1}{3}x^3 \sqrt{\sin^{-1}(ax)}$$

[Out] 1/72\*FresnelS(6^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*6^(1/2)\*Pi^(1/2)/a^3-1/8\*FresnelS(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)\*Pi^(1/2)/a^3+1/3\*x^3\*arcsin(a\*x)^(1/2)

**Rubi [A]** time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {4629, 4723, 3312, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{12a^3} + \frac{1}{3}x^3 \sqrt{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sqrt[ArcSin[a\*x]],x]

[Out] (x^3\*Sqrt[ArcSin[a\*x]])/3 - (Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcSin[a\*x]]])/(4\*a^3) + (Sqrt[Pi/6]\*FresnelS[Sqrt[6/Pi]\*Sqrt[ArcSin[a\*x]]])/(12\*a^3)

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

#### Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^(2))^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{\sin^{-1}(ax)} dx &= \frac{1}{3} x^3 \sqrt{\sin^{-1}(ax)} - \frac{1}{6} a \int \frac{x^3}{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}} dx \\
&= \frac{1}{3} x^3 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{6a^3} \\
&= \frac{1}{3} x^3 \sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4\sqrt{x}} - \frac{\sin(3x)}{4\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{6a^3} \\
&= \frac{1}{3} x^3 \sqrt{\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{24a^3} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^3} \\
&= \frac{1}{3} x^3 \sqrt{\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \sin(3x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{12a^3} - \frac{\text{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a^3} \\
&= \frac{1}{3} x^3 \sqrt{\sin^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^3} + \frac{\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{12a^3}
\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 126, normalized size = 1.47

$$\frac{9\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, -i \sin^{-1}(ax)\right) + 9\sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, i \sin^{-1}(ax)\right) - \sqrt{3} \left(\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, -3i \sin^{-1}(ax)\right)\right)}{72a^3 \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*Sqrt[ArcSin[a\*x]], x]

[Out] (9\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (-I)\*ArcSin[a\*x]] + 9\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, I\*ArcSin[a\*x]] - Sqrt[3]\*(Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (-3\*I)\*ArcSin[a\*x]] + Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, (3\*I)\*ArcSin[a\*x]]))/ (72\*a^3\*Sqrt[ArcSin[a\*x]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.27, size = 165, normalized size = 1.92

$$\frac{(i-1) \sqrt{6} \sqrt{\pi} \operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{6} \sqrt{\arcsin(ax)}\right)}{288 a^3} - \frac{(i+1) \sqrt{6} \sqrt{\pi} \operatorname{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{6} \sqrt{\arcsin(ax)}\right)}{288 a^3} - \frac{(i-1) \sqrt{2}}{288 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] (1/288\*I - 1/288)\*sqrt(6)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^3 - (1/288\*I + 1/288)\*sqrt(6)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^3 - (1/32\*I - 1/32)\*sqrt(2)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^3 + (1/32\*I + 1/32)\*sqrt(2)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^3 + 1/24\*I\*sqrt(arcsin(a\*x))\*e^(3\*I\*a

$\text{rcsin}(a*x))/a^3 - 1/8*I*\text{sqrt}(\text{arcsin}(a*x))*e^{(I*\text{arcsin}(a*x))/a^3} + 1/8*I*\text{sqrt}(\text{arcsin}(a*x))*e^{(-I*\text{arcsin}(a*x))/a^3} - 1/24*I*\text{sqrt}(\text{arcsin}(a*x))*e^{(-3*I*\text{arcsin}(a*x))/a^3}$

**maple** [A] time = 0.08, size = 96, normalized size = 1.12

$$\frac{-\sqrt{3} \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + 9\sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - 18ax \arcsin(ax)}{72a^3 \sqrt{\arcsin(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arcsin(a*x)^(1/2),x)`

[Out] `-1/72/a^3/arcsin(a*x)^(1/2)*(-3^(1/2)*2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcsin(a*x)^(1/2))+9*2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))-18*a*x*arcsin(a*x)+6*arcsin(a*x)*sin(3*arcsin(a*x))`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsin(a*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*asin(a*x)^(1/2),x)`

[Out] `int(x^2*asin(a*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*asin(a*x)**(1/2),x)`

[Out] `Integral(x**2*sqrt(asin(a*x)), x)`

### 3.77 $\int x\sqrt{\sin^{-1}(ax)} dx$

**Optimal.** Leaf size=59

$$\frac{\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2} - \frac{\sqrt{\sin^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)}$$

[Out]  $1/8*\text{FresnelC}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^2-1/4*\arcsin(a*x)^{(1/2)}/a^2+1/2*x^2*\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4629, 4723, 3312, 3304, 3352}

$$\frac{\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2} - \frac{\sqrt{\sin^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[ArcSin[a\*x]],x]

[Out]  $-\text{Sqrt}[\text{ArcSin}[a*x]]/(4*a^2) + (x^2*\text{Sqrt}[\text{ArcSin}[a*x]])/2 + (\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(8*a^2)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/ (f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

#### Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rubi steps

$$\begin{aligned}
\int x\sqrt{\sin^{-1}(ax)} dx &= \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)} - \frac{1}{4}a \int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}} dx \\
&= \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{4a^2} \\
&= \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cos(2x)}{2\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{4a^2} \\
&= -\frac{\sqrt{\sin^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^2} \\
&= -\frac{\sqrt{\sin^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)} + \frac{\text{Subst}\left(\int \cos(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a^2} \\
&= -\frac{\sqrt{\sin^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sin^{-1}(ax)} + \frac{\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^2}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 81, normalized size = 1.37

$$\frac{\sqrt{\sin^{-1}(ax)} \left( \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, -2i \sin^{-1}(ax)\right) + \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, 2i \sin^{-1}(ax)\right) \right)}{8\sqrt{2} a^2 \sqrt{\sin^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sqrt[ArcSin[a\*x]], x]

[Out] -1/8\*(Sqrt[ArcSin[a\*x]]\*(Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, (-2\*I)\*ArcSin[a\*x]] + Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (2\*I)\*ArcSin[a\*x]]))/(Sqrt[2]\*a^2\*Sqrt[ArcSin[a\*x]^2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.57, size = 71, normalized size = 1.20

$$-\frac{(i+1)\sqrt{\pi} \operatorname{erf}\left((i-1)\sqrt{\arcsin(ax)}\right)}{32a^2} + \frac{(i-1)\sqrt{\pi} \operatorname{erf}\left(-(i+1)\sqrt{\arcsin(ax)}\right)}{32a^2} - \frac{\sqrt{\arcsin(ax)} e^{2i \arcsin(ax)}}{8a^2} - \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] -(1/32\*I + 1/32)\*sqrt(pi)\*erf((I - 1)\*sqrt(arcsin(a\*x)))/a^2 + (1/32\*I - 1/32)\*sqrt(pi)\*erf(-(I + 1)\*sqrt(arcsin(a\*x)))/a^2 - 1/8\*sqrt(arcsin(a\*x))\*e^(2\*I\*arcsin(a\*x))/a^2 - 1/8\*sqrt(arcsin(a\*x))\*e^(-2\*I\*arcsin(a\*x))/a^2

**maple [A]** time = 0.06, size = 42, normalized size = 0.71

$$\frac{-2\sqrt{\arcsin(ax)} \sqrt{\pi} \cos(2 \arcsin(ax)) + \pi \operatorname{FresnelC}\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)}{8a^2\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsin(a*x)^(1/2),x)`

[Out] `1/8/a^2/Pi^(1/2)*(-2*arcsin(a*x)^(1/2)*Pi^(1/2)*cos(2*arcsin(a*x))+Pi*FresnelC(2*arcsin(a*x)^(1/2)/Pi^(1/2))`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsin(a*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \sqrt{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*asin(a*x)^(1/2),x)`

[Out] `int(x*asin(a*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*asin(a*x)**(1/2),x)`

[Out] `Integral(x*sqrt(asin(a*x)), x)`

### 3.78 $\int \sqrt{\sin^{-1}(ax)} dx$

**Optimal.** Leaf size=44

$$x\sqrt{\sin^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a}$$

[Out]  $-1/2*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a+x*\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4619, 4723, 3305, 3351}

$$x\sqrt{\sin^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[ArcSin[a\*x]],x]

[Out]  $x*\text{Sqrt}[\text{ArcSin}[a*x]] - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/a$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_], x\_Symbol] :> Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(x\_)^(m\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rubi steps



$$\begin{aligned}
\int \sqrt{\sin^{-1}(ax)} dx &= x\sqrt{\sin^{-1}(ax)} - \frac{1}{2}a \int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}} dx \\
&= x\sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{2a} \\
&= x\sqrt{\sin^{-1}(ax)} - \frac{\text{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a} \\
&= x\sqrt{\sin^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{a}
\end{aligned}$$

**Mathematica [C]** time = 0.03, size = 66, normalized size = 1.50

$$\frac{\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, -i \sin^{-1}(ax)\right) + \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{3}{2}, i \sin^{-1}(ax)\right)}{2a\sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[ArcSin[a\*x]], x]

[Out] (Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[3/2, (-I)\*ArcSin[a\*x]] + Sqrt[I\*ArcSin[a\*x]]\*Gamma[3/2, I\*ArcSin[a\*x]])/(2\*a\*Sqrt[ArcSin[a\*x]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.22, size = 83, normalized size = 1.89

$$-\frac{(i-1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arcsin(ax)}\right)}{8a} + \frac{(i+1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arcsin(ax)}\right)}{8a} - i\sqrt{\arcsin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] -(1/8\*I - 1/8)\*sqrt(2)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a + (1/8\*I + 1/8)\*sqrt(2)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a - 1/2\*I\*sqrt(arcsin(a\*x))\*e^(I\*arcsin(a\*x))/a + 1/2\*I\*sqrt(arcsin(a\*x))\*e^(-I\*arcsin(a\*x))/a

**maple [A]** time = 0.05, size = 49, normalized size = 1.11

$$\frac{-\sqrt{2}\sqrt{\arcsin(ax)}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + 2ax\arcsin(ax)}{2a\sqrt{\arcsin(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^(1/2), x)

[Out]  $\frac{1}{2} \frac{1}{a} \arcsin(ax)^{\frac{1}{2}} (-2)^{\frac{1}{2}} \arcsin(ax)^{\frac{1}{2}} \pi^{\frac{1}{2}} \text{FresnelS}\left(\frac{2^{\frac{1}{2}}}{\pi^{\frac{1}{2}}} \arcsin(ax)^{\frac{1}{2}}\right) + 2ax \arcsin(ax)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(a*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^(1/2),x)`

[Out] `int(asin(a*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)**(1/2),x)`

[Out] `Integral(sqrt(asin(a*x)), x)`

$$3.79 \quad \int \frac{\sqrt{\sin^{-1}(ax)}}{x} dx$$

**Optimal.** Leaf size=15

$$\text{Int}\left(\frac{\sqrt{\sin^{-1}(ax)}}{x}, x\right)$$

[Out] Unintegrable(arcsin(a\*x)^(1/2)/x, x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{\sin^{-1}(ax)}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[ArcSin[a\*x]]/x, x]

[Out] Defer[Int][Sqrt[ArcSin[a\*x]]/x, x]

Rubi steps

$$\int \frac{\sqrt{\sin^{-1}(ax)}}{x} dx = \int \frac{\sqrt{\sin^{-1}(ax)}}{x} dx$$

**Mathematica [A]** time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin^{-1}(ax)}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[ArcSin[a\*x]]/x, x]

[Out] Integrate[Sqrt[ArcSin[a\*x]]/x, x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(1/2)/x, x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arcsin(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(1/2)/x, x, algorithm="giac")

[Out] integrate(sqrt(arcsin(a\*x))/x, x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arcsin(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^(1/2)/x,x)

[Out] int(arcsin(a\*x)^(1/2)/x,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(1/2)/x,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sqrt{\arcsin(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^(1/2)/x,x)

[Out] int(asin(a\*x)^(1/2)/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arcsin(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*(1/2)/x,x)

[Out] Integral(sqrt(asin(a\*x))/x, x)

### 3.80 $\int x^4 \sin^{-1}(ax)^{3/2} dx$

**Optimal.** Leaf size=214

$$\frac{3\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^5} + \frac{\sqrt{\frac{\pi}{6}} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{32a^5} - \frac{3\sqrt{\frac{\pi}{10}} C\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{800a^5} + \frac{3x^4 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a}$$

[Out]  $\frac{1}{5}x^5 \arcsin(ax)^{3/2} - \frac{3}{8000} \text{FresnelC}\left(\frac{10^{1/2}}{\pi^{1/2}} \arcsin(ax)^{1/2}\right) \frac{10^{1/2} \pi^{1/2}}{a^5} + \frac{1}{192} \text{FresnelC}\left(\frac{6^{1/2}}{\pi^{1/2}} \arcsin(ax)^{1/2}\right) \frac{6^{1/2} \pi^{1/2}}{a^5} - \frac{3}{32} \text{FresnelC}\left(\frac{2^{1/2}}{\pi^{1/2}} \arcsin(ax)^{1/2}\right) \frac{2^{1/2} \pi^{1/2}}{a^5} + \frac{4}{25} (-a^2x^2+1)^{1/2} \arcsin(ax)^{1/2} \frac{1}{a^5} + \frac{2}{25} x^2 (-a^2x^2+1)^{1/2} \arcsin(ax)^{1/2} \frac{1}{a^3} + \frac{3}{50} x^4 (-a^2x^2+1)^{1/2} \arcsin(ax)^{1/2} \frac{1}{a}$

**Rubi [A]** time = 0.53, antiderivative size = 282, normalized size of antiderivative = 1.32, number of steps used = 23, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4629, 4707, 4677, 4623, 3304, 3352, 4635, 4406}

$$\frac{2\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{25a^5} - \frac{11\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{400a^5} + \frac{3\sqrt{\frac{3\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{800a^5}$$

Antiderivative was successfully verified.

[In] Int[x^4\*ArcSin[a\*x]^(3/2), x]

[Out]  $\frac{4\sqrt{1-a^2x^2} \sqrt{\text{ArcSin}[a*x]}}{(25a^5)} + \frac{2x^2 \sqrt{1-a^2x^2} \sqrt{\text{ArcSin}[a*x]}}{(25a^3)} + \frac{3x^4 \sqrt{1-a^2x^2} \sqrt{\text{ArcSin}[a*x]}}{(50a)} + \frac{x^5 \text{ArcSin}[a*x]^{3/2}}{5} - \frac{(11\sqrt{\pi/2} \text{FresnelC}[\sqrt{2/\pi} \sqrt{\text{ArcSin}[a*x]}}])}{(400a^5)} - \frac{(2\sqrt{2\pi} \text{FresnelC}[\sqrt{2/\pi} \sqrt{\text{ArcSin}[a*x]}}])}{(25a^5)} + \frac{(\sqrt{\pi/6} \text{FresnelC}[\sqrt{6/\pi} \sqrt{\text{ArcSin}[a*x]}}])}{(50a^5)} + \frac{(3\sqrt{(3\pi)/2} \text{FresnelC}[\sqrt{6/\pi} \sqrt{\text{ArcSin}[a*x]}}])}{(800a^5)} - \frac{(3\sqrt{\pi/10} \text{FresnelC}[\sqrt{10/\pi} \sqrt{\text{ArcSin}[a*x]}}])}{(800a^5)}$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n, x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n \* Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4629

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*(a + b*ArcSin[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[(x^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

#### Rule 4635

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

#### Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p + 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

#### Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[(f*x)^(m - 2)*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

#### Rubi steps

$$\begin{aligned}
 \int x^4 \sin^{-1}(ax)^{3/2} dx &= \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2} - \frac{1}{10}(3a) \int \frac{x^5 \sqrt{\sin^{-1}(ax)}}{\sqrt{1 - a^2x^2}} dx \\
 &= \frac{3x^4 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2} - \frac{3}{100} \int \frac{x^4}{\sqrt{\sin^{-1}(ax)}} dx - \frac{6}{25a} \int \frac{x^3 \sqrt{\sin^{-1}(ax)}}{\sqrt{1 - a^2x^2}} dx \\
 &= \frac{2x^2 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^3} + \frac{3x^4 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2} - \frac{3 \operatorname{Subst}\left(\int \frac{x^3 \sqrt{\sin^{-1}(ax)}}{\sqrt{1 - a^2x^2}} dx, x, \frac{x}{a}\right)}{25a} \\
 &= \frac{4\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^5} + \frac{2x^2 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^3} + \frac{3x^4 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2} \\
 &= \frac{4\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^5} + \frac{2x^2 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^3} + \frac{3x^4 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2} \\
 &= \frac{4\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^5} + \frac{2x^2 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^3} + \frac{3x^4 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2} \\
 &= \frac{4\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^5} + \frac{2x^2 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^3} + \frac{3x^4 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2} \\
 &= \frac{4\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^5} + \frac{2x^2 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{25a^3} + \frac{3x^4 \sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{50a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{3/2}
 \end{aligned}$$

**Mathematica [C]** time = 0.07, size = 202, normalized size = 0.94

$$\frac{\sqrt{\sin^{-1}(ax)} \left( 2250\sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, -i \sin^{-1}(ax)\right) + 2250\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, i \sin^{-1}(ax)\right) - 125\sqrt{3} \sqrt{i \sin^{-1}(ax)} \right)}{1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*ArcSin[a\*x]^(3/2), x]

[Out] (Sqrt[ArcSin[a\*x]]\*(2250\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (-I)\*ArcSin[a\*x]] + 2250\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, I\*ArcSin[a\*x]] - 125\*Sqrt[3]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (-3\*I)\*ArcSin[a\*x]] - 125\*Sqrt[3]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, (3\*I)\*ArcSin[a\*x]] + 9\*Sqrt[5]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (-5\*I)\*ArcSin[a\*x]] + 9\*Sqrt[5]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, (5\*I)\*ArcSin[a\*x]]))/(36000\*a^5\*Sqrt[ArcSin[a\*x]^2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.37, size = 355, normalized size = 1.66

$$\frac{i \arcsin(ax)^{\frac{3}{2}} e^{(5i \arcsin(ax))}}{160 a^5} + \frac{i \arcsin(ax)^{\frac{3}{2}} e^{(3i \arcsin(ax))}}{32 a^5} - \frac{i \arcsin(ax)^{\frac{3}{2}} e^{(i \arcsin(ax))}}{16 a^5} + \frac{i \arcsin(ax)^{\frac{3}{2}} e^{(-i \arcsin(ax))}}{16 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] -1/160\*I\*arcsin(a\*x)^(3/2)\*e^(5\*I\*arcsin(a\*x))/a^5 + 1/32\*I\*arcsin(a\*x)^(3/2)\*e^(3\*I\*arcsin(a\*x))/a^5 - 1/16\*I\*arcsin(a\*x)^(3/2)\*e^(I\*arcsin(a\*x))/a^5 + 1/16\*I\*arcsin(a\*x)^(3/2)\*e^(-I\*arcsin(a\*x))/a^5 - 1/32\*I\*arcsin(a\*x)^(3/2)\*e^(-3\*I\*arcsin(a\*x))/a^5 + 1/160\*I\*arcsin(a\*x)^(3/2)\*e^(-5\*I\*arcsin(a\*x))/a^5 + (3/32000\*I + 3/32000)\*sqrt(10)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(10)\*sqrt(arcsin(a\*x)))/a^5 - (3/32000\*I - 3/32000)\*sqrt(10)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(10)\*sqrt(arcsin(a\*x)))/a^5 - (1/768\*I + 1/768)\*sqrt(6)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^5 + (1/768\*I - 1/768)\*sqrt(6)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^5 + (3/128\*I + 3/128)\*sqrt(2)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^5 - (3/128\*I - 3/128)\*sqrt(2)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^5 + 3/1600\*sqrt(arcsin(a\*x))\*e^(5\*I\*arcsin(a\*x))/a^5 - 1/64\*sqrt(arcsin(a\*x))\*e^(3\*I\*arcsin(a\*x))/a^5 + 3/32\*sqrt(arcsin(a\*x))\*e^(I\*arcsin(a\*x))/a^5 + 3/32\*sqrt(arcsin(a\*x))\*e^(-I\*arcsin(a\*x))/a^5 - 1/64\*sqrt(arcsin(a\*x))\*e^(-3\*I\*arcsin(a\*x))/a^5 + 3/1600\*sqrt(arcsin(a\*x))\*e^(-5\*I\*arcsin(a\*x))/a^5

**maple [A]** time = 0.12, size = 193, normalized size = 0.90

$$\frac{-3000ax \arcsin(ax)^2 - 125 \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{3} \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} + 9 \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{5} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{5} \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arcsin(a*x)^(3/2),x)`

[Out] 
$$-1/24000/a^5/\arcsin(ax)^{(1/2)}*(-3000*ax*\arcsin(ax)^2-125*\operatorname{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}*\arcsin(ax)^{(1/2)})*3^{(1/2)}*2^{(1/2)}*\arcsin(ax)^{(1/2)}*\pi^{(1/2)}+9*\operatorname{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*5^{(1/2)}*\arcsin(ax)^{(1/2)})*5^{(1/2)}*2^{(1/2)}*\arcsin(ax)^{(1/2)}*\pi^{(1/2)}+1500*\arcsin(ax)^2*\sin(3*\arcsin(ax))-300*\arcsin(ax)^2*\sin(5*\arcsin(ax))+2250*\operatorname{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*\arcsin(ax)^{(1/2)})*2^{(1/2)}*\arcsin(ax)^{(1/2)}*\pi^{(1/2)}+750*\arcsin(ax)*\cos(3*\arcsin(ax))-90*\arcsin(ax)*\cos(5*\arcsin(ax))-4500*\arcsin(ax)*(-a^2*x^2+1)^{(1/2)})$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arcsin(a*x)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{asin}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*asin(a*x)^(3/2),x)`

[Out] `int(x^4*asin(a*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{asin}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*asin(a*x)**(3/2),x)`

[Out] `Integral(x**4*asin(a*x)**(3/2), x)`



### 3.81 $\int x^3 \sin^{-1}(ax)^{3/2} dx$

**Optimal.** Leaf size=157

$$\frac{3\sqrt{\frac{\pi}{2}} S\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{512a^4} - \frac{3\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^4} - \frac{3 \sin^{-1}(ax)^{3/2}}{32a^4} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} + \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3}$$

[Out]  $-3/32*\arcsin(a*x)^{(3/2)}/a^4+1/4*x^4*\arcsin(a*x)^{(3/2)}+3/1024*\text{FresnelS}(2*2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^4-3/64*\text{FresnelS}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^4+9/64*x*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)^{(1/2)}/a^3+3/32*x^3*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)^{(1/2)}/a$

**Rubi [A]** time = 0.38, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4629, 4707, 4641, 4635, 4406, 12, 3305, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}} S\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{512a^4} - \frac{3\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{64a^4} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} + \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3} - \frac{3 \sin^{-1}(ax)^{3/2}}{32a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcSin[a\*x]^(3/2),x]

[Out]  $(9*x*\text{Sqrt}[1-a^2*x^2]*\text{Sqrt}[\text{ArcSin}[a*x]])/(64*a^3) + (3*x^3*\text{Sqrt}[1-a^2*x^2]*\text{Sqrt}[\text{ArcSin}[a*x]])/(32*a) - (3*\text{ArcSin}[a*x]^{(3/2)})/(32*a^4) + (x^4*\text{ArcSin}[a*x]^{(3/2)})/4 + (3*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(512*a^4) - (3*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(64*a^4)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 3305

Int[sin[(e\_)+(f\_)\*(x\_)]/Sqrt[(c\_)+(d\_)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c+d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e-c\*f, 0]

#### Rule 3351

Int[Sin[(d\_)\*((e\_)+(f\_)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e+f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_)+(b\_)\*(x\_)]^(p\_)\*((c\_)+(d\_)\*(x\_))^(m\_)\*Sin[(a\_)+(b\_)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c+d\*x)^m, Sin[a+b\*x]^n\*cos[a+b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4629

Int[((a\_)+(ArcSin[(c\_)\*(x\_)]\*(b\_)))^(n\_)\*(x\_)^{(m\_)}, x\_Symbol] := Simp[(x^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(m+1), x] - Dist[(b\*c\*n)/(m+1), Int[(x^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 4635

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sin^{-1}(ax)^{3/2} dx &= \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2} - \frac{1}{8}(3a) \int \frac{x^4 \sqrt{\sin^{-1}(ax)}}{\sqrt{1-a^2x^2}} dx \\
&= \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} + \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2} - \frac{3}{64} \int \frac{x^3}{\sqrt{\sin^{-1}(ax)}} dx - \frac{9}{32a} \int \frac{x^2 \sqrt{\sin^{-1}(ax)}}{\sqrt{1-a^2x^2}} dx \\
&= \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} + \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2} - \frac{3}{32a} \text{Subst}\left(\int \frac{u^2 \sqrt{\sin^{-1}(u)}}{\sqrt{1-u^2}} du, ax\right) \\
&= \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} - \frac{3 \sin^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2} \\
&= \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} - \frac{3 \sin^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2} \\
&= \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} - \frac{3 \sin^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2} \\
&= \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} - \frac{3 \sin^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2} \\
&= \frac{9x \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{64a^3} + \frac{3x^3 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{32a} - \frac{3 \sin^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4 \sin^{-1}(ax)^{3/2}
\end{aligned}$$

**Mathematica** [C] time = 0.03, size = 130, normalized size = 0.83

$$\frac{8\sqrt{2} \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, -2i \sin^{-1}(ax)\right) + 8\sqrt{2} \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, 2i \sin^{-1}(ax)\right) - \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, -4i \sin^{-1}(ax)\right) - \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, 4i \sin^{-1}(ax)\right)}{512a^4 \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*ArcSin[a\*x]^(3/2),x]

[Out] (8\*Sqrt[2]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, (-2\*I)\*ArcSin[a\*x]] + 8\*Sqrt[2]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (2\*I)\*ArcSin[a\*x]] - Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, (-4\*I)\*ArcSin[a\*x]] - Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (4\*I)\*ArcSin[a\*x]])/(512\*a^4\*Sqrt[ArcSin[a\*x]])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 0.31, size = 225, normalized size = 1.43

$$\frac{\arcsin(ax)^{\frac{3}{2}} e^{4i \arcsin(ax)}}{64 a^4} - \frac{\arcsin(ax)^{\frac{3}{2}} e^{2i \arcsin(ax)}}{16 a^4} - \frac{\arcsin(ax)^{\frac{3}{2}} e^{-2i \arcsin(ax)}}{16 a^4} + \frac{\arcsin(ax)^{\frac{3}{2}} e^{-4i \arcsin(ax)}}{64 a^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(3/2),x, algorithm="giac")

[Out] 1/64\*arcsin(a\*x)^(3/2)\*e^(4\*I\*arcsin(a\*x))/a^4 - 1/16\*arcsin(a\*x)^(3/2)\*e^(2\*I\*arcsin(a\*x))/a^4 - 1/16\*arcsin(a\*x)^(3/2)\*e^(-2\*I\*arcsin(a\*x))/a^4 + 1/64\*arcsin(a\*x)^(3/2)\*e^(-4\*I\*arcsin(a\*x))/a^4 + (3/4096\*I - 3/4096)\*sqrt(2)\*sqrt(pi)\*erf((I - 1)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^4 - (3/4096\*I + 3/4096)\*sqrt(2)\*sqrt(pi)\*erf(-(I + 1)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^4 - (3/256\*I - 3/256)\*sqrt(pi)\*erf((I - 1)\*sqrt(arcsin(a\*x)))/a^4 + (3/256\*I + 3/256)\*sqrt(pi)\*erf(-(I + 1)\*sqrt(arcsin(a\*x)))/a^4 + 3/512\*I\*sqrt(arcsin(a\*x))\*e^(4\*I\*arcsin(a\*x))/a^4 - 3/64\*I\*sqrt(arcsin(a\*x))\*e^(2\*I\*arcsin(a\*x))/a^4 + 3/64\*I\*sqrt(arcsin(a\*x))\*e^(-2\*I\*arcsin(a\*x))/a^4 - 3/512\*I\*sqrt(arcsin(a\*x))\*e^(-4\*I\*arcsin(a\*x))/a^4

**maple** [A] time = 0.09, size = 121, normalized size = 0.77

$$\frac{128 \arcsin(ax)^2 \cos(2 \arcsin(ax)) - 32 \arcsin(ax)^2 \cos(4 \arcsin(ax)) - 3\sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{2\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)}{1024a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arcsin(a\*x)^(3/2),x)

[Out] -1/1024/a^4/arcsin(a\*x)^(1/2)\*(128\*arcsin(a\*x)^2\*cos(2\*arcsin(a\*x))-32\*arcsin(a\*x)^2\*cos(4\*arcsin(a\*x))-3\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*FresnelS(2\*2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))+12\*arcsin(a\*x)\*sin(4\*arcsin(a\*x))-96\*arcsin(a\*x)\*sin(2\*arcsin(a\*x))+48\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*FresnelS(2\*arcsin(a\*x)^(1/2)/Pi^(1/2)))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asin}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*asin(a*x)^(3/2),x)`

[Out] `int(x^3*asin(a*x)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asin}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*asin(a*x)**(3/2),x)`

[Out] `Integral(x**3*asin(a*x)**(3/2), x)`

### 3.82 $\int x^2 \sin^{-1}(ax)^{3/2} dx$

**Optimal.** Leaf size=147

$$-\frac{3\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{6}} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{24a^3} + \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{3a^3} + \frac{1}{3}$$

[Out]  $1/3*x^3*\arcsin(a*x)^{(3/2)}+1/144*\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^3-3/16*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^3+1/3*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)^{(1/2)}/a^3+1/6*x^2*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)^{(1/2)}/a$

**Rubi [A]** time = 0.30, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4629, 4707, 4677, 4623, 3304, 3352, 4635, 4406}

$$-\frac{3\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{6}} \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{24a^3} + \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{\sqrt{1-a^2x^2}}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcSin[a\*x]^(3/2), x]

[Out]  $(\text{Sqrt}[1 - a^2*x^2]*\text{Sqrt}[\text{ArcSin}[a*x]])/(3*a^3) + (x^2*\text{Sqrt}[1 - a^2*x^2]*\text{Sqrt}[\text{ArcSin}[a*x]])/(6*a) + (x^3*\text{ArcSin}[a*x]^{(3/2)})/3 - (3*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(8*a^3) + (\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(24*a^3)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n\*cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n], x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(m+1), x] - Dist[(b\*c\*n)/(m+1), Int[(x^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 4635

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p + 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[(f*x)^(m - 2)*(a + b*ArcSin[c*x])^n]/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int x^2 \sin^{-1}(ax)^{3/2} dx &= \frac{1}{3}x^3 \sin^{-1}(ax)^{3/2} - \frac{1}{2}a \int \frac{x^3 \sqrt{\sin^{-1}(ax)}}{\sqrt{1-a^2x^2}} dx \\ &= \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \sin^{-1}(ax)^{3/2} - \frac{1}{12} \int \frac{x^2}{\sqrt{\sin^{-1}(ax)}} dx - \frac{\int \frac{x \sqrt{\sin^{-1}(ax)}}{\sqrt{1-a^2x^2}} dx}{3a} \\ &= \frac{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{3a^3} + \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \sin^{-1}(ax)^{3/2} - \frac{\text{Subst}\left(\int \frac{\cos(x) \sin^{-1}(ax)}{\sqrt{x}} dx\right)}{1} \\ &= \frac{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{3a^3} + \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \sin^{-1}(ax)^{3/2} - \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{4\sqrt{x}}\right) dx\right)}{4} \\ &= \frac{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{3a^3} + \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \sin^{-1}(ax)^{3/2} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx\right)}{48a} \\ &= \frac{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{3a^3} + \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \sin^{-1}(ax)^{3/2} - \frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^3} \\ &= \frac{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{3a^3} + \frac{x^2 \sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}}{6a} + \frac{1}{3}x^3 \sin^{-1}(ax)^{3/2} - \frac{3\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^3} \end{aligned}$$

**Mathematica** [C] time = 0.05, size = 136, normalized size = 0.93

$$\frac{\sqrt{\sin^{-1}(ax)} \left(27\sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, -i \sin^{-1}(ax)\right) + 27\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, i \sin^{-1}(ax)\right) - \sqrt{3} \left(\sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, -3\sqrt{i \sin^{-1}(ax)}\right) + \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, 3\sqrt{-i \sin^{-1}(ax)}\right)\right)\right)}{216a^3 \sqrt{\sin^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcSin[a\*x]^(3/2),x]

[Out] (Sqrt[ArcSin[a\*x]]\*(27\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (-I)\*ArcSin[a\*x]] + 27\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, I\*ArcSin[a\*x]] - Sqrt[3]\*(Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (-3\*I)\*ArcSin[a\*x]] + Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, (3\*I)\*ArcSin[a\*x]])))/(216\*a^3\*Sqrt[ArcSin[a\*x]^2])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 0.47, size = 237, normalized size = 1.61

$$\frac{i \arcsin(ax)^{\frac{3}{2}} e^{3i \arcsin(ax)}}{24 a^3} - \frac{i \arcsin(ax)^{\frac{3}{2}} e^{i \arcsin(ax)}}{8 a^3} + \frac{i \arcsin(ax)^{\frac{3}{2}} e^{-i \arcsin(ax)}}{8 a^3} - \frac{i \arcsin(ax)^{\frac{3}{2}} e^{-3i \arcsin(ax)}}{24 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^(3/2),x, algorithm="giac")

[Out] 1/24\*I\*arcsin(a\*x)^(3/2)\*e^(3\*I\*arcsin(a\*x))/a^3 - 1/8\*I\*arcsin(a\*x)^(3/2)\*e^(I\*arcsin(a\*x))/a^3 + 1/8\*I\*arcsin(a\*x)^(3/2)\*e^(-I\*arcsin(a\*x))/a^3 - 1/24\*I\*arcsin(a\*x)^(3/2)\*e^(-3\*I\*arcsin(a\*x))/a^3 - (1/576\*I + 1/576)\*sqrt(6)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^3 + (1/576\*I - 1/576)\*sqrt(6)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^3 + (3/64\*I + 3/64)\*sqrt(2)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^3 - (3/64\*I - 3/64)\*sqrt(2)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^3 - 1/48\*sqrt(arcsin(a\*x))\*e^(3\*I\*arcsin(a\*x))/a^3 + 3/16\*sqrt(arcsin(a\*x))\*e^(I\*arcsin(a\*x))/a^3 + 3/16\*sqrt(arcsin(a\*x))\*e^(-I\*arcsin(a\*x))/a^3 - 1/48\*sqrt(arcsin(a\*x))\*e^(-3\*I\*arcsin(a\*x))/a^3

**maple** [A] time = 0.08, size = 131, normalized size = 0.89

$$\frac{-36ax \arcsin(ax)^2 - \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{3} \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} + 12 \arcsin(ax)^2 \sin(3 \arcsin(ax))}{144}$$

144

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arcsin(a\*x)^(3/2),x)

[Out] -1/144/a^3/arcsin(a\*x)^(1/2)\*(-36\*a\*x\*arcsin(a\*x)^2-FresnelC(2^(1/2)/Pi^(1/2))\*3^(1/2)\*arcsin(a\*x)^(1/2))\*3^(1/2)\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)+12\*arcsin(a\*x)^2\*sin(3\*arcsin(a\*x))+27\*FresnelC(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)+6\*arcsin(a\*x)\*cos(3\*arcsin(a\*x))-54\*arcsin(a\*x)\*(-a^2\*x^2+1)^(1/2))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asin}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*asin(a*x)^(3/2),x)`

[Out] `int(x^2*asin(a*x)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asin}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*asin(a*x)**(3/2),x)`

[Out] `Integral(x**2*asin(a*x)**(3/2), x)`



### 3.83 $\int x \sin^{-1}(ax)^{3/2} dx$

**Optimal.** Leaf size=89

$$-\frac{3\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2} + \frac{3x\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{8a} - \frac{\sin^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2}$$

[Out]  $-1/4*\arcsin(a*x)^{(3/2)}/a^2+1/2*x^2*\arcsin(a*x)^{(3/2)}-3/32*\text{FresnelS}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^2+3/8*x*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)^{(1/2)}/a$

**Rubi [A]** time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$ , Rules used = {4629, 4707, 4641, 4635, 4406, 12, 3305, 3351}

$$-\frac{3\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2} + \frac{3x\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{8a} - \frac{\sin^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcSin[a*x]^(3/2),x]`

[Out]  $(3*x*\text{Sqrt}[1-a^2*x^2]*\text{Sqrt}[\text{ArcSin}[a*x]])/(8*a) - \text{ArcSin}[a*x]^{(3/2)}/(4*a^2) + (x^2*\text{ArcSin}[a*x]^{(3/2)})/2 - (3*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(32*a^2)$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 3305

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

#### Rule 3351

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]`

#### Rule 4406

`Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]]^n*Cos[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

#### Rule 4629

`Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n*(x_)^(m_.), x_Symbol] := Simp[(x^(m+1)*(a + b*ArcSin[c*x])^n)/(m+1), x] - Dist[(b*c*n)/(m+1), Int[(x^(m+1)*(a + b*ArcSin[c*x])^(n-1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]`

#### Rule 4635

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

#### Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/Sqrt[(d_.) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

#### Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)))/Sqrt[(d_.)
+ (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

#### Rubi steps

$$\begin{aligned}
\int x \sin^{-1}(ax)^{3/2} dx &= \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2} - \frac{1}{4}(3a) \int \frac{x^2 \sqrt{\sin^{-1}(ax)}}{\sqrt{1 - a^2x^2}} dx \\
&= \frac{3x\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{8a} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2} - \frac{3}{16} \int \frac{x}{\sqrt{\sin^{-1}(ax)}} dx - \frac{3 \int \frac{\sqrt{\sin^{-1}(ax)}}{\sqrt{1 - a^2x^2}} dx}{8a} \\
&= \frac{3x\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{8a} - \frac{\sin^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2} - \frac{3 \operatorname{Subst}\left(\int \frac{\cos(x) \sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{16a^2} \\
&= \frac{3x\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{8a} - \frac{\sin^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2} - \frac{3 \operatorname{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{16a^2} \\
&= \frac{3x\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{8a} - \frac{\sin^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2} - \frac{3 \operatorname{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{32a^2} \\
&= \frac{3x\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{8a} - \frac{\sin^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2} - \frac{3 \operatorname{Subst}\left(\int \sin(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{16a^2} \\
&= \frac{3x\sqrt{1 - a^2x^2} \sqrt{\sin^{-1}(ax)}}{8a} - \frac{\sin^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{3/2} - \frac{3\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{32a^2}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 71, normalized size = 0.80

$$\frac{\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, -2i \sin^{-1}(ax)\right) + \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, 2i \sin^{-1}(ax)\right)}{16\sqrt{2} a^2 \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSin[a*x]^(3/2), x]
```

```
[Out] (Sqrt[(-I)*ArcSin[a*x]]*Gamma[5/2, (-2*I)*ArcSin[a*x]] + Sqrt[I*ArcSin[a*x]
]*Gamma[5/2, (2*I)*ArcSin[a*x]])/(16*Sqrt[2]*a^2*Sqrt[ArcSin[a*x]])
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 0.27, size = 107, normalized size = 1.20

$$\frac{\arcsin(ax)^{\frac{3}{2}} e^{2i \arcsin(ax)}}{8a^2} - \frac{\arcsin(ax)^{\frac{3}{2}} e^{-2i \arcsin(ax)}}{8a^2} - \frac{(3i-3)\sqrt{\pi} \operatorname{erf}\left((i-1)\sqrt{\arcsin(ax)}\right)}{128a^2} + \frac{(3i+3)\sqrt{\pi}}{128a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(3/2),x, algorithm="giac")

[Out]  $-1/8 \arcsin(ax)^{3/2} e^{2i \arcsin(ax)} / a^2 - 1/8 \arcsin(ax)^{3/2} e^{-2i \arcsin(ax)} / a^2 - (3/128 i - 3/128) \sqrt{\pi} \operatorname{erf}\left((i-1)\sqrt{\arcsin(ax)}\right) / a^2 + (3/128 i + 3/128) \sqrt{\pi} \operatorname{erf}\left(-i\sqrt{\arcsin(ax)}\right) / a^2 - 3/32 i \sqrt{\arcsin(ax)} e^{2i \arcsin(ax)} / a^2 + 3/32 i \sqrt{\arcsin(ax)} e^{-2i \arcsin(ax)} / a^2$

**maple** [A] time = 0.08, size = 64, normalized size = 0.72

$$\frac{8 \arcsin(ax)^2 \cos(2 \arcsin(ax)) + 3 \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - 6 \arcsin(ax) \sin(2 \arcsin(ax))}{32a^2 \sqrt{\arcsin(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arcsin(a\*x)^(3/2),x)

[Out]  $-1/32/a^2*(8*\arcsin(a*x)^2*\cos(2*\arcsin(a*x))+3*\arcsin(a*x)^(1/2)*\text{Pi}^(1/2)*\text{FresnelS}(2*\arcsin(a*x)^(1/2)/\text{Pi}^(1/2))-6*\arcsin(a*x)*\sin(2*\arcsin(a*x)))/\arcsin(a*x)^(1/2)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{asin}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*asin(a\*x)^(3/2),x)

[Out] int(x\*asin(a\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asin}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(a*x)**(3/2),x)
```

```
[Out] Integral(x*asin(a*x)**(3/2), x)
```

### 3.84 $\int \sin^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=75

$$\frac{3\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{2a} - \frac{3\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{2a} + x\sin^{-1}(ax)^{3/2}$$

[Out] x\*arcsin(a\*x)^(3/2)-3/4\*FresnelC(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)\*Pi^(1/2)/a+3/2\*(-a^2\*x^2+1)^(1/2)\*arcsin(a\*x)^(1/2)/a

**Rubi [A]** time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4619, 4677, 4623, 3304, 3352}

$$\frac{3\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{2a} - \frac{3\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{2a} + x\sin^{-1}(ax)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(3/2), x]

[Out] (3\*Sqrt[1 - a^2\*x^2]\*Sqrt[ArcSin[a\*x]])/(2\*a) + x\*ArcSin[a\*x]^(3/2) - (3\*Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcSin[a\*x]]])/(2\*a)

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n, x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p+1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p+1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p+1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p+1/2)\*(a + b\*ArcSin[c\*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \sin^{-1}(ax)^{3/2} dx &= x \sin^{-1}(ax)^{3/2} - \frac{1}{2}(3a) \int \frac{x\sqrt{\sin^{-1}(ax)}}{\sqrt{1-a^2x^2}} dx \\
&= \frac{3\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{2a} + x \sin^{-1}(ax)^{3/2} - \frac{3}{4} \int \frac{1}{\sqrt{\sin^{-1}(ax)}} dx \\
&= \frac{3\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{2a} + x \sin^{-1}(ax)^{3/2} - \frac{3 \operatorname{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{4a} \\
&= \frac{3\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{2a} + x \sin^{-1}(ax)^{3/2} - \frac{3 \operatorname{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{2a} \\
&= \frac{3\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}}{2a} + x \sin^{-1}(ax)^{3/2} - \frac{3\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a}
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 76, normalized size = 1.01

$$\frac{\sqrt{\sin^{-1}(ax)} \left( \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, -i \sin^{-1}(ax)\right) + \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{5}{2}, i \sin^{-1}(ax)\right) \right)}{2a \sqrt{\sin^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^(3/2), x]

[Out] (Sqrt[ArcSin[a\*x]]\*(Sqrt[I\*ArcSin[a\*x]]\*Gamma[5/2, (-I)\*ArcSin[a\*x]] + Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[5/2, I\*ArcSin[a\*x]]))/(2\*a\*Sqrt[ArcSin[a\*x]^2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.33, size = 119, normalized size = 1.59

$$-\frac{i \arcsin(ax)^{\frac{3}{2}} e^{i \arcsin(ax)}}{2a} + \frac{i \arcsin(ax)^{\frac{3}{2}} e^{-i \arcsin(ax)}}{2a} + \frac{(3i+3) \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\arcsin(ax)}\right)}{16a} (3i - \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] -1/2\*I\*arcsin(a\*x)^(3/2)\*e^(I\*arcsin(a\*x))/a + 1/2\*I\*arcsin(a\*x)^(3/2)\*e^(-I\*arcsin(a\*x))/a + (3/16\*I + 3/16)\*sqrt(2)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a - (3/16\*I - 3/16)\*sqrt(2)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a + 3/4\*sqrt(arcsin(a\*x))\*e^(I\*arcsin(a\*x))/a + 3/4\*sqrt(arcsin(a\*x))\*e^(-I\*arcsin(a\*x))/a

**maple [A]** time = 0.05, size = 72, normalized size = 0.96

$$\frac{\sqrt{2} \left( 2 \arcsin(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} xa + 3\sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} \sqrt{-a^2x^2+1} - 3\pi \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \right)}{4a\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(a*x)^(3/2),x)`

[Out] `1/4/a*2^(1/2)*(2*arcsin(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x+a+3*2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)-3*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2)))/Pi^(1/2)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(a*x)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asin}(ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^(3/2),x)`

[Out] `int(asin(a*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asin}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(a*x)**(3/2),x)`

[Out] `Integral(asin(a*x)**(3/2), x)`

$$3.85 \quad \int \frac{\sin^{-1}(ax)^{3/2}}{x} dx$$

**Optimal.** Leaf size=15

$$\text{Int}\left(\frac{\sin^{-1}(ax)^{3/2}}{x}, x\right)$$

[Out] Unintegrable(arcsin(a\*x)^(3/2)/x, x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sin^{-1}(ax)^{3/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcSin[a\*x]^(3/2)/x, x]

[Out] Defer[Int][ArcSin[a\*x]^(3/2)/x, x]

Rubi steps

$$\int \frac{\sin^{-1}(ax)^{3/2}}{x} dx = \int \frac{\sin^{-1}(ax)^{3/2}}{x} dx$$

**Mathematica [A]** time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(ax)^{3/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcSin[a\*x]^(3/2)/x, x]

[Out] Integrate[ArcSin[a\*x]^(3/2)/x, x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(3/2)/x, x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(3/2)/x, x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^(3/2)/x, x)



**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^(3/2)/x,x)

[Out] int(arcsin(a\*x)^(3/2)/x,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(3/2)/x,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{asin}(ax)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^(3/2)/x,x)

[Out] int(asin(a\*x)^(3/2)/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^{\frac{3}{2}}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*(3/2)/x,x)

[Out] Integral(asin(a\*x)\*\*(3/2)/x, x)

### 3.86 $\int x^4 \sin^{-1}(ax)^{5/2} dx$

**Optimal.** Leaf size=263

$$\frac{15\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{32a^5} - \frac{5\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{192a^5} + \frac{3\sqrt{\frac{\pi}{10}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{1600a^5} - \frac{2x\sqrt{\sin^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sin^{-1}(ax)}}{15a^2}$$

[Out]  $1/5*x^5*\arcsin(a*x)^{(5/2)}+3/16000*\text{FresnelS}(10^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*10^{(1/2)}*\text{Pi}^{(1/2)}/a^5-5/1152*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^5+15/64*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^5+4/15*\arcsin(a*x)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a^5+2/15*x^2*\arcsin(a*x)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a^3+1/10*x^4*\arcsin(a*x)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a^2+5*x*\arcsin(a*x)^{(1/2)}/a^4-1/15*x^3*\arcsin(a*x)^{(1/2)}/a^2-3/100*x^5*\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.80, antiderivative size = 298, normalized size of antiderivative = 1.13, number of steps used = 26, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4629, 4707, 4677, 4619, 4723, 3305, 3351, 3312}

$$\frac{15\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{320a^5} - \frac{\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{60a^5} + \frac{3\sqrt{\frac{\pi}{10}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{1600a^5} + x^3\sqrt{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*\text{ArcSin}[a*x]^{(5/2)}, x]$

[Out]  $(-2*x*\text{Sqrt}[\text{ArcSin}[a*x]])/(5*a^4) - (x^3*\text{Sqrt}[\text{ArcSin}[a*x]])/(15*a^2) - (3*x^5*\text{Sqrt}[\text{ArcSin}[a*x]])/100 + (4*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(15*a^5) + (2*x^2*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(15*a^3) + (x^4*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(10*a) + (x^5*\text{ArcSin}[a*x]^{(5/2)})/5 + (15*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(32*a^5) - (\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(60*a^5) - (\text{Sqrt}[(3*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(320*a^5) + (3*\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(1600*a^5)$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3312

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{ :> Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x\} \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ (!\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 1]))$

#### Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \text{ :> Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f, x\}$

#### Rule 4619

$\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \text{ :> Simp}[x*(a + b*\text{ArcSin}[c*x])^n, x] - \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{GtQ}[n, 0]$

#### Rule 4629

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[(
x^(m + 1)*(a + b*ArcSin[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[(x
^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

#### Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p + 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

#### Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[(f*x)^(m - 2)*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]
```

#### Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x]^(2*p + 1), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])
```

#### Rubi steps

$$\begin{aligned}
\int x^4 \sin^{-1}(ax)^{5/2} dx &= \frac{1}{5}x^5 \sin^{-1}(ax)^{5/2} - \frac{1}{2}a \int \frac{x^5 \sin^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}} dx \\
&= \frac{x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{5/2} - \frac{3}{20} \int x^4 \sqrt{\sin^{-1}(ax)} dx - \frac{2 \int \frac{x^3 \sin^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}}}{5a} \\
&= -\frac{3}{100}x^5 \sqrt{\sin^{-1}(ax)} + \frac{2x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^3} + \frac{x^4 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5 \sin^{-1}(ax)^{5/2} \\
&= -\frac{x^3 \sqrt{\sin^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5 \sqrt{\sin^{-1}(ax)} + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^3} \\
&= -\frac{2x \sqrt{\sin^{-1}(ax)}}{5a^4} - \frac{x^3 \sqrt{\sin^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5 \sqrt{\sin^{-1}(ax)} + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^3} \\
&= -\frac{2x \sqrt{\sin^{-1}(ax)}}{5a^4} - \frac{x^3 \sqrt{\sin^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5 \sqrt{\sin^{-1}(ax)} + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^3} \\
&= -\frac{2x \sqrt{\sin^{-1}(ax)}}{5a^4} - \frac{x^3 \sqrt{\sin^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5 \sqrt{\sin^{-1}(ax)} + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^3} \\
&= -\frac{2x \sqrt{\sin^{-1}(ax)}}{5a^4} - \frac{x^3 \sqrt{\sin^{-1}(ax)}}{15a^2} - \frac{3}{100}x^5 \sqrt{\sin^{-1}(ax)} + \frac{4\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2 \sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{15a^3}
\end{aligned}$$

**Mathematica [C]** time = 0.07, size = 204, normalized size = 0.78

$$i\sqrt{\sin^{-1}(ax)} \left( 33750\sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, -i \sin^{-1}(ax)\right) - 33750\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, i \sin^{-1}(ax)\right) - 625\sqrt{3} \sqrt{i \sin^{-1}(ax)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*ArcSin[a\*x]^(5/2),x]

[Out] ((I/540000)\*Sqrt[ArcSin[a\*x]]\*(33750\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[7/2, (-I)\*ArcSin[a\*x]] - 33750\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[7/2, I\*ArcSin[a\*x]] - 625\*Sqrt[3]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[7/2, (-3\*I)\*ArcSin[a\*x]] + 625\*Sqrt[3]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[7/2, (3\*I)\*ArcSin[a\*x]] + 27\*Sqrt[5]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[7/2, (-5\*I)\*ArcSin[a\*x]] - 27\*Sqrt[5]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[7/2, (5\*I)\*ArcSin[a\*x]]))/(a^5\*Sqrt[ArcSin[a\*x]^2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.32, size = 463, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^(5/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/160*I*\arcsin(a*x)^{(5/2)}*e^{(5*I*\arcsin(a*x))}/a^5 + 1/32*I*\arcsin(a*x)^{(5/2)}*e^{(3*I*\arcsin(a*x))}/a^5 - 1/16*I*\arcsin(a*x)^{(5/2)}*e^{(I*\arcsin(a*x))}/a^5 \\ & + 1/16*I*\arcsin(a*x)^{(5/2)}*e^{(-I*\arcsin(a*x))}/a^5 - 1/32*I*\arcsin(a*x)^{(5/2)}*e^{(-3*I*\arcsin(a*x))}/a^5 + 1/160*I*\arcsin(a*x)^{(5/2)}*e^{(-5*I*\arcsin(a*x))}/a^5 \\ & + 1/320*\arcsin(a*x)^{(3/2)}*e^{(5*I*\arcsin(a*x))}/a^5 - 5/192*\arcsin(a*x)^{(3/2)}*e^{(3*I*\arcsin(a*x))}/a^5 + 5/32*\arcsin(a*x)^{(3/2)}*e^{(I*\arcsin(a*x))}/a^5 \\ & + 5/32*\arcsin(a*x)^{(3/2)}*e^{(-I*\arcsin(a*x))}/a^5 - 5/192*\arcsin(a*x)^{(3/2)}*e^{(-3*I*\arcsin(a*x))}/a^5 + 1/320*\arcsin(a*x)^{(3/2)}*e^{(-5*I*\arcsin(a*x))}/a^5 \\ & + (3/64000*I - 3/64000)*\sqrt{10}*\sqrt{\pi}*\operatorname{erf}((1/2*I - 1/2)*\sqrt{10}*\sqrt{\arcsin(a*x)})/a^5 - (3/64000*I + 3/64000)*\sqrt{10}*\sqrt{\pi}*\operatorname{erf}(-(1/2*I + 1/2)*\sqrt{10}*\sqrt{\arcsin(a*x)})/a^5 \\ & - (5/4608*I - 5/4608)*\sqrt{6}*\sqrt{\pi}*\operatorname{erf}((1/2*I - 1/2)*\sqrt{6}*\sqrt{\arcsin(a*x)})/a^5 + (5/4608*I + 5/4608)*\sqrt{6}*\sqrt{\pi}*\operatorname{erf}(-(1/2*I + 1/2)*\sqrt{6}*\sqrt{\arcsin(a*x)})/a^5 \\ & + (15/256*I - 15/256)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}((1/2*I - 1/2)*\sqrt{2}*\sqrt{\arcsin(a*x)})/a^5 - (15/256*I + 15/256)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}(-(1/2*I + 1/2)*\sqrt{2}*\sqrt{\arcsin(a*x)})/a^5 \\ & + 3/3200*I*\sqrt{\arcsin(a*x)}*e^{(5*I*\arcsin(a*x))}/a^5 - 5/384*I*\sqrt{\arcsin(a*x)}*e^{(3*I*\arcsin(a*x))}/a^5 + 15/64*I*\sqrt{\arcsin(a*x)}*e^{(I*\arcsin(a*x))}/a^5 - 15/64*I*\sqrt{\arcsin(a*x)}*e^{(-I*\arcsin(a*x))}/a^5 \\ & + 5/384*I*\sqrt{\arcsin(a*x)}*e^{(-3*I*\arcsin(a*x))}/a^5 - 3/3200*I*\sqrt{\arcsin(a*x)}*e^{(-5*I*\arcsin(a*x))}/a^5 \end{aligned}$$

**maple [A]** time = 0.14, size = 233, normalized size = 0.89

---


$$-18000ax \arcsin(ax)^3 + 9000 \arcsin(ax)^3 \sin(3 \arcsin(ax)) - 1800 \arcsin(ax)^3 \sin(5 \arcsin(ax)) - 27\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arcsin(a\*x)^(5/2),x)

[Out] 
$$\begin{aligned} & -1/144000/a^5/\arcsin(a*x)^{(1/2)}*(-18000*a*x*\arcsin(a*x)^3+9000*\arcsin(a*x)^3*\sin(3*\arcsin(a*x))-1800*\arcsin(a*x)^3*\sin(5*\arcsin(a*x))-27*5^{(1/2)}*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*\pi^{(1/2)}*\operatorname{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*5^{(1/2)}*\arcsin(a*x)^{(1/2)})+625*3^{(1/2)}*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*\pi^{(1/2)}*\operatorname{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}*\arcsin(a*x)^{(1/2)})-45000*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}+7500*\arcsin(a*x)^2*\cos(3*\arcsin(a*x))-900*\arcsin(a*x)^2*\cos(5*\arcsin(a*x))-33750*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*\pi^{(1/2)}*\operatorname{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*\arcsin(a*x)^{(1/2)})+67500*a*x*\arcsin(a*x)-3750*\arcsin(a*x)*\sin(3*\arcsin(a*x))+270*\arcsin(a*x)*\sin(5*\arcsin(a*x))) \end{aligned}$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arcsin(a\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \operatorname{asin}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asin(a*x)^(5/2),x)
```

```
[Out] int(x^4*asin(a*x)^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asin(a*x)**(5/2),x)
```

```
[Out] Timed out
```

### 3.87 $\int x^3 \sin^{-1}(ax)^{5/2} dx$

**Optimal.** Leaf size=205

$$\frac{15\sqrt{\frac{\pi}{2}} C\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4096a^4} - \frac{15\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^4} - \frac{3 \sin^{-1}(ax)^{5/2}}{32a^4} + \frac{225\sqrt{\sin^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sin^{-1}(ax)}}{256a^2} + \frac{5x^3\sqrt{\sin^{-1}(ax)}}{256a^2}$$

[Out]  $-3/32*\arcsin(ax)^{(5/2)}/a^4+1/4*x^4*\arcsin(ax)^{(5/2)}+15/8192*\text{FresnelC}(2*2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(ax)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^4-15/256*\text{FresnelC}(2*\arcsin(ax)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^4+15/64*x*\arcsin(ax)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a^3+5/32*x^3*\arcsin(ax)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a+225/2048*\arcsin(ax)^{(1/2)}/a^4-45/256*x^2*\arcsin(ax)^{(1/2)}/a^2-15/256*x^4*\arcsin(ax)^{(1/2)}$

**Rubi [A]** time = 0.60, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {4629, 4707, 4641, 4723, 3312, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4096a^4} - \frac{15\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{256a^4} + \frac{5x^3\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} - \frac{45x^2\sqrt{\sin^{-1}(ax)}}{256a^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcSin[a\*x]^(5/2), x]

[Out]  $(225*\text{Sqrt}[\text{ArcSin}[a*x]])/(2048*a^4) - (45*x^2*\text{Sqrt}[\text{ArcSin}[a*x]])/(256*a^2) - (15*x^4*\text{Sqrt}[\text{ArcSin}[a*x]])/256 + (15*x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(64*a^3) + (5*x^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(32*a) - (3*\text{ArcSin}[a*x]^{(5/2)})/(32*a^4) + (x^4*\text{ArcSin}[a*x]^{(5/2)})/4 + (15*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(4096*a^4) - (15*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(256*a^4)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(m+1), x] - Dist[(b\*c\*n)/(m+1), Int[(x^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

#### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_.))^(m\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rubi steps

$$\begin{aligned}
 \int x^3 \sin^{-1}(ax)^{5/2} dx &= \frac{1}{4}x^4 \sin^{-1}(ax)^{5/2} - \frac{1}{8}(5a) \int \frac{x^4 \sin^{-1}(ax)^{3/2}}{\sqrt{1 - a^2x^2}} dx \\
 &= \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4 \sin^{-1}(ax)^{5/2} - \frac{15}{64} \int x^3 \sqrt{\sin^{-1}(ax)} dx - \frac{15}{32a} \int \frac{x^2 \sin^{-1}(ax)}{\sqrt{1 - a^2x^2}} dx \\
 &= -\frac{15}{256}x^4 \sqrt{\sin^{-1}(ax)} + \frac{15x \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{64a^3} + \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4 \sin^{-1}(ax)^{5/2} \\
 &= -\frac{45x^2 \sqrt{\sin^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4 \sqrt{\sin^{-1}(ax)} + \frac{15x \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{64a^3} + \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} \\
 &= -\frac{45x^2 \sqrt{\sin^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4 \sqrt{\sin^{-1}(ax)} + \frac{15x \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{64a^3} + \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} \\
 &= \frac{45 \sqrt{\sin^{-1}(ax)}}{2048a^4} - \frac{45x^2 \sqrt{\sin^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4 \sqrt{\sin^{-1}(ax)} + \frac{15x \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{64a^3} + \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} \\
 &= \frac{225 \sqrt{\sin^{-1}(ax)}}{2048a^4} - \frac{45x^2 \sqrt{\sin^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4 \sqrt{\sin^{-1}(ax)} + \frac{15x \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{64a^3} + \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} \\
 &= \frac{225 \sqrt{\sin^{-1}(ax)}}{2048a^4} - \frac{45x^2 \sqrt{\sin^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4 \sqrt{\sin^{-1}(ax)} + \frac{15x \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{64a^3} + \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a} \\
 &= \frac{225 \sqrt{\sin^{-1}(ax)}}{2048a^4} - \frac{45x^2 \sqrt{\sin^{-1}(ax)}}{256a^2} - \frac{15}{256}x^4 \sqrt{\sin^{-1}(ax)} + \frac{15x \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{64a^3} + \frac{5x^3 \sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{32a}
 \end{aligned}$$

**Mathematica** [C] time = 0.04, size = 140, normalized size = 0.68

$$\frac{\sqrt{\sin^{-1}(ax)} \left( 16\sqrt{2} \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, -2i \sin^{-1}(ax)\right) + 16\sqrt{2} \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, 2i \sin^{-1}(ax)\right) - \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, -2i \sin^{-1}(ax)\right) - \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, 2i \sin^{-1}(ax)\right) \right)}{2048a^4 \sqrt{\sin^{-1}(ax)^2}}$$



Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*ArcSin[a\*x]^(5/2),x]

[Out] (Sqrt[ArcSin[a\*x]]\*(16\*Sqrt[2]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[7/2, (-2\*I)\*ArcSin[a\*x]] + 16\*Sqrt[2]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[7/2, (2\*I)\*ArcSin[a\*x]] - Sqrt[I\*ArcSin[a\*x]]\*Gamma[7/2, (-4\*I)\*ArcSin[a\*x]] - Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[7/2, (4\*I)\*ArcSin[a\*x]]))/(2048\*a^4\*Sqrt[ArcSin[a\*x]^2])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 0.31, size = 297, normalized size = 1.45

$$\frac{\arcsin(ax)^{\frac{5}{2}} e^{4i \arcsin(ax)}}{64 a^4} - \frac{\arcsin(ax)^{\frac{5}{2}} e^{2i \arcsin(ax)}}{16 a^4} - \frac{\arcsin(ax)^{\frac{5}{2}} e^{-2i \arcsin(ax)}}{16 a^4} + \frac{\arcsin(ax)^{\frac{5}{2}} e^{-4i \arcsin(ax)}}{64 a^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^(5/2),x, algorithm="giac")

[Out] 1/64\*arcsin(a\*x)^(5/2)\*e^(4\*I\*arcsin(a\*x))/a^4 - 1/16\*arcsin(a\*x)^(5/2)\*e^(2\*I\*arcsin(a\*x))/a^4 - 1/16\*arcsin(a\*x)^(5/2)\*e^(-2\*I\*arcsin(a\*x))/a^4 + 1/64\*arcsin(a\*x)^(5/2)\*e^(-4\*I\*arcsin(a\*x))/a^4 + 5/512\*I\*arcsin(a\*x)^(3/2)\*e^(4\*I\*arcsin(a\*x))/a^4 - 5/64\*I\*arcsin(a\*x)^(3/2)\*e^(2\*I\*arcsin(a\*x))/a^4 + 5/64\*I\*arcsin(a\*x)^(3/2)\*e^(-2\*I\*arcsin(a\*x))/a^4 - 5/512\*I\*arcsin(a\*x)^(3/2)\*e^(-4\*I\*arcsin(a\*x))/a^4 - (15/32768\*I + 15/32768)\*sqrt(2)\*sqrt(pi)\*erf((I - 1)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^4 + (15/32768\*I - 15/32768)\*sqrt(2)\*sqrt(pi)\*erf(-(I + 1)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^4 + (15/1024\*I + 15/1024)\*sqrt(pi)\*erf((I - 1)\*sqrt(arcsin(a\*x)))/a^4 - (15/1024\*I - 15/1024)\*sqrt(pi)\*erf(-(I + 1)\*sqrt(arcsin(a\*x)))/a^4 - 15/4096\*sqrt(arcsin(a\*x))\*e^(4\*I\*arcsin(a\*x))/a^4 + 15/256\*sqrt(arcsin(a\*x))\*e^(2\*I\*arcsin(a\*x))/a^4 + 15/256\*sqrt(arcsin(a\*x))\*e^(-2\*I\*arcsin(a\*x))/a^4 - 15/4096\*sqrt(arcsin(a\*x))\*e^(-4\*I\*arcsin(a\*x))/a^4

**maple** [A] time = 0.10, size = 154, normalized size = 0.75

$$\frac{1024 \arcsin(ax)^{\frac{5}{2}} \cos(2 \arcsin(ax)) \sqrt{\pi} - 256 \arcsin(ax)^{\frac{5}{2}} \cos(4 \arcsin(ax)) \sqrt{\pi} - 1280 \arcsin(ax)^{\frac{3}{2}} \sin(\dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arcsin(a\*x)^(5/2),x)

[Out] -1/8192/a^4/Pi^(1/2)\*(1024\*arcsin(a\*x)^(5/2)\*cos(2\*arcsin(a\*x))\*Pi^(1/2)-256\*arcsin(a\*x)^(5/2)\*cos(4\*arcsin(a\*x))\*Pi^(1/2)-1280\*arcsin(a\*x)^(3/2)\*sin(2\*arcsin(a\*x))\*Pi^(1/2)+160\*arcsin(a\*x)^(3/2)\*sin(4\*arcsin(a\*x))\*Pi^(1/2)-15\*Pi\*FresnelC(2\*2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)-960\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*cos(2\*arcsin(a\*x))+480\*Pi\*FresnelC(2\*arcsin(a\*x)^(1/2)/Pi^(1/2))+60\*cos(4\*arcsin(a\*x))\*arcsin(a\*x)^(1/2)\*Pi^(1/2))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(a*x)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{asin}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*asin(a*x)^(5/2),x)`

[Out] `int(x^3*asin(a*x)^(5/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asin}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*asin(a*x)**(5/2),x)`

[Out] `Integral(x**3*asin(a*x)**(5/2), x)`

### 3.88 $\int x^2 \sin^{-1}(ax)^{5/2} dx$

**Optimal.** Leaf size=178

$$\frac{15\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^3} - \frac{5\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{144a^3} + \frac{5x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} - \frac{5x\sqrt{\sin^{-1}(ax)}}{6a^2} + \frac{5\sqrt{1-a^2x^2}}{9a^3}$$

[Out]  $1/3*x^3*\arcsin(a*x)^{(5/2)}-5/864*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^3+15/32*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})^2*\text{Pi}^{(1/2)}/a^3+5/9*\arcsin(a*x)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a^3+5/18*x^2*\arcsin(a*x)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a-5/6*x*\arcsin(a*x)^{(1/2)}/a^2-5/36*x^3*\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.47, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4629, 4707, 4677, 4619, 4723, 3305, 3351, 3312}

$$\frac{15\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^3} - \frac{5\sqrt{\frac{\pi}{6}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{144a^3} + \frac{5x^2\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} + \frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{9a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcSin[a\*x]^(5/2), x]

[Out]  $(-5*x*\text{Sqrt}[\text{ArcSin}[a*x]])/(6*a^2) - (5*x^3*\text{Sqrt}[\text{ArcSin}[a*x]])/36 + (5*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(9*a^3) + (5*x^2*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(18*a) + (x^3*\text{ArcSin}[a*x]^{(5/2)})/3 + (15*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])]/(16*a^3) - (5*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])]/(144*a^3)$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 4707

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_)^(m\_))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/Sqrt[d + e\*x^2], x], x) + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
 \int x^2 \sin^{-1}(ax)^{5/2} dx &= \frac{1}{3}x^3 \sin^{-1}(ax)^{5/2} - \frac{1}{6}(5a) \int \frac{x^3 \sin^{-1}(ax)^{3/2}}{\sqrt{1 - a^2x^2}} dx \\
 &= \frac{5x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3 \sin^{-1}(ax)^{5/2} - \frac{5}{12} \int x^2 \sqrt{\sin^{-1}(ax)} dx - \frac{5 \int \frac{x \sin^{-1}(ax)^{3/2}}{\sqrt{1 - a^2x^2}} dx}{9a} \\
 &= -\frac{5}{36}x^3 \sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{9a^3} + \frac{5x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3 \sin^{-1}(ax) \\
 &= -\frac{5x\sqrt{\sin^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3 \sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{9a^3} + \frac{5x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} \\
 &= -\frac{5x\sqrt{\sin^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3 \sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{9a^3} + \frac{5x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} \\
 &= -\frac{5x\sqrt{\sin^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3 \sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{9a^3} + \frac{5x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} \\
 &= -\frac{5x\sqrt{\sin^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3 \sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{9a^3} + \frac{5x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{18a} \\
 &= -\frac{5x\sqrt{\sin^{-1}(ax)}}{6a^2} - \frac{5}{36}x^3 \sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{9a^3} + \frac{5x^2\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{18a}
 \end{aligned}$$

**Mathematica [C]** time = 0.06, size = 125, normalized size = 0.70

$$\frac{-81\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, -i \sin^{-1}(ax)\right) - 81\sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, i \sin^{-1}(ax)\right) + \sqrt{3} \left(\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, -3i \sin^{-1}(ax)\right) + \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, 3i \sin^{-1}(ax)\right)\right)}{648a^3 \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcSin[a\*x]^(5/2),x]

[Out]  $(-81\sqrt{(-1)\text{ArcSin}[a*x]}\Gamma[7/2, (-1)\text{ArcSin}[a*x]] - 81\sqrt{I\text{ArcSin}[a*x]}\Gamma[7/2, I\text{ArcSin}[a*x]] + \sqrt{3}(\sqrt{(-1)\text{ArcSin}[a*x]}\Gamma[7/2, (-3*I)\text{ArcSin}[a*x]] + \sqrt{I\text{ArcSin}[a*x]}\Gamma[7/2, (3*I)\text{ArcSin}[a*x]]))/ (648*a^3\sqrt{\text{ArcSin}[a*x]})$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 0.54, size = 309, normalized size = 1.74

$$\frac{i \arcsin(ax)^{\frac{5}{2}} e^{3i \arcsin(ax)}}{24 a^3} - \frac{i \arcsin(ax)^{\frac{5}{2}} e^{i \arcsin(ax)}}{8 a^3} + \frac{i \arcsin(ax)^{\frac{5}{2}} e^{-i \arcsin(ax)}}{8 a^3} - \frac{i \arcsin(ax)^{\frac{5}{2}} e^{-3i \arcsin(ax)}}{24 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^(5/2),x, algorithm="giac")

[Out]  $1/24*I\arcsin(a*x)^{(5/2)}*e^{(3*I\arcsin(a*x))/a^3} - 1/8*I\arcsin(a*x)^{(5/2)}*e^{(I\arcsin(a*x))/a^3} + 1/8*I\arcsin(a*x)^{(5/2)}*e^{(-I\arcsin(a*x))/a^3} - 1/24*I\arcsin(a*x)^{(5/2)}*e^{(-3*I\arcsin(a*x))/a^3} - 5/144*\arcsin(a*x)^{(3/2)}*e^{(3*I\arcsin(a*x))/a^3} + 5/16*\arcsin(a*x)^{(3/2)}*e^{(I\arcsin(a*x))/a^3} + 5/16*\arcsin(a*x)^{(3/2)}*e^{(-I\arcsin(a*x))/a^3} - 5/144*\arcsin(a*x)^{(3/2)}*e^{(-3*I\arcsin(a*x))/a^3} - (5/3456*I - 5/3456)*\sqrt{6}*\sqrt{\pi}*\text{erf}((1/2*I - 1/2)*\sqrt{6}*\sqrt{\arcsin(a*x)})/a^3 + (5/3456*I + 5/3456)*\sqrt{6}*\sqrt{\pi}*\text{erf}(-(1/2*I + 1/2)*\sqrt{6}*\sqrt{\arcsin(a*x)})/a^3 + (15/128*I - 15/128)*\sqrt{2}*\sqrt{\pi}*\text{erf}((1/2*I - 1/2)*\sqrt{2}*\sqrt{\arcsin(a*x)})/a^3 - (15/128*I + 15/128)*\sqrt{2}*\sqrt{\pi}*\text{erf}(-(1/2*I + 1/2)*\sqrt{2}*\sqrt{\arcsin(a*x)})/a^3 - 5/288*I*\sqrt{\arcsin(a*x)}*e^{(3*I\arcsin(a*x))/a^3} + 15/32*I*\sqrt{\arcsin(a*x)}*e^{(I\arcsin(a*x))/a^3} - 15/32*I*\sqrt{\arcsin(a*x)}*e^{(-I\arcsin(a*x))/a^3} + 5/288*I*\sqrt{\arcsin(a*x)}*e^{(-3*I\arcsin(a*x))/a^3}$

**maple** [A] time = 0.09, size = 156, normalized size = 0.88

$$\frac{-216ax \arcsin(ax)^3 + 72 \arcsin(ax)^3 \sin(3 \arcsin(ax)) + 5\sqrt{3} \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arcsin(a\*x)^(5/2),x)

[Out]  $-1/864/a^3/\arcsin(a*x)^{(1/2)}*(-216*a*x*\arcsin(a*x)^3+72*\arcsin(a*x)^3*\sin(3*\arcsin(a*x))+5*3^{(1/2)}*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*\pi^{(1/2)}*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}*\arcsin(a*x)^{(1/2)})+60*\arcsin(a*x)^2*\cos(3*\arcsin(a*x))-540*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}-405*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*\pi^{(1/2)}*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*\arcsin(a*x)^{(1/2)})+810*a*x*\arcsin(a*x)-30*\arcsin(a*x)*\sin(3*\arcsin(a*x))$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsin(a*x)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asin}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*asin(a*x)^(5/2),x)`

[Out] `int(x^2*asin(a*x)^(5/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asin}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*asin(a*x)**(5/2),x)`

[Out] `Integral(x**2*asin(a*x)**(5/2), x)`

### 3.89 $\int x \sin^{-1}(ax)^{5/2} dx$

**Optimal.** Leaf size=119

$$-\frac{15\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^2} + \frac{5x\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{15\sqrt{\sin^{-1}(ax)}}{64a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} - \frac{15}{32}x^2 \sqrt{\sin^{-1}(ax)}$$

[Out]  $-1/4*\arcsin(a*x)^{(5/2)}/a^2+1/2*x^2*\arcsin(a*x)^{(5/2)}-15/128*\text{FresnelC}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^2+5/8*x*\arcsin(a*x)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a+15/64*\arcsin(a*x)^{(1/2)}/a^2-15/32*x^2*\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.31, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {4629, 4707, 4641, 4723, 3312, 3304, 3352}

$$-\frac{15\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{128a^2} + \frac{5x\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{15\sqrt{\sin^{-1}(ax)}}{64a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} - \frac{15}{32}x^2 \sqrt{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcSin[a\*x]^(5/2),x]

[Out]  $(15*\text{Sqrt}[\text{ArcSin}[a*x]])/(64*a^2) - (15*x^2*\text{Sqrt}[\text{ArcSin}[a*x]])/32 + (5*x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(8*a) - \text{ArcSin}[a*x]^{(5/2)}/(4*a^2) + (x^2*\text{ArcSin}[a*x]^{(5/2)})/2 - (15*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(128*a^2)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] :> Simp[(x^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(m+1), x] - Dist[(b\*c\*n)/(m+1), Int[(x^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

#### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[(a + b\*ArcSin[c\*x])^(n+1)/(b\*c\*Sqrt[d]\*(n+1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

## Rule 4707

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.)*(x_.))^ (m_.))/Sqrt[(d_.
+ (e_.)*(x_)^2], x_Symbol] := Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

## Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*C
os[x]^(2*p + 1), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &&
EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

## Rubi steps

$$\begin{aligned}
\int x \sin^{-1}(ax)^{5/2} dx &= \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} - \frac{1}{4}(5a) \int \frac{x^2 \sin^{-1}(ax)^{3/2}}{\sqrt{1 - a^2x^2}} dx \\
&= \frac{5x\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} - \frac{15}{16} \int x\sqrt{\sin^{-1}(ax)} dx - \frac{5 \int \frac{\sin^{-1}(ax)^{3/2}}{\sqrt{1 - a^2x^2}} dx}{8a} \\
&= -\frac{15}{32}x^2\sqrt{\sin^{-1}(ax)} + \frac{5x\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} + \frac{1}{64}(15\sqrt{\sin^{-1}(ax)}) \\
&= -\frac{15}{32}x^2\sqrt{\sin^{-1}(ax)} + \frac{5x\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} + \frac{15\sqrt{\sin^{-1}(ax)}}{64a^2} \\
&= -\frac{15}{32}x^2\sqrt{\sin^{-1}(ax)} + \frac{5x\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} + \frac{15\sqrt{\sin^{-1}(ax)}}{64a^2} \\
&= \frac{15\sqrt{\sin^{-1}(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\sin^{-1}(ax)} + \frac{5x\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} \\
&= \frac{15\sqrt{\sin^{-1}(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\sin^{-1}(ax)} + \frac{5x\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2} \\
&= \frac{15\sqrt{\sin^{-1}(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\sin^{-1}(ax)} + \frac{5x\sqrt{1 - a^2x^2} \sin^{-1}(ax)^{3/2}}{8a} - \frac{\sin^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2 \sin^{-1}(ax)^{5/2}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 81, normalized size = 0.68

$$\frac{\sqrt{\sin^{-1}(ax)} \left( \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, -2i \sin^{-1}(ax)\right) + \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, 2i \sin^{-1}(ax)\right) \right)}{32\sqrt{2} a^2 \sqrt{\sin^{-1}(ax)^2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*ArcSin[a\*x]^(5/2), x]

[Out] (Sqrt[ArcSin[a\*x]]\*(Sqrt[I\*ArcSin[a\*x]]\*Gamma[7/2, (-2\*I)\*ArcSin[a\*x]] + Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[7/2, (2\*I)\*ArcSin[a\*x]]))/(32\*Sqrt[2]\*a^2\*Sqrt[ArcSin[a\*x]^2])



**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 0.28, size = 143, normalized size = 1.20

$$\frac{\arcsin(ax)^{\frac{5}{2}} e^{2i \arcsin(ax)}}{8a^2} - \frac{\arcsin(ax)^{\frac{5}{2}} e^{-2i \arcsin(ax)}}{8a^2} - \frac{5i \arcsin(ax)^{\frac{3}{2}} e^{2i \arcsin(ax)}}{32a^2} + \frac{5i \arcsin(ax)^{\frac{3}{2}} e^{-2i \arcsin(ax)}}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(5/2), x, algorithm="giac")

[Out]  $-1/8 \arcsin(ax)^{5/2} e^{2i \arcsin(ax)} / a^2 - 1/8 \arcsin(ax)^{5/2} e^{-2i \arcsin(ax)} / a^2 - 5/32 i \arcsin(ax)^{3/2} e^{2i \arcsin(ax)} / a^2 + 5/32 i \arcsin(ax)^{3/2} e^{-2i \arcsin(ax)} / a^2 + (15/512 i + 15/512) \sqrt{\pi} \operatorname{erf}((i-1) \sqrt{\arcsin(ax)}) / a^2 - (15/512 i - 15/512) \sqrt{\pi} \operatorname{erf}((-i+1) \sqrt{\arcsin(ax)}) / a^2 + 15/128 \sqrt{\arcsin(ax)} e^{2i \arcsin(ax)} / a^2 + 15/128 \sqrt{\arcsin(ax)} e^{-2i \arcsin(ax)} / a^2$

**maple** [A] time = 0.07, size = 79, normalized size = 0.66

$$\frac{32 \arcsin(ax)^{\frac{5}{2}} \cos(2 \arcsin(ax)) \sqrt{\pi} - 40 \arcsin(ax)^{\frac{3}{2}} \sin(2 \arcsin(ax)) \sqrt{\pi} - 30 \sqrt{\arcsin(ax)} \sqrt{\pi} \cos(2 \arcsin(ax))}{128 a^2 \sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arcsin(a\*x)^(5/2), x)

[Out]  $-1/128 a^{-2} \pi^{1/2} (32 \arcsin(ax)^{5/2} \cos(2 \arcsin(ax)) \pi^{1/2} - 40 \arcsin(ax)^{3/2} \sin(2 \arcsin(ax)) \pi^{1/2} - 30 \arcsin(ax)^{1/2} \pi^{1/2} \cos(2 \arcsin(ax)) + 15 \pi \operatorname{FresnelC}(2 \arcsin(ax)^{1/2} / \pi^{1/2}))$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^(5/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{asin}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*asin(a\*x)^(5/2), x)

[Out] int(x\*asin(a\*x)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asin}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(a*x)**(5/2),x)
```

```
[Out] Integral(x*asin(a*x)**(5/2), x)
```

### 3.90 $\int \sin^{-1}(ax)^{5/2} dx$

**Optimal.** Leaf size=88

$$\frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{2a} + \frac{15\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a} + x \sin^{-1}(ax)^{5/2} - \frac{15}{4} x \sqrt{\sin^{-1}(ax)}$$

[Out]  $x*\arcsin(a*x)^{(5/2)}+15/8*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a+5/2*\arcsin(a*x)^{(3/2)}*(-a^2*x^2+1)^{(1/2)}/a-15/4*x*\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.16, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4619, 4677, 4723, 3305, 3351}

$$\frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{2a} + \frac{15\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a} + x \sin^{-1}(ax)^{5/2} - \frac{15}{4} x \sqrt{\sin^{-1}(ax)}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(5/2), x]

[Out]  $(-15*x*\text{Sqrt}[\text{ArcSin}[a*x]])/4 + (5*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^{(3/2)})/(2*a) + x*\text{ArcSin}[a*x]^{(5/2)} + (15*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])]/(4*a)$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*(x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p+1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p+1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p+1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p+1/2)\*(a + b\*ArcSin[c\*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*(x\_.)^(m\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m+1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p+1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
\int \sin^{-1}(ax)^{5/2} dx &= x \sin^{-1}(ax)^{5/2} - \frac{1}{2}(5a) \int \frac{x \sin^{-1}(ax)^{3/2}}{\sqrt{1-a^2x^2}} dx \\
&= \frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{2a} + x \sin^{-1}(ax)^{5/2} - \frac{15}{4} \int \sqrt{\sin^{-1}(ax)} dx \\
&= -\frac{15}{4}x\sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{2a} + x \sin^{-1}(ax)^{5/2} + \frac{1}{8}(15a) \int \frac{x}{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}} dx \\
&= -\frac{15}{4}x\sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{2a} + x \sin^{-1}(ax)^{5/2} + \frac{15 \operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a} \\
&= -\frac{15}{4}x\sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{2a} + x \sin^{-1}(ax)^{5/2} + \frac{15 \operatorname{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a} \\
&= -\frac{15}{4}x\sqrt{\sin^{-1}(ax)} + \frac{5\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}}{2a} + x \sin^{-1}(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}} \operatorname{Si}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a}
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 68, normalized size = 0.77

$$\frac{-\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, -i \sin^{-1}(ax)\right) - \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{7}{2}, i \sin^{-1}(ax)\right)}{2a\sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^(5/2), x]

[Out]  $(-\sqrt{-i \operatorname{ArcSin}[a*x]} \Gamma[7/2, (-i) \operatorname{ArcSin}[a*x]] - \sqrt{i \operatorname{ArcSin}[a*x]} \Gamma[7/2, i \operatorname{ArcSin}[a*x]]) / (2*a*\sqrt{\operatorname{ArcSin}[a*x]})$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.35, size = 155, normalized size = 1.76

$$-\frac{i \arcsin(ax)^{\frac{5}{2}} e^{i \arcsin(ax)}}{2a} + \frac{i \arcsin(ax)^{\frac{5}{2}} e^{-i \arcsin(ax)}}{2a} + \frac{5 \arcsin(ax)^{\frac{3}{2}} e^{i \arcsin(ax)}}{4a} + \frac{5 \arcsin(ax)^{\frac{3}{2}} e^{-i \arcsin(ax)}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(5/2), x, algorithm="giac")

[Out]  $-1/2*I*\arcsin(a*x)^{(5/2)}*e^{(I*\arcsin(a*x))}/a + 1/2*I*\arcsin(a*x)^{(5/2)}*e^{(-I*\arcsin(a*x))}/a + 5/4*\arcsin(a*x)^{(3/2)}*e^{(I*\arcsin(a*x))}/a + 5/4*\arcsin(a*x)^{(3/2)}*e^{(-I*\arcsin(a*x))}/a + (15/32*I - 15/32)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}((1/2*I - 1/2)*\sqrt{2}*\sqrt{\arcsin(a*x)})/a - (15/32*I + 15/32)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}(-(1/2*I + 1/2)*\sqrt{2}*\sqrt{\arcsin(a*x)})/a + 15/8*I*\sqrt{\arcsin(a*x)}*e^{(I*\arcsin(a*x))}/a - 15/8*I*\sqrt{\arcsin(a*x)}*e^{(-I*\arcsin(a*x))}/a$

**maple** [A] time = 0.06, size = 88, normalized size = 1.00

$$\frac{\sqrt{2} \left( 4 \arcsin(ax)^{\frac{5}{2}} \sqrt{2} \sqrt{\pi} xa + 10 \arcsin(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} \sqrt{-a^2x^2+1} - 15\sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} xa + 15\pi S \left( \frac{2\sqrt{2}\sqrt{\pi} \arcsin(ax)}{\sqrt{-a^2x^2+1}} \right) \right)}{8a\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^(5/2),x)

[Out] 1/8/a\*2^(1/2)/Pi^(1/2)\*(4\*arcsin(a\*x)^(5/2)\*2^(1/2)\*Pi^(1/2)\*x\*a+10\*arcsin(a\*x)^(3/2)\*2^(1/2)\*Pi^(1/2)\*(-a^2\*x^2+1)^(1/2)-15\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*x\*a+15\*Pi\*FresnelS(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2)))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asin}(ax)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^(5/2),x)

[Out] int(asin(a\*x)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asin}^{\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*(5/2),x)

[Out] Integral(asin(a\*x)\*\*(5/2), x)

$$3.91 \quad \int \frac{\sin^{-1}(ax)^{5/2}}{x} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\sin^{-1}(ax)^{5/2}}{x}, x\right)$$

[Out] Unintegrable(arcsin(a\*x)^(5/2)/x, x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sin^{-1}(ax)^{5/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcSin[a\*x]^(5/2)/x, x]

[Out] Defer[Int][ArcSin[a\*x]^(5/2)/x, x]

Rubi steps

$$\int \frac{\sin^{-1}(ax)^{5/2}}{x} dx = \int \frac{\sin^{-1}(ax)^{5/2}}{x} dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(ax)^{5/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcSin[a\*x]^(5/2)/x, x]

[Out] Integrate[ArcSin[a\*x]^(5/2)/x, x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(5/2)/x, x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^{5/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(5/2)/x, x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^(5/2)/x, x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^(5/2)/x,x)

[Out] int(arcsin(a\*x)^(5/2)/x,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^(5/2)/x,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{asin}(ax)^{5/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^(5/2)/x,x)

[Out] int(asin(a\*x)^(5/2)/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^{\frac{5}{2}}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*(5/2)/x,x)

[Out] Integral(asin(a\*x)\*\*(5/2)/x, x)

$$3.92 \quad \int \frac{x^4}{\sqrt{\sin^{-1}(ax)}} dx$$

**Optimal.** Leaf size=106

$$\frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5} + \frac{\sqrt{\frac{\pi}{10}} C\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5}$$

[Out] 1/80\*FresnelC(10^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*10^(1/2)\*Pi^(1/2)/a^5+1/8\*FresnelC(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)\*Pi^(1/2)/a^5-1/16\*FresnelC(6^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*6^(1/2)\*Pi^(1/2)/a^5

**Rubi [A]** time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4635, 4406, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5} + \frac{\sqrt{\frac{\pi}{10}} \text{FresnelC}\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[ArcSin[a\*x]],x]

[Out] (Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcSin[a\*x]]])/(4\*a^5) - (Sqrt[(3\*Pi)/2]\*FresnelC[Sqrt[6/Pi]\*Sqrt[ArcSin[a\*x]]])/(8\*a^5) + (Sqrt[Pi/10]\*FresnelC[Sqrt[10/Pi]\*Sqrt[ArcSin[a\*x]]])/(8\*a^5)

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_.)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n\*Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rubi steps



$$\begin{aligned}
\int \frac{x^4}{\sqrt{\sin^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^4(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{8\sqrt{x}} - \frac{3\cos(3x)}{16\sqrt{x}} + \frac{\cos(5x)}{16\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^5} - \frac{3\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{16a^5} \\
&= \frac{\text{Subst}\left(\int \cos(5x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{8a^5} + \frac{\text{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{3\text{Subst}\left(\int \cos(3x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{8a^5} \\
&= \frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5} + \frac{\sqrt{\frac{\pi}{10}} C\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^5}
\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 193, normalized size = 1.82

$$\frac{i\left(10\sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -i\sin^{-1}(ax)\right) - 10\sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, i\sin^{-1}(ax)\right) - 5\sqrt{3}\sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -3i\sin^{-1}(ax)\right) + 5\sqrt{3}\sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, 3i\sin^{-1}(ax)\right)\right)}{a^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/Sqrt[ArcSin[a\*x]], x]

[Out]  $((-1/160*I)*(10*\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (-I)*\text{ArcSin}[a*x]] - 10*\text{Sqrt}[I*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, I*\text{ArcSin}[a*x]] - 5*\text{Sqrt}[3]*\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (-3*I)*\text{ArcSin}[a*x]] + 5*\text{Sqrt}[3]*\text{Sqrt}[I*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (3*I)*\text{ArcSin}[a*x]] + \text{Sqrt}[5]*\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (-5*I)*\text{ArcSin}[a*x]] - \text{Sqrt}[5]*\text{Sqrt}[I*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (5*I)*\text{ArcSin}[a*x]]))/(a^5*\text{Sqrt}[\text{ArcSin}[a*x]])$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.56, size = 139, normalized size = 1.31

$$\frac{(i+1)\sqrt{10}\sqrt{\pi}\text{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{10}\sqrt{\arcsin(ax)}\right)}{320a^5} + \frac{(i-1)\sqrt{10}\sqrt{\pi}\text{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{10}\sqrt{\arcsin(ax)}\right)}{320a^5} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out]  $-(1/320*I + 1/320)*\text{sqrt}(10)*\text{sqrt}(\pi)*\text{erf}((1/2*I - 1/2)*\text{sqrt}(10)*\text{sqrt}(\arcsin(a*x)))/a^5 + (1/320*I - 1/320)*\text{sqrt}(10)*\text{sqrt}(\pi)*\text{erf}(-1/2*I + 1/2)*\text{sqrt}(10)*\text{sqrt}(\arcsin(a*x))/a^5 + (1/64*I + 1/64)*\text{sqrt}(6)*\text{sqrt}(\pi)*\text{erf}((1/2*I - 1/2)*\text{sqrt}(6)*\text{sqrt}(\arcsin(a*x)))/a^5 - (1/64*I - 1/64)*\text{sqrt}(6)*\text{sqrt}(\pi)*\text{erf}(-1/2*I + 1/2)*\text{sqrt}(6)*\text{sqrt}(\arcsin(a*x))/a^5 - (1/32*I + 1/32)*\text{sqrt}(2)*\text{sqrt}(\pi)*\text{erf}((1/2*I - 1/2)*\text{sqrt}(2)*\text{sqrt}(\arcsin(a*x)))/a^5 + (1/32*I - 1/32)*\text{sqrt}(2)*\text{sqrt}(\pi)*\text{erf}(-1/2*I + 1/2)*\text{sqrt}(2)*\text{sqrt}(\arcsin(a*x))/a^5$

$(\pi) \cdot \operatorname{erf}\left(\frac{1}{2} \sqrt{2} \sqrt{\arcsin(ax)}\right) / a^5 + (1/32) \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} \sqrt{\arcsin(ax)}\right) / a^5$

**maple** [A] time = 0.10, size = 72, normalized size = 0.68

$$\frac{\sqrt{2} \sqrt{\pi} \left( -5\sqrt{3} \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + \sqrt{5} \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{5} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + 10 \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \right)}{80a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/arcsin(a*x)^(1/2),x)`

[Out] `1/80/a^5*2^(1/2)*Pi^(1/2)*(-5*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arcsin(a*x)^(1/2))+5^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arcsin(a*x)^(1/2))+10*FresnelC(2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/arcsin(a*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/asin(a*x)^(1/2),x)`

[Out] `int(x^4/asin(a*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/asin(a*x)**(1/2),x)`

[Out] `Integral(x**4/sqrt(asin(a*x)), x)`

$$3.93 \quad \int \frac{x^3}{\sqrt{\sin^{-1}(ax)}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}} S\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^4}$$

[Out] -1/16\*FresnelS(2\*2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)\*Pi^(1/2)/a^4+1/4\*FresnelS(2\*arcsin(a\*x)^(1/2)/Pi^(1/2))\*Pi^(1/2)/a^4

**Rubi [A]** time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4635, 4406, 3305, 3351}

$$\frac{\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}} S\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[ArcSin[a\*x]],x]

[Out] -(Sqrt[Pi/2]\*FresnelS[2\*Sqrt[2/Pi]\*Sqrt[ArcSin[a\*x]]])/(8\*a^4) + (Sqrt[Pi]\*FresnelS[(2\*Sqrt[ArcSin[a\*x]])/Sqrt[Pi]])/(4\*a^4)

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n\*Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{\sin^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^3(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}} - \frac{\sin(4x)}{8\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{4a^4} \\
&= -\frac{\text{Subst}\left(\int \sin(4x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a^4} + \frac{\text{Subst}\left(\int \sin(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{2a^4} \\
&= -\frac{\sqrt{\frac{\pi}{2}} S\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{4a^4}
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 128, normalized size = 1.97

$$\frac{-2\sqrt{2}\sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -2i\sin^{-1}(ax)\right) - 2\sqrt{2}\sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, 2i\sin^{-1}(ax)\right) + \sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -4i\sin^{-1}(ax)\right) + \sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, 4i\sin^{-1}(ax)\right)}{32a^4\sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/Sqrt[ArcSin[a\*x]], x]

[Out]  $(-2\sqrt{2}\sqrt{-i\text{ArcSin}[a*x]}\Gamma[1/2, (-2i)\text{ArcSin}[a*x]] - 2\sqrt{2}\sqrt{i\text{ArcSin}[a*x]}\Gamma[1/2, (2i)\text{ArcSin}[a*x]] + \sqrt{-i\text{ArcSin}[a*x]}\Gamma[1/2, (-4i)\text{ArcSin}[a*x]] + \sqrt{i\text{ArcSin}[a*x]}\Gamma[1/2, (4i)\text{ArcSin}[a*x]])/(32a^4\sqrt{\text{ArcSin}[a*x]})$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.31, size = 81, normalized size = 1.25

$$-\frac{(i-1)\sqrt{2}\sqrt{\pi}\text{erf}\left((i-1)\sqrt{2}\sqrt{\arcsin(ax)}\right)}{64a^4} + \frac{(i+1)\sqrt{2}\sqrt{\pi}\text{erf}\left(-(i+1)\sqrt{2}\sqrt{\arcsin(ax)}\right)}{64a^4} + \frac{(i-1)\sqrt{\pi}\text{erf}\left(i\sqrt{\arcsin(ax)}\right)}{64a^4} + \frac{(i+1)\sqrt{\pi}\text{erf}\left(-i\sqrt{\arcsin(ax)}\right)}{64a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out]  $(-1/64i - 1/64)\sqrt{2}\sqrt{\pi}\text{erf}\left((i-1)\sqrt{2}\sqrt{\arcsin(ax)}\right)/a^4 + (1/64i + 1/64)\sqrt{2}\sqrt{\pi}\text{erf}\left(-(i+1)\sqrt{2}\sqrt{\arcsin(ax)}\right)/a^4 + (1/16i - 1/16)\sqrt{\pi}\text{erf}\left(i\sqrt{\arcsin(ax)}\right)/a^4 - (1/16i + 1/16)\sqrt{\pi}\text{erf}\left(-i\sqrt{\arcsin(ax)}\right)/a^4$

**maple [A]** time = 0.07, size = 44, normalized size = 0.68

$$\frac{\sqrt{\pi}\left(-\sqrt{2}S\left(\frac{2\sqrt{2}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + 4S\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)\right)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/arcsin(a*x)^(1/2),x)`

[Out] `1/16/a^4*Pi^(1/2)*(-2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))+4*FresnelS(2*arcsin(a*x)^(1/2)/Pi^(1/2))`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arcsin(a*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{\sin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/asin(a*x)^(1/2),x)`

[Out] `int(x^3/asin(a*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\sin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/asin(a*x)**(1/2),x)`

[Out] `Integral(x**3/sqrt(asin(a*x)), x)`

$$3.94 \quad \int \frac{x^2}{\sqrt{\sin^{-1}(ax)}} dx$$

**Optimal.** Leaf size=71

$$\frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^3}$$

[Out]  $-1/12*\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(ax)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^{3+1/4}$   
 $*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(ax)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^3$

**Rubi [A]** time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4635, 4406, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}} \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[ArcSin[a\*x]],x]

[Out] (Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcSin[a\*x]]])/(2\*a^3) - (Sqrt[Pi/6]\*FresnelC[Sqrt[6/Pi]\*Sqrt[ArcSin[a\*x]]])/(2\*a^3)

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_.)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n\*Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{\sin^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{4\sqrt{x}} - \frac{\cos(3x)}{4\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{4a^3} \\
&= \frac{\text{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{2a^3} - \frac{\text{Subst}\left(\int \cos(3x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{2a^3} \\
&= \frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^3}
\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 128, normalized size = 1.80

$$\frac{i\left(3\sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -i\sin^{-1}(ax)\right) - 3\sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, i\sin^{-1}(ax)\right) + \sqrt{3}\left(\sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, 3i\sin^{-1}(ax)\right)\right)\right)}{24a^3\sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/Sqrt[ArcSin[a\*x]], x]

[Out] ((-1/24\*I)\*(3\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[1/2, (-I)\*ArcSin[a\*x]] - 3\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[1/2, I\*ArcSin[a\*x]] + Sqrt[3]\*(-(Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[1/2, (-3\*I)\*ArcSin[a\*x]]) + Sqrt[I\*ArcSin[a\*x]]\*Gamma[1/2, (3\*I)\*ArcSin[a\*x]])))/(a^3\*Sqrt[ArcSin[a\*x]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [C]** time = 0.37, size = 93, normalized size = 1.31

$$\frac{(i+1)\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{6}\sqrt{\arcsin(ax)}\right)}{48a^3} - \frac{(i-1)\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{6}\sqrt{\arcsin(ax)}\right)}{48a^3} - \frac{(i+1)\sqrt{2}}{48a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] (1/48\*I + 1/48)\*sqrt(6)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^3 - (1/48\*I - 1/48)\*sqrt(6)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(6)\*sqrt(arcsin(a\*x)))/a^3 - (1/16\*I + 1/16)\*sqrt(2)\*sqrt(pi)\*erf((1/2\*I - 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^3 + (1/16\*I - 1/16)\*sqrt(2)\*sqrt(pi)\*erf(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(arcsin(a\*x)))/a^3

**maple** [A] time = 0.07, size = 51, normalized size = 0.72

$$\frac{\sqrt{2} \sqrt{\pi} \left( -\sqrt{3} \operatorname{FresnelC} \left( \frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}} \right) + 3 \operatorname{FresnelC} \left( \frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}} \right) \right)}{12a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arcsin(a\*x)^(1/2),x)

[Out] 1/12/a^3\*2^(1/2)\*Pi^(1/2)\*(-3^(1/2)\*FresnelC(2^(1/2)/Pi^(1/2)\*3^(1/2)\*arcsin(a\*x)^(1/2))+3\*FresnelC(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/asin(a\*x)^(1/2),x)

[Out] int(x^2/asin(a\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/asin(a\*x)\*\*(1/2),x)

[Out] Integral(x\*\*2/sqrt(asin(a\*x)), x)



$$3.95 \quad \int \frac{x}{\sqrt{\sin^{-1}(ax)}} dx$$

Optimal. Leaf size=28

$$\frac{\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

[Out] 1/2\*FresnelS(2\*arcsin(a\*x)^(1/2)/Pi^(1/2))\*Pi^(1/2)/a^2

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4635, 4406, 12, 3305, 3351}

$$\frac{\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[ArcSin[a\*x]],x]

[Out] (Sqrt[Pi]\*FresnelS[(2\*Sqrt[ArcSin[a\*x]])/Sqrt[Pi]])/(2\*a^2)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)]]/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^(n)\*Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{\sin^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{2a^2} \\
&= \frac{\text{Subst}\left(\int \sin(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a^2} \\
&= \frac{\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{2a^2}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 71, normalized size = 2.54

$$\frac{\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2i \sin^{-1}(ax)\right) + \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, 2i \sin^{-1}(ax)\right)}{4\sqrt{2} a^2 \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/Sqrt[ArcSin[a\*x]], x]

[Out]  $-1/4*(\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (-2*I)*\text{ArcSin}[a*x]] + \text{Sqrt}[I*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (2*I)*\text{ArcSin}[a*x]])/(\text{Sqrt}[2]*a^2*\text{Sqrt}[\text{ArcSin}[a*x]])$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [C]** time = 0.31, size = 35, normalized size = 1.25

$$\frac{(i-1)\sqrt{\pi} \operatorname{erf}\left((i-1)\sqrt{\arcsin(ax)}\right)}{8a^2} - \frac{(i+1)\sqrt{\pi} \operatorname{erf}\left(-(i+1)\sqrt{\arcsin(ax)}\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out]  $(1/8*I - 1/8)*\text{sqrt}(\pi)*\text{erf}((I - 1)*\text{sqrt}(\arcsin(a*x)))/a^2 - (1/8*I + 1/8)*\text{sqrt}(\pi)*\text{erf}(-(I + 1)*\text{sqrt}(\arcsin(a*x)))/a^2$

**maple [A]** time = 0.05, size = 21, normalized size = 0.75

$$\frac{S\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/arcsin(a*x)^(1/2),x)`

[Out] `1/2*FresnelS(2*arcsin(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/arcsin(a*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/asin(a*x)^(1/2),x)`

[Out] `int(x/asin(a*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/asin(a*x)**(1/2),x)`

[Out] `Integral(x/sqrt(asin(a*x)), x)`

$$3.96 \quad \int \frac{1}{\sqrt{\sin^{-1}(ax)}} dx$$

**Optimal.** Leaf size=30

$$\frac{\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a}$$

[Out] FresnelC(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)\*Pi^(1/2)/a

**Rubi [A]** time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4623, 3304, 3352}

$$\frac{\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[ArcSin[a\*x]], x]

[Out] (Sqrt[2\*Pi]\*FresnelC[Sqrt[2/Pi]\*Sqrt[ArcSin[a\*x]]])/a

Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\sin^{-1}(ax)}} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a} \\ &= \frac{2 \text{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a} \\ &= \frac{\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a} \end{aligned}$$

**Mathematica [C]** time = 0.02, size = 69, normalized size = 2.30

$$\frac{i\left(\sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -i \sin^{-1}(ax)\right) - \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, i \sin^{-1}(ax)\right)\right)}{2a \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[ArcSin[a\*x]],x]

[Out]  $((-1/2*I)*(Sqrt[(-I)*ArcSin[a*x]]*Gamma[1/2, (-I)*ArcSin[a*x]] - Sqrt[I*ArcSin[a*x]]*Gamma[1/2, I*ArcSin[a*x]]))/(a*Sqrt[ArcSin[a*x]])$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 0.59, size = 47, normalized size = 1.57

$$\frac{(i+1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arcsin(ax)}\right)}{4a} + \frac{(i-1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arcsin(ax)}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(1/2),x, algorithm="giac")

[Out]  $-(1/4*I + 1/4)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}\left(\left(\frac{1}{2}I - \frac{1}{2}\right)*\sqrt{2}*\sqrt{\arcsin(a*x)}\right)/a + (1/4*I - 1/4)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}\left(-\left(\frac{1}{2}I + \frac{1}{2}\right)*\sqrt{2}*\sqrt{\arcsin(a*x)}\right)/a$

**maple** [A] time = 0.05, size = 25, normalized size = 0.83

$$\frac{\operatorname{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\sqrt{\pi}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arcsin(a\*x)^(1/2),x)

[Out]  $\operatorname{FresnelC}(2^{1/2}/\pi^{1/2}*\arcsin(a*x)^{1/2})*2^{1/2}*\pi^{1/2}/a$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/asin(a\*x)^(1/2),x)

[Out] int(1/asin(a\*x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asin(a*x)**(1/2), x)
```

```
[Out] Integral(1/sqrt(asin(a*x)), x)
```

$$3.97 \quad \int \frac{1}{x\sqrt{\sin^{-1}(ax)}} dx$$

**Optimal.** Leaf size=15

$$\text{Int}\left(\frac{1}{x\sqrt{\sin^{-1}(ax)}}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x)^(1/2), x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x\sqrt{\sin^{-1}(ax)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*Sqrt[ArcSin[a\*x]]), x]

[Out] Defer[Int][1/(x\*Sqrt[ArcSin[a\*x]]), x]

Rubi steps

$$\int \frac{1}{x\sqrt{\sin^{-1}(ax)}} dx = \int \frac{1}{x\sqrt{\sin^{-1}(ax)}} dx$$

**Mathematica [A]** time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sin^{-1}(ax)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*Sqrt[ArcSin[a\*x]]), x]

[Out] Integrate[1/(x\*Sqrt[ArcSin[a\*x]]), x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] integrate(1/(x\*sqrt(arcsin(a\*x))), x)

**maple** [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x)^(1/2), x)

[Out] int(1/x/arcsin(a\*x)^(1/2), x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(1/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)^(1/2)), x)

[Out] int(1/(x\*asin(a\*x)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x)\*\*(1/2), x)

[Out] Integral(1/(x\*sqrt(asin(a\*x))), x)



$$3.98 \quad \int \frac{1}{x^2 \sqrt{\sin^{-1}(ax)}} dx$$

**Optimal.** Leaf size=15

$$\text{Int}\left(\frac{1}{x^2 \sqrt{\sin^{-1}(ax)}}, x\right)$$

[Out] Unintegrable(1/x^2/arcsin(a\*x)^(1/2), x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 \sqrt{\sin^{-1}(ax)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*Sqrt[ArcSin[a\*x]]), x]

[Out] Defer[Int][1/(x^2\*Sqrt[ArcSin[a\*x]]), x]

Rubi steps

$$\int \frac{1}{x^2 \sqrt{\sin^{-1}(ax)}} dx = \int \frac{1}{x^2 \sqrt{\sin^{-1}(ax)}} dx$$

**Mathematica [A]** time = 2.97, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\sin^{-1}(ax)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*Sqrt[ArcSin[a\*x]]), x]

[Out] Integrate[1/(x^2\*Sqrt[ArcSin[a\*x]]), x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] integrate(1/(x^2\*sqrt(arcsin(a\*x))), x)

**maple** [A] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arcsin(a\*x)^(1/2),x)

[Out] int(1/x^2/arcsin(a\*x)^(1/2),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arcsin(a\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x^2 \sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*asin(a\*x)^(1/2)),x)

[Out] int(1/(x^2\*asin(a\*x)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/asin(a\*x)\*\*(1/2),x)

[Out] Integral(1/(x\*\*2\*sqrt(asin(a\*x))), x)

$$3.99 \quad \int \frac{x^6}{\sin^{-1}(ax)^{3/2}} dx$$

**Optimal.** Leaf size=171

$$\frac{5\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7} + \frac{9\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7} + \frac{\sqrt{\frac{7\pi}{2}} S\left(\sqrt{\frac{14}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7}$$

[Out]  $-5/32*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^{7+9/3}$   
 $2*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^{7-5/32}*\text{Fr}$   
 $\text{esnelS}(10^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*10^{(1/2)}*\text{Pi}^{(1/2)}/a^{7+1/32}*\text{Fres}$   
 $\text{nelS}(14^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*14^{(1/2)}*\text{Pi}^{(1/2)}/a^{7-2*x^6*(-a^2$   
 $*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4631, 3305, 3351}

$$\frac{5\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7} + \frac{9\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7} + \frac{\sqrt{\frac{7\pi}{2}} S\left(\sqrt{\frac{14}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{16a^7}$$

Antiderivative was successfully verified.

[In] Int[x^6/ArcSin[a\*x]^(3/2), x]

[Out]  $(-2*x^6*\text{Sqrt}[1 - a^2*x^2])/(a*\text{Sqrt}[\text{ArcSin}[a*x]]) - (5*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(16*a^7) + (9*\text{Sqrt}[(3*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(16*a^7) - (5*\text{Sqrt}[(5*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(16*a^7) + (\text{Sqrt}[(7*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[14/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(16*a^7)$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^m, x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sin^{-1}(ax)^{3/2}} dx &= -\frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2 \operatorname{Subst}\left(\int\left(-\frac{5\sin(x)}{64\sqrt{x}} + \frac{27\sin(3x)}{64\sqrt{x}} - \frac{25\sin(5x)}{64\sqrt{x}} + \frac{7\sin(7x)}{64\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^7} \\
&= -\frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{5 \operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{32a^7} + \frac{7 \operatorname{Subst}\left(\int \frac{\sin(7x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{32a^7} \\
&= -\frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{5 \operatorname{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{16a^7} + \frac{7 \operatorname{Subst}\left(\int \sin(7x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{16a^7} \\
&= -\frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{5\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{16a^7} + \frac{9\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{16a^7}
\end{aligned}$$

**Mathematica [C]** time = 0.22, size = 427, normalized size = 2.50

$$\frac{5\left(e^{i\sin^{-1}(ax)} - \sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -i\sin^{-1}(ax)\right)\right)}{64\sqrt{\sin^{-1}(ax)}} - \frac{5\left(e^{-i\sin^{-1}(ax)} - \sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, i\sin^{-1}(ax)\right)\right)}{64\sqrt{\sin^{-1}(ax)}} + \frac{9\left(e^{3i\sin^{-1}(ax)} - \sqrt{3}\sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -3i\sin^{-1}(ax)\right)\right)}{64\sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/ArcSin[a\*x]^(3/2), x]

[Out]  $\left(\frac{-5\left(E^{i\operatorname{ArcSin}[a*x]} - \sqrt{-i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, -i\operatorname{ArcSin}[a*x]\right)\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}} - \frac{5\left(E^{-i\operatorname{ArcSin}[a*x]} - \sqrt{i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, i\operatorname{ArcSin}[a*x]\right)\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}} + \frac{9\left(E^{3i\operatorname{ArcSin}[a*x]} - \sqrt{3}\sqrt{-i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, -3i\operatorname{ArcSin}[a*x]\right)\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}} + \frac{9\left(E^{-3i\operatorname{ArcSin}[a*x]} - \sqrt{3}\sqrt{i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, 3i\operatorname{ArcSin}[a*x]\right)\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}} - \frac{5\left(E^{5i\operatorname{ArcSin}[a*x]} - \sqrt{5}\sqrt{-i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, -5i\operatorname{ArcSin}[a*x]\right)\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}} - \frac{5\left(E^{-5i\operatorname{ArcSin}[a*x]} - \sqrt{5}\sqrt{i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, 5i\operatorname{ArcSin}[a*x]\right)\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}} + \frac{E^{7i\operatorname{ArcSin}[a*x]} - \sqrt{7}\sqrt{-i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, -7i\operatorname{ArcSin}[a*x]\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}} + \frac{E^{-7i\operatorname{ArcSin}[a*x]} - \sqrt{7}\sqrt{i\operatorname{ArcSin}[a*x]}\Gamma\left(\frac{1}{2}, 7i\operatorname{ArcSin}[a*x]\right)}{64\sqrt{\operatorname{ArcSin}[a*x]}}\right)/a^7$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate(x^6/arcsin(a\*x)^(3/2), x)

**maple** [A] time = 0.14, size = 184, normalized size = 1.08

$$-\sqrt{2} \sqrt{\pi} \sqrt{7} S\left(\frac{\sqrt{2} \sqrt{7} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{\arcsin(ax)} + 5\sqrt{5} \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{5} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - 9\sqrt{3} \sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/arcsin(a\*x)^(3/2), x)

[Out]  $-1/32/a^7*(-2^{(1/2)}*Pi^{(1/2)}*7^{(1/2)}*FresnelS(2^{(1/2)}/Pi^{(1/2)}*7^{(1/2)}*\arcsin(a*x)^{(1/2)}*\arcsin(a*x)^{(1/2)}+5*5^{(1/2)}*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*Pi^{(1/2)}*FresnelS(2^{(1/2)}/Pi^{(1/2)}*5^{(1/2)}*\arcsin(a*x)^{(1/2)})-9*3^{(1/2)}*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*Pi^{(1/2)}*FresnelS(2^{(1/2)}/Pi^{(1/2)}*3^{(1/2)}*\arcsin(a*x)^{(1/2)})+5*2^{(1/2)}*\arcsin(a*x)^{(1/2)}*Pi^{(1/2)}*FresnelS(2^{(1/2)}/Pi^{(1/2)}*\arcsin(a*x)^{(1/2)})+5*(-a^2*x^2+1)^{(1/2)}-9*\cos(3*\arcsin(a*x))+5*\cos(5*\arcsin(a*x))- \cos(7*\arcsin(a*x)))/\arcsin(a*x)^{(1/2)}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/arcsin(a\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\operatorname{asin}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/asin(a\*x)^(3/2), x)

[Out] int(x^6/asin(a\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/asin(a\*x)\*\*(3/2), x)

[Out] Integral(x\*\*6/asin(a\*x)\*\*(3/2), x)

$$3.100 \quad \int \frac{x^5}{\sin^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=127

$$-\frac{\sqrt{\frac{\pi}{2}} C\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^6} + \frac{\sqrt{3\pi} C\left(2\sqrt{\frac{3}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^6} + \frac{5\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^6} - \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-1/2*\text{FresnelC}(2*2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^6+5/8*\text{FresnelC}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^6+1/8*\text{FresnelC}(2*3^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*3^{(1/2)}*\text{Pi}^{(1/2)}/a^6-2*x^5*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4631, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^6} + \frac{\sqrt{3\pi} \text{FresnelC}\left(2\sqrt{\frac{3}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^6} + \frac{5\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{8a^6} - \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x^5/ArcSin[a\*x]^(3/2), x]

[Out]  $(-2*x^5*\text{Sqrt}[1 - a^2*x^2])/(a*\text{Sqrt}[\text{ArcSin}[a*x]]) - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/a^6 + (\text{Sqrt}[3*\text{Pi}]*\text{FresnelC}[2*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(8*a^6) + (5*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(8*a^6)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)]/(f\*Rt[d, 2])), x] /; FreeQ[{d, e, f}, x]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sin^{-1}(ax)^{3/2}} dx &= -\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2 \operatorname{Subst}\left(\int \left(\frac{5\cos(2x)}{16\sqrt{x}} - \frac{\cos(4x)}{2\sqrt{x}} + \frac{3\cos(6x)}{16\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^6} \\
&= -\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{3 \operatorname{Subst}\left(\int \frac{\cos(6x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^6} + \frac{5 \operatorname{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^6} \\
&= -\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{3 \operatorname{Subst}\left(\int \cos(6x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a^6} + \frac{5 \operatorname{Subst}\left(\int \cos(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a^6} \\
&= -\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}} C\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^6} + \frac{\sqrt{3\pi} C\left(2\sqrt{\frac{3}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^6} + \frac{5\sqrt{\pi} C\left(2\sqrt{\frac{1}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{8a^6}
\end{aligned}$$

**Mathematica [C]** time = 0.14, size = 231, normalized size = 1.82

$$10 \sin\left(2 \sin^{-1}(ax)\right) - 8 \sin\left(4 \sin^{-1}(ax)\right) + 2 \sin\left(6 \sin^{-1}(ax)\right) + 5i\sqrt{2} \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2i \sin^{-1}(ax)\right) - 5$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5/ArcSin[a\*x]^(3/2), x]

[Out]  $-1/32*((5*I)*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[(-I)*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (-2*I)*\operatorname{ArcSin}[a*x]] - (5*I)*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[I*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (2*I)*\operatorname{ArcSin}[a*x]] - (8*I)*\operatorname{Sqrt}[(-I)*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (-4*I)*\operatorname{ArcSin}[a*x]] + (8*I)*\operatorname{Sqrt}[I*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (4*I)*\operatorname{ArcSin}[a*x]] + I*\operatorname{Sqrt}[6]*\operatorname{Sqrt}[(-I)*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (-6*I)*\operatorname{ArcSin}[a*x]] - I*\operatorname{Sqrt}[6]*\operatorname{Sqrt}[I*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (6*I)*\operatorname{ArcSin}[a*x]] + 10*\operatorname{Sin}[2*\operatorname{ArcSin}[a*x]] - 8*\operatorname{Sin}[4*\operatorname{ArcSin}[a*x]] + 2*\operatorname{Sin}[6*\operatorname{ArcSin}[a*x]])/(a^6*\operatorname{Sqrt}[\operatorname{ArcSin}[a*x]])$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vector & l) Error: Bad Argument Value

**maple [A]** time = 0.11, size = 121, normalized size = 0.95

$$8 \operatorname{FresnelC}\left(\frac{2\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} - 2\sqrt{\pi} \sqrt{3} \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{6} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{\arcsin(ax)} - 5 \operatorname{FresnelC}\left(\frac{2\sqrt{\pi} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{\arcsin(ax)} - 5$$

$16a^6\sqrt{\arcsin(ax)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/arcsin(a*x)^(3/2),x)`

[Out] `-1/16/a^6*(8*FresnelC(2*2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))*2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)-2*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)*arcsin(a*x)^(1/2))*arcsin(a*x)^(1/2)-10*FresnelC(2*arcsin(a*x)^(1/2)/Pi^(1/2))*arcsin(a*x)^(1/2)*Pi^(1/2)+5*sin(2*arcsin(a*x))-4*sin(4*arcsin(a*x))+sin(6*arcsin(a*x)))/arcsin(a*x)^(1/2)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/arcsin(a*x)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\operatorname{asin}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/asin(a*x)^(3/2),x)`

[Out] `int(x^5/asin(a*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/asin(a*x)**(3/2),x)`

[Out] `Integral(x**5/asin(a*x)**(3/2), x)`



$$3.101 \quad \int \frac{x^4}{\sin^{-1}(ax)^{3/2}} dx$$

**Optimal.** Leaf size=136

$$\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^5} + \frac{3\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-1/4*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^5+3/8*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^5-1/8*\text{FresnelS}(10^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*10^{(1/2)}*\text{Pi}^{(1/2)}/a^5-2*x^4*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4631, 3305, 3351}

$$\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^5} + \frac{3\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcSin[a\*x]^(3/2), x]

[Out]  $(-2*x^4*\text{Sqrt}[1 - a^2*x^2])/(a*\text{Sqrt}[\text{ArcSin}[a*x]]) - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(2*a^5) + (3*\text{Sqrt}[(3*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(4*a^5) - (\text{Sqrt}[(5*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(4*a^5)$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^2)], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sin^{-1}(ax)^{3/2}} dx &= -\frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2 \operatorname{Subst}\left(\int \left(-\frac{\sin(x)}{8\sqrt{x}} + \frac{9\sin(3x)}{16\sqrt{x}} - \frac{5\sin(5x)}{16\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^5} \\
&= -\frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{4a^5} - \frac{5 \operatorname{Subst}\left(\int \frac{\sin(5x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{8a^5} + \frac{9}{16a^5} \\
&= -\frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\operatorname{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{2a^5} - \frac{5 \operatorname{Subst}\left(\int \sin(5x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{4a^5} \\
&= -\frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^5} + \frac{3\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{4a^5}
\end{aligned}$$

**Mathematica [C]** time = 0.12, size = 319, normalized size = 2.35

$$\frac{e^{i\sin^{-1}(ax)} - \sqrt{-i\sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -i\sin^{-1}(ax)\right)}{8\sqrt{\sin^{-1}(ax)}} - \frac{e^{-i\sin^{-1}(ax)} - \sqrt{i\sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, i\sin^{-1}(ax)\right)}{8\sqrt{\sin^{-1}(ax)}} + \frac{3\left(e^{3i\sin^{-1}(ax)} - \sqrt{3}\sqrt{-i\sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -3i\sin^{-1}(ax)\right)\right)}{16\sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/ArcSin[a\*x]^(3/2), x]

[Out]  $(-1/8*(E^{(I*ArcSin[a*x])} - Sqrt[(-I)*ArcSin[a*x]]*Gamma[1/2, (-I)*ArcSin[a*x]])/Sqrt[ArcSin[a*x]] - (E^{((-I)*ArcSin[a*x])} - Sqrt[I*ArcSin[a*x]]*Gamma[1/2, I*ArcSin[a*x]])/(8*Sqrt[ArcSin[a*x]]) + (3*(E^{((3*I)*ArcSin[a*x])} - Sqrt[3]*Sqrt[(-I)*ArcSin[a*x]]*Gamma[1/2, (-3*I)*ArcSin[a*x]])/(16*Sqrt[ArcSin[a*x]]) + (3*(E^{((-3*I)*ArcSin[a*x])} - Sqrt[3]*Sqrt[I*ArcSin[a*x]]*Gamma[1/2, (3*I)*ArcSin[a*x]])/(16*Sqrt[ArcSin[a*x]]) - (E^{((5*I)*ArcSin[a*x])} - Sqrt[5]*Sqrt[(-I)*ArcSin[a*x]]*Gamma[1/2, (-5*I)*ArcSin[a*x]])/(16*Sqrt[ArcSin[a*x]]) - (E^{((-5*I)*ArcSin[a*x])} - Sqrt[5]*Sqrt[I*ArcSin[a*x]]*Gamma[1/2, (5*I)*ArcSin[a*x]])/(16*Sqrt[ArcSin[a*x]])/a^5$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate(x^4/arcsin(a\*x)^(3/2), x)

**maple** [A] time = 0.08, size = 141, normalized size = 1.04

$$\frac{3\sqrt{3}\sqrt{2}\sqrt{\arcsin(ax)}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - \sqrt{5}\sqrt{2}\sqrt{\arcsin(ax)}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{5}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - 2\sqrt{2}\sqrt{\arcsin(ax)}}{8a^5\sqrt{\arcsin(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arcsin(a\*x)^(3/2),x)

[Out] 1/8/a^5\*(3\*3^(1/2)\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*FresnelS(2^(1/2)/Pi^(1/2)\*3^(1/2)\*arcsin(a\*x)^(1/2))-5^(1/2)\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*FresnelS(2^(1/2)/Pi^(1/2)\*5^(1/2)\*arcsin(a\*x)^(1/2))-2\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*FresnelS(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))-2\*(-a^2\*x^2+1)^(1/2)+3\*cos(3\*arcsin(a\*x))-cos(5\*arcsin(a\*x)))/arcsin(a\*x)^(1/2)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{asin}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/asin(a\*x)^(3/2),x)

[Out] int(x^4/asin(a\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/asin(a\*x)\*\*(3/2),x)

[Out] Integral(x\*\*4/asin(a\*x)\*\*(3/2), x)

$$3.102 \quad \int \frac{x^3}{\sin^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=90

$$-\frac{\sqrt{\frac{\pi}{2}} C\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^4} + \frac{\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4} - \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-1/2*\text{FresnelC}(2*2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^4+\text{FresnelC}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^4-2*x^3*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4631, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^4} + \frac{\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4} - \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/\text{ArcSin}[a*x]^{(3/2)}, x]$

[Out]  $(-2*x^3*\text{Sqrt}[1 - a^2*x^2])/(a*\text{Sqrt}[\text{ArcSin}[a*x]]) - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/a^4 + (\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/a^4$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \text{ :> Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(\text{f}*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

#### Rule 4631

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)}*(x_.)^{(m_.)}, x\_Symbol] \text{ :> Simp}[(x^m*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^{(n+1)})/(b*c*(n+1)), x] - \text{Dist}[1/(b*c^{(m+1)}*(n+1)), \text{Subst}[\text{Int}[\text{ExpandTrigReduce}[(a + b*x)^{(n+1)}, \text{Sin}[x]^{(m-1)}*(m - (m+1)*\text{Sin}[x]^2), x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -2] \ \&\& \ \text{LtQ}[n, -1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sin^{-1}(ax)^{3/2}} dx &= -\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2 \operatorname{Subst}\left(\int \left(\frac{\cos(2x)}{2\sqrt{x}} - \frac{\cos(4x)}{2\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^4} \\
&= -\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{\operatorname{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^4} - \frac{\operatorname{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^4} \\
&= -\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2 \operatorname{Subst}\left(\int \cos(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a^4} - \frac{2 \operatorname{Subst}\left(\int \cos(4x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a^4} \\
&= -\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}} C\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^4} + \frac{\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^4}
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 154, normalized size = 1.71

$$\frac{-2 \sin\left(2 \sin^{-1}(ax)\right) + \sin\left(4 \sin^{-1}(ax)\right) - i\sqrt{2} \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2i \sin^{-1}(ax)\right) + i\sqrt{2} \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, 2i \sin^{-1}(ax)\right)}{4a^4 \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/ArcSin[a\*x]^(3/2), x]

[Out]  $((-1)*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[(-1)*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (-2*I)*\operatorname{ArcSin}[a*x]] + I*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[I*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (2*I)*\operatorname{ArcSin}[a*x]] + I*\operatorname{Sqrt}[(-1)*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (-4*I)*\operatorname{ArcSin}[a*x]] - I*\operatorname{Sqrt}[I*\operatorname{ArcSin}[a*x]]*\operatorname{Gamma}[1/2, (4*I)*\operatorname{ArcSin}[a*x]] - 2*\operatorname{Sin}[2*\operatorname{ArcSin}[a*x]] + \operatorname{Sin}[4*\operatorname{ArcSin}[a*x]])/(4*a^4*\operatorname{Sqrt}[\operatorname{ArcSin}[a*x]])$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple [A]** time = 0.06, size = 83, normalized size = 0.92

$$\frac{2 \operatorname{FresnelC}\left(\frac{2\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} - 4 \operatorname{FresnelC}\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{\arcsin(ax)} \sqrt{\pi} + 2 \sin(2 \arcsin(ax))}{4a^4 \sqrt{\arcsin(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/arcsin(a*x)^(3/2),x)`

[Out] `-1/4/a^4/arcsin(a*x)^(1/2)*(2*FresnelC(2*2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))*2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)-4*FresnelC(2*arcsin(a*x)^(1/2)/Pi^(1/2))*arcsin(a*x)^(1/2)*Pi^(1/2)+2*sin(2*arcsin(a*x))-sin(4*arcsin(a*x))`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arcsin(a*x)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{asin}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/asin(a*x)^(3/2),x)`

[Out] `int(x^3/asin(a*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/asin(a*x)**(3/2),x)`

[Out] `Integral(x**3/asin(a*x)**(3/2), x)`

$$3.103 \quad \int \frac{x^2}{\sin^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=96

$$-\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3} + \frac{\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3} - \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-1/2*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\text{arcsin}(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^{3+1/2}*$   
 $\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\text{arcsin}(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^{3-2*x^2*(-a$   
 $^2*x^2+1)^{(1/2)}/a/\text{arcsin}(a*x)^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4631, 3305, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3} + \frac{\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3} - \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcSin[a\*x]^(3/2), x]

[Out]  $(-2*x^2*\text{Sqrt}[1 - a^2*x^2])/(a*\text{Sqrt}[\text{ArcSin}[a*x]]) - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/a^3 + (\text{Sqrt}[(3*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/a^3$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/ (f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^ (n\_)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sin^{-1}(ax)^{3/2}} dx &= -\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2 \operatorname{Subst}\left(\int \left(-\frac{\sin(x)}{4\sqrt{x}} + \frac{3\sin(3x)}{4\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^3} \\
&= -\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{2a^3} + \frac{3 \operatorname{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{2a^3} \\
&= -\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\operatorname{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a^3} + \frac{3 \operatorname{Subst}\left(\int \sin(3x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a^3} \\
&= -\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3} + \frac{\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3}
\end{aligned}$$

**Mathematica [C]** time = 0.07, size = 211, normalized size = 2.20

$$\frac{e^{i\sin^{-1}(ax)} - \sqrt{-i\sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -i\sin^{-1}(ax)\right)}{4\sqrt{\sin^{-1}(ax)}} - \frac{e^{-i\sin^{-1}(ax)} - \sqrt{i\sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, i\sin^{-1}(ax)\right)}{4\sqrt{\sin^{-1}(ax)}} + \frac{e^{3i\sin^{-1}(ax)} - \sqrt{3} \sqrt{-i\sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -3i\sin^{-1}(ax)\right)}{4\sqrt{\sin^{-1}(ax)}} + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/ArcSin[a\*x]^(3/2), x]

[Out] (-1/4\*(E^(I\*ArcSin[a\*x]) - Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[1/2, (-I)\*ArcSin[a\*x]])/Sqrt[ArcSin[a\*x]] - (E^((-I)\*ArcSin[a\*x]) - Sqrt[I\*ArcSin[a\*x]]\*Gamma[1/2, I\*ArcSin[a\*x]])/(4\*Sqrt[ArcSin[a\*x]]) + (E^((3\*I)\*ArcSin[a\*x]) - Sqrt[3]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[1/2, (-3\*I)\*ArcSin[a\*x]])/(4\*Sqrt[ArcSin[a\*x]]) + (E^((-3\*I)\*ArcSin[a\*x]) - Sqrt[3]\*Sqrt[I\*ArcSin[a\*x]]\*Gamma[1/2, (3\*I)\*ArcSin[a\*x]])/(4\*Sqrt[ArcSin[a\*x]])/a^3

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate(x^2/arcsin(a\*x)^(3/2), x)

**maple [A]** time = 0.06, size = 95, normalized size = 0.99

$$\frac{\sqrt{3} \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - \sqrt{-a^2x^2 + 1} + \cos(3 \arcsin(ax))}{2a^3 \sqrt{\arcsin(ax)}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/arcsin(a*x)^(3/2),x)`

[Out]  $\frac{1}{2}a^{-3}(3^{(1/2)}2^{(1/2)}\arcsin(ax)^{(1/2)}\pi^{(1/2)}\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)})3^{(1/2)}\arcsin(ax)^{(1/2)})-2^{(1/2)}\arcsin(ax)^{(1/2)}\pi^{(1/2)}\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}\arcsin(ax)^{(1/2)})-(-a^2x^2+1)^{(1/2)}+\cos(3\arcsin(ax)))/a\arcsin(ax)^{(1/2)}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/arcsin(a*x)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\arcsin(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/asin(a*x)^(3/2),x)`

[Out] `int(x^2/asin(a*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\arcsin^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/asin(a*x)**(3/2),x)`

[Out] `Integral(x**2/asin(a*x)**(3/2), x)`

### 3.104 $\int \frac{x}{\sin^{-1}(ax)^{3/2}} dx$

**Optimal.** Leaf size=55

$$\frac{2\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2} - \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

[Out]  $2*\text{FresnelC}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^2-2*x*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4631, 3304, 3352}

$$\frac{2\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2} - \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSin[a\*x]^(3/2), x]

[Out]  $(-2*x*\text{Sqrt}[1 - a^2*x^2])/(a*\text{Sqrt}[\text{ArcSin}[a*x]]) + (2*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/a^2$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/ (f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{x}{\sin^{-1}(ax)^{3/2}} dx &= -\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2 \text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^2} \\ &= -\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{4 \text{Subst}\left(\int \cos(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a^2} \\ &= -\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} + \frac{2\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{a^2} \end{aligned}$$

**Mathematica [C]** time = 0.03, size = 91, normalized size = 1.65

$$\frac{2 \sin\left(2 \sin^{-1}(ax)\right) + i\sqrt{2} \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2i \sin^{-1}(ax)\right) - i\sqrt{2} \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, 2i \sin^{-1}(ax)\right)}{2a^2 \sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/ArcSin[a\*x]^(3/2), x]

[Out]  $-1/2*(I*\text{Sqrt}[2]*\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (-2*I)*\text{ArcSin}[a*x]] - I*\text{Sqrt}[2]*\text{Sqrt}[I*\text{ArcSin}[a*x]]*\text{Gamma}[1/2, (2*I)*\text{ArcSin}[a*x]] + 2*\text{Sin}[2*\text{ArcSin}[a*x]])/(a^2*\text{Sqrt}[\text{ArcSin}[a*x]])$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate(x/arcsin(a\*x)^(3/2), x)

**maple [A]** time = 0.05, size = 43, normalized size = 0.78

$$\frac{-2 \text{FresnelC}\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \sqrt{\arcsin(ax)} \sqrt{\pi} + \sin(2 \arcsin(ax))}{a^2 \sqrt{\arcsin(ax)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arcsin(a\*x)^(3/2), x)

[Out]  $-1/a^2*(-2*\text{FresnelC}(2*\arcsin(a*x)^(1/2)/\text{Pi}^(1/2))*\arcsin(a*x)^(1/2)*\text{Pi}^(1/2) + \sin(2*\arcsin(a*x)))/\arcsin(a*x)^(1/2)$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\arcsin(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/asin(a*x)^(3/2),x)`

[Out] `int(x/asin(a*x)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/asin(a*x)**(3/2),x)`

[Out] `Integral(x/asin(a*x)**(3/2), x)`

### 3.105 $\int \frac{1}{\sin^{-1}(ax)^{3/2}} dx$

**Optimal.** Leaf size=59

$$-\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{2\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{a}$$

[Out]  $-2*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a-2*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4621, 4723, 3305, 3351}

$$-\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{2\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(-3/2), x]

[Out]  $(-2*\text{Sqrt}[1 - a^2*x^2])/(a*\text{Sqrt}[\text{ArcSin}[a*x]]) - (2*\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/a$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sin^{-1}(ax)^{3/2}} dx &= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - (2a) \int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\sin^{-1}(ax)}} dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{2 \operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{4 \operatorname{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\sin^{-1}(ax)}} - \frac{2\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a}
\end{aligned}$$

**Mathematica** [C] time = 0.09, size = 87, normalized size = 1.47

$$\frac{-e^{-i \sin^{-1}(ax)} (1 + e^{2i \sin^{-1}(ax)}) + \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -i \sin^{-1}(ax)\right) + \sqrt{i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, i \sin^{-1}(ax)\right)}{a\sqrt{\sin^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^(-3/2), x]

[Out]  $(-(1 + E^{((2I) \operatorname{ArcSin}[a*x])})/E^{(I \operatorname{ArcSin}[a*x])}) + \operatorname{Sqrt}[(-I) \operatorname{ArcSin}[a*x]] * \operatorname{Gamma}[1/2, (-I) \operatorname{ArcSin}[a*x]] + \operatorname{Sqrt}[I \operatorname{ArcSin}[a*x]] * \operatorname{Gamma}[1/2, I \operatorname{ArcSin}[a*x]]) / (a * \operatorname{Sqrt}[\operatorname{ArcSin}[a*x]])$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^(-3/2), x)

**maple** [A] time = 0.06, size = 65, normalized size = 1.10

$$-\frac{\sqrt{2} \left( 2 \arcsin(ax) \pi S\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} \sqrt{-a^2x^2 + 1} \right)}{a\sqrt{\pi} \arcsin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arcsin(a\*x)^(3/2), x)

```
[Out] -1/a*2^(1/2)/Pi^(1/2)*(2*arcsin(a*x)*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))+2^(1/2)*arcsin(a*x)^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2))/arcsin(a*x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsin(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asin}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asin(a*x)^(3/2),x)
```

```
[Out] int(1/asin(a*x)^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asin(a*x)**(3/2),x)
```

```
[Out] Integral(asin(a*x)**(-3/2), x)
```

$$3.106 \quad \int \frac{1}{x \sin^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sin^{-1}(ax)^{3/2}}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x)^(3/2), x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x \sin^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*ArcSin[a\*x]^(3/2)), x]

[Out] Defer[Int][1/(x\*ArcSin[a\*x]^(3/2)), x]

Rubi steps

$$\int \frac{1}{x \sin^{-1}(ax)^{3/2}} dx = \int \frac{1}{x \sin^{-1}(ax)^{3/2}} dx$$

Mathematica [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sin^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*ArcSin[a\*x]^(3/2)), x]

[Out] Integrate[1/(x\*ArcSin[a\*x]^(3/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate(1/(x\*arcsin(a\*x)^(3/2)), x)



**maple** [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x)^(3/2),x)

[Out] int(1/x/arcsin(a\*x)^(3/2),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asin}(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)^(3/2)),x)

[Out] int(1/(x\*asin(a\*x)^(3/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x)\*\*(3/2),x)

[Out] Integral(1/(x\*asin(a\*x)\*\*(3/2)), x)

$$3.107 \quad \int \frac{x^4}{\sin^{-1}(ax)^{5/2}} dx$$

**Optimal.** Leaf size=171

$$\frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^5} + \frac{3\sqrt{\frac{3\pi}{2}} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{2a^5} - \frac{5\sqrt{\frac{5\pi}{2}} C\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{6a^5} - \frac{16x^3}{3a^2\sqrt{\sin^{-1}(ax)}} - \frac{2x^4\sqrt{1-}}{3a\sin^{-1}(ax)}$$

[Out]  $-1/6*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^{5+3/4}*$   
 $\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^{5-5/12}*\text{Fres}$   
 $\text{nelC}(10^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*10^{(1/2)}*\text{Pi}^{(1/2)}/a^{5-2/3}*x^4*(-a$   
 $^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(3/2)}-16/3*x^3/a^2/\arcsin(a*x)^{(1/2)}+20/3*x^5$   
 $/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.43, antiderivative size = 235, normalized size of antiderivative = 1.37, number of steps used = 19, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4633, 4719, 4635, 4406, 3304, 3352}

$$\frac{4\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^5} - \frac{25\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^5} - \frac{4\sqrt{\frac{2\pi}{3}} \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^5} + \dots$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcSin[a\*x]^(5/2), x]

[Out]  $(-2*x^4*\text{Sqrt}[1 - a^2*x^2])/(3*a*\text{ArcSin}[a*x]^{(3/2)}) - (16*x^3)/(3*a^2*\text{Sqrt}[A$   
 $\text{rcSin}[a*x]) + (20*x^5)/(3*\text{Sqrt}[\text{ArcSin}[a*x]]) - (25*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqr}$   
 $\text{t}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(3*a^5) + (4*\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Sqr}$   
 $\text{t}[\text{ArcSin}[a*x]])/a^5 + (25*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]$   
 $])/ (2*a^5) - (4*\text{Sqrt}[(2*\text{Pi})/3]*\text{FresnelC}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/a^5$   
 $- (5*\text{Sqrt}[(5*\text{Pi})/2]*\text{FresnelC}[\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]])/(6*a^5)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/ (f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n \* Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_.))^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{\sin^{-1}(ax)^{5/2}} dx &= -\frac{2x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} + \frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{3/2}} dx}{3a} - \frac{1}{3}(10a) \int \frac{x^5}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{3/2}} dx \\ &= -\frac{2x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\sin^{-1}(ax)}} - \frac{100}{3} \int \frac{x^4}{\sqrt{\sin^{-1}(ax)}} dx + \frac{16\int \frac{x^5}{\sqrt{\sin^{-1}(ax)}} dx}{a} \\ &= -\frac{2x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\sin^{-1}(ax)}} + \frac{16\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^5} \\ &= -\frac{2x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\sin^{-1}(ax)}} + \frac{16\text{Subst}\left(\int \left(\frac{\cos(x)}{4\sqrt{x}} - \frac{\cos(3x)}{4\sqrt{x}}\right) dx, x, \sin^{-1}(ax)\right)}{a^5} \\ &= -\frac{2x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\sin^{-1}(ax)}} - \frac{25\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{12a^5} \\ &= -\frac{2x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\sin^{-1}(ax)}} - \frac{25\text{Subst}\left(\int \cos(5x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{6a^5} \\ &= -\frac{2x^4\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{20x^5}{3\sqrt{\sin^{-1}(ax)}} - \frac{25\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{3a^5} + \frac{4\sqrt{2}}{3a^5} \end{aligned}$$

**Mathematica [C]** time = 0.30, size = 418, normalized size = 2.44

$$\frac{i e^{i \sin^{-1}(ax)} (-2 \sin^{-1}(ax) + i) - 2 (-i \sin^{-1}(ax))^{3/2} \Gamma\left(\frac{1}{2}, -i \sin^{-1}(ax)\right)}{24 \sin^{-1}(ax)^{3/2}} - \frac{e^{-i \sin^{-1}(ax)} (-2i \sin^{-1}(ax) + 2e^{i \sin^{-1}(ax)} (i \sin^{-1}(ax))^{3/2} \Gamma\left(\frac{1}{2}, i \sin^{-1}(ax)\right) + 1)}{24 \sin^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/ArcSin[a\*x]^(5/2), x]

[Out] ((I\*E^(I\*ArcSin[a\*x]))\*(I - 2\*ArcSin[a\*x]) - 2\*((-I)\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (-I)\*ArcSin[a\*x]])/(24\*ArcSin[a\*x]^(3/2)) - (1 - (2\*I)\*ArcSin[a\*x] + 2\*E^(I\*ArcSin[a\*x]))\*(I\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, I\*ArcSin[a\*x]]/(24\*E^(I\*ArcSin[a\*x])\*ArcSin[a\*x]^(3/2)) - (I\*E^((3\*I)\*ArcSin[a\*x]))\*(I - 6\*ArcSin[a\*x]) - 6\*Sqrt[3]\*((-I)\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (-3\*I)\*ArcSin[a\*x]]/(16\*ArcSin[a\*x]^(3/2)) + (1 - (6\*I)\*ArcSin[a\*x] + 6\*Sqrt[3]\*E^((3\*I)\*ArcSin[a\*x]))\*(I\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (3\*I)\*ArcSin[a\*x]]/(16\*E^((3\*I)\*

$\text{ArcSin}[a*x])*\text{ArcSin}[a*x]^{(3/2)} + (I*\text{E}^{((5*I)*\text{ArcSin}[a*x])}*(I - 10*\text{ArcSin}[a*x]) - 10*\text{Sqrt}[5]*((-I)*\text{ArcSin}[a*x])^{(3/2)}*\text{Gamma}[1/2, (-5*I)*\text{ArcSin}[a*x]])/(48*\text{ArcSin}[a*x]^{(3/2)}) - (1 - (10*I)*\text{ArcSin}[a*x] + 10*\text{Sqrt}[5]*\text{E}^{((5*I)*\text{ArcSin}[a*x])}*(I*\text{ArcSin}[a*x])^{(3/2)}*\text{Gamma}[1/2, (5*I)*\text{ArcSin}[a*x]])/(48*\text{E}^{((5*I)*\text{ArcSin}[a*x])}*\text{ArcSin}[a*x]^{(3/2)})/a^5$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\arcsin(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(5/2),x, algorithm="giac")

[Out] integrate(x^4/arcsin(a\*x)^(5/2), x)

**maple** [A] time = 0.12, size = 173, normalized size = 1.01

---


$$10\sqrt{2} \sqrt{\pi} \sqrt{5} \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{5} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{3}{2}} - 18\sqrt{2} \sqrt{\pi} \sqrt{3} \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{3}{2}}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arcsin(a\*x)^(5/2),x)

[Out]  $-1/24/a^5*(10*2^{(1/2)}*\text{Pi}^{(1/2)}*5^{(1/2)}*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*5^{(1/2)}*\arcsin(a*x)^{(1/2)})*\arcsin(a*x)^{(3/2)} - 18*2^{(1/2)}*\text{Pi}^{(1/2)}*3^{(1/2)}*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*3^{(1/2)}*\arcsin(a*x)^{(1/2)})*\arcsin(a*x)^{(3/2)} + 4*2^{(1/2)}*\text{Pi}^{(1/2)}*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*\arcsin(a*x)^{(3/2)} - 4*a*x*\arcsin(a*x) + 18*\arcsin(a*x)*\sin(3*\arcsin(a*x)) - 10*\arcsin(a*x)*\sin(5*\arcsin(a*x)) + 2*(-a^2*x^2+1)^{(1/2)} - 3*\cos(3*\arcsin(a*x)) + \cos(5*\arcsin(a*x)))/\arcsin(a*x)^{(3/2)}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\text{asin}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asin(a*x)^(5/2),x)
```

```
[Out] int(x^4/asin(a*x)^(5/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^4}{\operatorname{asin}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asin(a*x)**(5/2),x)
```

```
[Out] Integral(x**4/asin(a*x)**(5/2), x)
```

$$3.108 \quad \int \frac{x^3}{\sin^{-1}(ax)^{5/2}} dx$$

**Optimal.** Leaf size=126

$$\frac{4\sqrt{2\pi} S\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^4} - \frac{4\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^4} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} - \frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-4/3*\text{FresnelS}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^4+4/3*\text{FresnelS}(2*2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^4-2/3*x^3*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(3/2)}-4*x^2/a^2/\arcsin(a*x)^{(1/2)}+16/3*x^4/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.34, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {4633, 4719, 4635, 4406, 3305, 3351, 12}

$$\frac{4\sqrt{2\pi} S\left(2\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^4} - \frac{4\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^4} - \frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] `Int[x^3/ArcSin[a*x]^(5/2), x]`

[Out]  $(-2*x^3*\text{Sqrt}[1 - a^2*x^2])/(3*a*\text{ArcSin}[a*x]^{(3/2)}) - (4*x^2)/(a^2*\text{Sqrt}[\text{ArcSin}[a*x]]) + (16*x^4)/(3*\text{Sqrt}[\text{ArcSin}[a*x]]) + (4*\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(3*a^4) - (4*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(3*a^4)$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 3305

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

#### Rule 3351

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]`

#### Rule 4406

`Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^(n)*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

#### Rule 4633

`Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSin[c*x])^(n + 1))/Sqrt`

$[1 - c^2 x^2], x], x] - \text{Dist}[m/(b*c*(n + 1)), \text{Int}[(x^{(m - 1)}*(a + b*\text{ArcSin}[c*x])^{(n + 1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{LtQ}[n, -2]$

### Rule 4635

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*(x^m), x\_Symbol] := \text{Dist}[1/c^{(m + 1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sin}[x]^m*\text{Cos}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 4719

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*((f*x)^m)/\text{Sqrt}[d + e*x^2], x\_Symbol] := \text{Simp}[(f*x)^m*(a + b*\text{ArcSin}[c*x])^{(n + 1)}/(b*c*\text{Sqrt}[d]*(n + 1)), x] - \text{Dist}[(f*m)/(b*c*\text{Sqrt}[d]*(n + 1)), \text{Int}[(f*x)^{(m - 1)}*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{LtQ}[n, -1] \&\& \text{GtQ}[d, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sin^{-1}(ax)^{5/2}} dx &= -\frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{3/2}} dx}{a} - \frac{1}{3}(8a) \int \frac{x^4}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{3/2}} dx \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}} - \frac{64}{3} \int \frac{x^3}{\sqrt{\sin^{-1}(ax)}} dx + \frac{8\int \frac{x}{\sqrt{\sin^{-1}(ax)}} dx}{a^2} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}} + \frac{8\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^4} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}} + \frac{8\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{a^4} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}} + \frac{8\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{3a^4} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}} + \frac{16\text{Subst}\left(\int \sin(4x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{3a^4} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sin^{-1}(ax)}} + \frac{16x^4}{3\sqrt{\sin^{-1}(ax)}} + \frac{4\sqrt{2\pi} S\left(2\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{3a^4} - \frac{4\sqrt{\pi}}{3a^4} \end{aligned}$$

**Mathematica [C]** time = 0.38, size = 200, normalized size = 1.59

$$-2 \sin\left(2 \sin^{-1}(ax)\right) + \sin\left(4 \sin^{-1}(ax)\right) - 4 \sin^{-1}(ax) \left(e^{-2i \sin^{-1}(ax)} + e^{2i \sin^{-1}(ax)} - \sqrt{2} \sqrt{-i \sin^{-1}(ax)} \Gamma\left(\frac{1}{2}, -2i \sin^{-1}(ax)\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/ArcSin[a\*x]^(5/2), x]

[Out] (-4\*ArcSin[a\*x]\*(E^((-2\*I)\*ArcSin[a\*x]) + E^((2\*I)\*ArcSin[a\*x])) - Sqrt[2]\*Sqrt[(-I)\*ArcSin[a\*x]]\*Gamma[1/2, (-2\*I)\*ArcSin[a\*x]] - Sqrt[2]\*Sqrt[I\*ArcSin[a\*x]])

```
n[a*x]]*Gamma[1/2, (2*I)*ArcSin[a*x]]) + 4*ArcSin[a*x]*(E^((-4*I)*ArcSin[a*x]) + E^((4*I)*ArcSin[a*x]) - 2*Sqrt[(-I)*ArcSin[a*x]]*Gamma[1/2, (-4*I)*ArcSin[a*x]] - 2*Sqrt[I*ArcSin[a*x]]*Gamma[1/2, (4*I)*ArcSin[a*x]]) - 2*Sin[2*ArcSin[a*x]] + Sin[4*ArcSin[a*x]])/(12*a^4*ArcSin[a*x]^(3/2))
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsin(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsin(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value
```

**maple** [A] time = 0.08, size = 109, normalized size = 0.87

$$\frac{-16\sqrt{2}\sqrt{\pi} S\left(\frac{2\sqrt{2}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{3}{2}} + 16\sqrt{\pi} S\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{3}{2}} + 8\arcsin(ax)\cos(2\arcsin(ax))}{12a^4 \arcsin(ax)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsin(a*x)^(5/2),x)
```

```
[Out] -1/12/a^4*(-16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))*arcsin(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arcsin(a*x)^(1/2)/Pi^(1/2))*arcsin(a*x)^(3/2)+8*arcsin(a*x)*cos(2*arcsin(a*x))-8*arcsin(a*x)*cos(4*arcsin(a*x))+2*sin(2*arcsin(a*x))-sin(4*arcsin(a*x)))/arcsin(a*x)^(3/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsin(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\arcsin(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asin(a*x)^(5/2),x)
```



```
[Out] int(x^3/asin(a*x)^(5/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3}{\operatorname{asin}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asin(a*x)**(5/2), x)
```

```
[Out] Integral(x**3/asin(a*x)**(5/2), x)
```

$$3.109 \quad \int \frac{x^2}{\sin^{-1}(ax)^{5/2}} dx$$

**Optimal.** Leaf size=125

$$-\frac{\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi} C\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3} - \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-1/3*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^3+\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^3-2/3*x^2*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(3/2)}-8/3*x/a^2/\arcsin(a*x)^{(1/2)}+4*x^3/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.30, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {4633, 4719, 4635, 4406, 3304, 3352, 4623}

$$-\frac{\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi} \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^3} - \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/\text{ArcSin}[a*x]^{(5/2)}, x]$

[Out]  $(-2*x^2*\text{Sqrt}[1 - a^2*x^2])/(3*a*\text{ArcSin}[a*x]^{(3/2)}) - (8*x)/(3*a^2*\text{Sqrt}[\text{ArcSin}[a*x]]) + (4*x^3)/\text{Sqrt}[\text{ArcSin}[a*x]] - (\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(3*a^3) + (\text{Sqrt}[6*\text{Pi}]*\text{FresnelC}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/a^3$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] \text{ /; FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \text{ :> Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] \text{ /; FreeQ}\{d, e, f\}, x]$

#### Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{ :> Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]]^n*\text{Cos}[a + b*x]^p, x], x] \text{ /; FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 4623

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}, x\_Symbol] \text{ :> Dist}[1/(b*c), \text{Subst}[\text{Int}[x^n*\text{Cos}[a/b - x/b], x], x, a + b*\text{ArcSin}[c*x]], x] \text{ /; FreeQ}\{a, b, c, n\}, x]$

#### Rule 4633

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)^{(m_.)}, x\_Symbol] \text{ :> Simp}[(x^m*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^{(n+1)})/(b*c*(n+1)), x] + (\text{Dist}[(c*(m+1))/(b*(n+1)), \text{Int}[(x^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n+1)})/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Dist}[m/(b*c*(n+1)), \text{Int}[(x^{(m-1)}*(a + b*\text{ArcSin}[$

$c*x])^{(n + 1)}/\text{Sqrt}[1 - c^2*x^2], x], x]) /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{IGtQ}\{m, 0\} \&\& \text{LtQ}\{n, -2\}$

### Rule 4635

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*(x)^m, x\_Symbol] := \text{Dist}[1/c^{(m + 1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sin}[x]^m*\text{Cos}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, n\}, x\} \&\& \text{IGtQ}\{m, 0\}$

### Rule 4719

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*((f*x)^m)/\text{Sqrt}[d + (e*x)^2], x\_Symbol] := \text{Simp}[(f*x)^m*(a + b*\text{ArcSin}[c*x])^{(n + 1)}/(b*c*\text{Sqrt}[d]*(n + 1)), x] - \text{Dist}[(f*m)/(b*c*\text{Sqrt}[d]*(n + 1)), \text{Int}[(f*x)^{m-1}*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \& \& \text{EqQ}[c^2*d + e, 0] \&\& \text{LtQ}\{n, -1\} \&\& \text{GtQ}\{d, 0\}$

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sin^{-1}(ax)^{5/2}} dx &= \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} + \frac{4 \int \frac{x}{\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}} dx}{3a} - (2a) \int \frac{x^3}{\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}} dx \\ &= \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}} - 12 \int \frac{x^2}{\sqrt{\sin^{-1}(ax)}} dx + \frac{8 \int \frac{1}{\sqrt{\sin^{-1}(ax)}}}{3a^2} \\ &= \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}} + \frac{8 \text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{3a^3} \\ &= \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}} + \frac{16 \text{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{3a^3} \\ &= \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}} + \frac{8\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^3} - \frac{3 \text{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, \sqrt{\sin^{-1}(ax)}\right)}{3a^3} \\ &= \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}} + \frac{8\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^3} - \frac{6 \text{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, \sqrt{\sin^{-1}(ax)}\right)}{3a^3} \\ &= \frac{2x^2\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{4x^3}{\sqrt{\sin^{-1}(ax)}} - \frac{\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^3} \end{aligned}$$

**Mathematica [C]** time = 0.15, size = 277, normalized size = 2.22

$$\frac{i e^{i \sin^{-1}(ax)} (-2 \sin^{-1}(ax) + i) - 2 (-i \sin^{-1}(ax))^{3/2} \Gamma\left(\frac{1}{2}, -i \sin^{-1}(ax)\right)}{12 \sin^{-1}(ax)^{3/2}} - \frac{e^{-i \sin^{-1}(ax)} (-2i \sin^{-1}(ax) + 2e^{i \sin^{-1}(ax)} (i \sin^{-1}(ax))^{3/2} \Gamma\left(\frac{1}{2}, i \sin^{-1}(ax)\right) + 1)}{12 \sin^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/ArcSin[a\*x]^(5/2), x]

[Out] ((I\*E^(I\*ArcSin[a\*x]))\*(I - 2\*ArcSin[a\*x]) - 2\*((-I)\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (-I)\*ArcSin[a\*x]])/(12\*ArcSin[a\*x]^(3/2)) - (1 - (2\*I)\*ArcSin[a\*x] + 2\*E^(I\*ArcSin[a\*x]))\*(I\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, I\*ArcSin[a\*x]]/(12\*E^(I\*ArcSin[a\*x])\*ArcSin[a\*x]^(3/2)) - (I\*E^((3\*I)\*ArcSin[a\*x]))\*(I - 6\*ArcSi

$$\frac{n[ax] - 6\sqrt{3} * ((-I) * \text{ArcSin}[ax])^{3/2} * \text{Gamma}[1/2, (-3I) * \text{ArcSin}[ax]]}{(12 * \text{ArcSin}[ax]^{3/2}) + (1 - (6I) * \text{ArcSin}[ax] + 6\sqrt{3} * E^{((3I) * \text{ArcSin}[ax])} * (I * \text{ArcSin}[ax])^{3/2} * \text{Gamma}[1/2, (3I) * \text{ArcSin}[ax]]) / (12 * E^{((3I) * \text{ArcSin}[ax])} * \text{ArcSin}[ax]^{3/2}))} / a^3$$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\arcsin(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(5/2),x, algorithm="giac")

[Out] integrate(x^2/arcsin(a\*x)^(5/2), x)

**maple** [A] time = 0.07, size = 117, normalized size = 0.94

$$\frac{-6\sqrt{2} \sqrt{\pi} \sqrt{3} \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{3}{2}} + 2\sqrt{2} \sqrt{\pi} \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{3}{2}} - 2}{6a^3 \arcsin(ax)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arcsin(a\*x)^(5/2),x)

[Out] 
$$\frac{-1/6/a^3 * (-6 * 2^{(1/2)} * \text{Pi}^{(1/2)} * 3^{(1/2)} * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)} * 3^{(1/2)} * \arcsin(a*x)^{(1/2)}) * \arcsin(a*x)^{(3/2)} + 2 * 2^{(1/2)} * \text{Pi}^{(1/2)} * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)} * \arcsin(a*x)^{(1/2)}) * \arcsin(a*x)^{(3/2)} - 2 * a * x * \arcsin(a*x) + 6 * \arcsin(a*x) * \sin(3 * \arcsin(a*x)) + (-a^2 * x^2 + 1)^{(1/2)} - \cos(3 * \arcsin(a*x))}{\arcsin(a*x)^{(3/2)}}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsin(a\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\text{asin}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/asin(a\*x)^(5/2),x)

[Out] int(x^2/asin(a\*x)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asin}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/asin(a\*x)\*\*(5/2), x)

[Out] Integral(x\*\*2/asin(a\*x)\*\*(5/2), x)

$$3.110 \quad \int \frac{x}{\sin^{-1}(ax)^{5/2}} dx$$

**Optimal.** Leaf size=89

$$-\frac{8\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-8/3*\text{FresnelS}(2*\arcsin(ax)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^2-2/3*x*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(ax)^{(3/2)}-4/3/a^2/\arcsin(ax)^{(1/2)}+8/3*x^2/\arcsin(ax)^{(1/2)}$

**Rubi [A]** time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$ , Rules used = {4633, 4719, 4635, 4406, 12, 3305, 3351, 4641}

$$-\frac{8\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSin[a\*x]^(5/2), x]

[Out]  $(-2*x*\text{Sqrt}[1-a^2*x^2])/(3*a*\text{ArcSin}[a*x]^{(3/2)}) - 4/(3*a^2*\text{Sqrt}[\text{ArcSin}[a*x]]) + (8*x^2)/(3*\text{Sqrt}[\text{ArcSin}[a*x]]) - (8*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(3*a^2)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)]/(f\*Rt[d, 2])), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^(n)\*Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_))^(n\_)\*(x\_)^m, x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m,

0] && LtQ[n, -2]

### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x]; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x]; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

### Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_.)^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x]; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{\sin^{-1}(ax)^{5/2}} dx &= -\frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{3/2}} dx}{3a} - \frac{1}{3}(4a) \int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{3/2}} dx \\
 &= -\frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}} - \frac{16}{3} \int \frac{x}{\sqrt{\sin^{-1}(ax)}} dx \\
 &= -\frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}} - \frac{16 \operatorname{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{3a^2} \\
 &= -\frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}} - \frac{16 \operatorname{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{3a^2} \\
 &= -\frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}} - \frac{8 \operatorname{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{3a^2} \\
 &= -\frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}} - \frac{16 \operatorname{Subst}\left(\int \sin(2x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{3a^2} \\
 &= -\frac{2x\sqrt{1-a^2x^2}}{3a\sin^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sin^{-1}(ax)}} + \frac{8x^2}{3\sqrt{\sin^{-1}(ax)}} - \frac{8\sqrt{\pi} S\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{3a^2}
 \end{aligned}$$

**Mathematica [C]** time = 0.21, size = 112, normalized size = 1.26

$$\frac{\sin(2\sin^{-1}(ax)) + 2\sin^{-1}(ax)\left(e^{-2i\sin^{-1}(ax)} + e^{2i\sin^{-1}(ax)} - \sqrt{2}\sqrt{-i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, -2i\sin^{-1}(ax)\right) - \sqrt{2}\sqrt{i\sin^{-1}(ax)}\Gamma\left(\frac{1}{2}, 2i\sin^{-1}(ax)\right)\right)}{3a^2\sin^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/ArcSin[a\*x]^(5/2), x]

```
[Out] -1/3*(2*ArcSin[a*x]*(E^((-2*I)*ArcSin[a*x]) + E^((2*I)*ArcSin[a*x]) - Sqrt[2]*Sqrt[(-I)*ArcSin[a*x]]*Gamma[1/2, (-2*I)*ArcSin[a*x]] - Sqrt[2]*Sqrt[I*ArcSin[a*x]]*Gamma[1/2, (2*I)*ArcSin[a*x]]) + Sin[2*ArcSin[a*x]])/(a^2*ArcSin[a*x]^(3/2))
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsin(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\arcsin(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsin(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsin(a*x)^(5/2), x)
```

**maple** [A] time = 0.06, size = 56, normalized size = 0.63

$$\frac{8\sqrt{\pi} S\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{3}{2}} + 4 \arcsin(ax) \cos(2 \arcsin(ax)) + \sin(2 \arcsin(ax))}{3a^2 \arcsin(ax)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsin(a*x)^(5/2),x)
```

```
[Out] -1/3/a^2*(8*Pi^(1/2)*FresnelS(2*arcsin(a*x)^(1/2)/Pi^(1/2))*arcsin(a*x)^(3/2)+4*arcsin(a*x)*cos(2*arcsin(a*x))+sin(2*arcsin(a*x)))/arcsin(a*x)^(3/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsin(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asin}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asin(a*x)^(5/2),x)
```

```
[Out] int(x/asin(a*x)^(5/2), x)
```



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asin}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/asin(a\*x)\*\*(5/2), x)

[Out] Integral(x/asin(a\*x)\*\*(5/2), x)

$$3.111 \quad \int \frac{1}{\sin^{-1}(ax)^{5/2}} dx$$

**Optimal.** Leaf size=76

$$-\frac{2\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^{3/2}} - \frac{4\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a} + \frac{4x}{3\sqrt{\sin^{-1}(ax)}}$$

[Out]  $-4/3*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a-2/3*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(3/2)}+4/3*x/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4621, 4719, 4623, 3304, 3352}

$$-\frac{2\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^{3/2}} - \frac{4\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a} + \frac{4x}{3\sqrt{\sin^{-1}(ax)}}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(-5/2), x]

[Out]  $(-2*\text{Sqrt}[1 - a^2*x^2])/(3*a*\text{ArcSin}[a*x]^{(3/2)}) + (4*x)/(3*\text{Sqrt}[\text{ArcSin}[a*x]]) - (4*\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(3*a)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n], x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n], x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4719

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*((f\_.)\*(x\_))^m]/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sin^{-1}(ax)^{5/2}} dx &= -\frac{2\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^{3/2}} - \frac{1}{3}(2a) \int \frac{x}{\sqrt{1-a^2x^2} \sin^{-1}(ax)^{3/2}} dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^{3/2}} + \frac{4x}{3\sqrt{\sin^{-1}(ax)}} - \frac{4}{3} \int \frac{1}{\sqrt{\sin^{-1}(ax)}} dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^{3/2}} + \frac{4x}{3\sqrt{\sin^{-1}(ax)}} - \frac{4 \operatorname{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{3a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^{3/2}} + \frac{4x}{3\sqrt{\sin^{-1}(ax)}} - \frac{8 \operatorname{Subst}\left(\int \cos(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{3a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{3a \sin^{-1}(ax)^{3/2}} + \frac{4x}{3\sqrt{\sin^{-1}(ax)}} - \frac{4\sqrt{2\pi} C\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a}
\end{aligned}$$

**Mathematica [C]** time = 0.17, size = 138, normalized size = 1.82

$$\frac{-2ie^{i \sin^{-1}(ax)} (2 \sin^{-1}(ax) - i) - 4(-i \sin^{-1}(ax))^{3/2} \Gamma\left(\frac{1}{2}, -i \sin^{-1}(ax)\right) e^{-i \sin^{-1}(ax)} (4i \sin^{-1}(ax) - 4e^{i \sin^{-1}(ax)} (i \sin^{-1}(ax) - 1))}{6a \sin^{-1}(ax)^{3/2}} + \frac{e^{-i \sin^{-1}(ax)} (4i \sin^{-1}(ax) - 4e^{i \sin^{-1}(ax)} (i \sin^{-1}(ax) - 1))}{6a \sin^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSin[a\*x]^(-5/2), x]

[Out]  $((-2*I)*E^{(I*ArcSin[a*x])}*(-I + 2*ArcSin[a*x]) - 4*((-I)*ArcSin[a*x])^{(3/2)} *Gamma[1/2, (-I)*ArcSin[a*x]])/(6*a*ArcSin[a*x]^{(3/2)}) + (-2 + (4*I)*ArcSin[a*x] - 4*E^{(I*ArcSin[a*x])}*(I*ArcSin[a*x])^{(3/2)}*Gamma[1/2, I*ArcSin[a*x]])/(6*a*E^{(I*ArcSin[a*x])}*ArcSin[a*x]^{(3/2)})$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\arcsin(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(5/2), x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^(-5/2), x)

**maple [A]** time = 0.06, size = 83, normalized size = 1.09

$$\frac{\sqrt{2} \left( 4 \arcsin(ax)^2 \pi \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) - 2 \arcsin(ax)^{3/2} \sqrt{2} \sqrt{\pi} xa + \sqrt{2} \sqrt{\arcsin(ax)} \sqrt{\pi} \sqrt{-a^2x^2} \right)}{3a\sqrt{\pi} \arcsin(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/arcsin(a*x)^(5/2),x)`

[Out]  $-1/3/a*2^{(1/2)}/\text{Pi}^{(1/2)}*(4*\arcsin(a*x)^2*\text{Pi}*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})-2*\arcsin(a*x)^{(3/2)}*2^{(1/2)}*\text{Pi}^{(1/2)}*x*a+2^{(1/2)}*\arcsin(a*x)^{(1/2)}*\text{Pi}^{(1/2)}*(-a^2*x^2+1)^{(1/2)})/\arcsin(a*x)^2$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arcsin(a*x)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\arcsin(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/asin(a*x)^(5/2),x)`

[Out] `int(1/asin(a*x)^(5/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\arcsin^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/asin(a*x)**(5/2),x)`

[Out] `Integral(asin(a*x)**(-5/2), x)`

$$3.112 \quad \int \frac{1}{x \sin^{-1}(ax)^{5/2}} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sin^{-1}(ax)^{5/2}}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x)^(5/2), x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x \sin^{-1}(ax)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int [1/(x\*ArcSin[a\*x]^(5/2)), x]

[Out] Defer[Int] [1/(x\*ArcSin[a\*x]^(5/2)), x]

Rubi steps

$$\int \frac{1}{x \sin^{-1}(ax)^{5/2}} dx = \int \frac{1}{x \sin^{-1}(ax)^{5/2}} dx$$

Mathematica [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sin^{-1}(ax)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*ArcSin[a\*x]^(5/2)), x]

[Out] Integrate[1/(x\*ArcSin[a\*x]^(5/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(5/2), x, algorithm="giac")

[Out] integrate(1/(x\*arcsin(a\*x)^(5/2)), x)

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x)^(5/2),x)

[Out] int(1/x/arcsin(a\*x)^(5/2),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asin}(ax)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)^(5/2)),x)

[Out] int(1/(x\*asin(a\*x)^(5/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asin}^{\frac{5}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x)\*\*(5/2),x)

[Out] Integral(1/(x\*asin(a\*x)\*\*(5/2)), x)

$$3.113 \quad \int \frac{x^4}{\sin^{-1}(ax)^{7/2}} dx$$

**Optimal.** Leaf size=264

$$\frac{\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{15a^5} + \frac{8\sqrt{6\pi} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{5a^5} - \frac{5\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^5} + \frac{5\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^5}$$

[Out]  $-16/15*x^3/a^2/\arcsin(a*x)^{(3/2)}+4/3*x^5/\arcsin(a*x)^{(3/2)}-9/10*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^5+1/15*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^5+5/6*\text{FresnelS}(10^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(a*x)^{(1/2)})*10^{(1/2)}*\text{Pi}^{(1/2)}/a^5-2/5*x^4*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(5/2)}-32/5*x^2*(-a^2*x^2+1)^{(1/2)}/a^3/\arcsin(a*x)^{(1/2)}+40/3*x^4*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.40, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {4633, 4719, 4631, 3305, 3351}

$$\frac{\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{15a^5} + \frac{8\sqrt{6\pi} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{5a^5} - \frac{5\sqrt{\frac{3\pi}{2}} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{a^5} + \frac{5\sqrt{\frac{5\pi}{2}} S\left(\sqrt{\frac{10}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{3a^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcSin[a\*x]^(7/2), x]

[Out]  $(-2*x^4*\text{Sqrt}[1 - a^2*x^2])/(5*a*\text{ArcSin}[a*x]^{(5/2)}) - (16*x^3)/(15*a^2*\text{ArcSin}[a*x]^{(3/2)}) + (4*x^5)/(3*\text{ArcSin}[a*x]^{(3/2)}) - (32*x^2*\text{Sqrt}[1 - a^2*x^2])/(5*a^3*\text{Sqrt}[\text{ArcSin}[a*x]]) + (40*x^4*\text{Sqrt}[1 - a^2*x^2])/(3*a*\text{Sqrt}[\text{ArcSin}[a*x]]) + (\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(15*a^5) - (5*\text{Sqrt}[(3*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/a^5 + (8*\text{Sqrt}[6*\text{Pi}]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(5*a^5) + (5*\text{Sqrt}[(5*\text{Pi})/2]*\text{FresnelS}[\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(3*a^5)$

#### Rule 3305

Int[sin[(e.) + (f.)\*(x.)]/Sqrt[(c.) + (d.)\*(x.)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d.)\*((e.) + (f.)\*(x.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4631

Int[((a.) + ArcSin[(c.)\*(x.)]\*(b.))^(n.)\*(x.)^(m.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rule 4633

Int[((a.) + ArcSin[(c.)\*(x.)]\*(b.))^(n.)\*(x.)^(m.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]

$c*x)^{(n + 1)}/\text{Sqrt}[1 - c^2*x^2], x], x]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{LtQ}[n, -2]$

Rule 4719

$\text{Int}[(((a_.) + \text{ArcSin}[c_.)*(x_.)]*(b_.))^{(n_.)}*((f_.)*(x_.))^{(m_.)})/\text{Sqrt}[(d_.) + (e_.)*(x_.)^2], x\_Symbol] \text{:>} \text{Simp}[(f*x)^m*(a + b*\text{ArcSin}[c*x])^{(n + 1)}/(b*c*\text{Sqrt}[d]*(n + 1)), x] - \text{Dist}[(f*m)/(b*c*\text{Sqrt}[d]*(n + 1)), \text{Int}[(f*x)^{(m - 1)}*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \& \& \text{EqQ}[c^2*d + e, 0] \&\& \text{LtQ}[n, -1] \&\& \text{GtQ}[d, 0]$

Rubi steps

$$\int \frac{x^4}{\sin^{-1}(ax)^{7/2}} dx = -\frac{2x^4\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} + \frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx}{5a} - (2a)\int \frac{x^5}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx$$

$$= -\frac{2x^4\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^5}{3\sin^{-1}(ax)^{3/2}} - \frac{20}{3}\int \frac{x^4}{\sin^{-1}(ax)^{3/2}} dx + \frac{16\int \frac{x^2}{\sin^{-1}(ax)} dx}{5a^2}$$

$$= -\frac{2x^4\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^5}{3\sin^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\sin^{-1}(ax)}}$$

$$= -\frac{2x^4\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^5}{3\sin^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\sin^{-1}(ax)}}$$

$$= -\frac{2x^4\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^5}{3\sin^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\sin^{-1}(ax)}}$$

$$= -\frac{2x^4\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^5}{3\sin^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\sin^{-1}(ax)}}$$

**Mathematica [C]** time = 0.74, size = 417, normalized size = 1.58

$$9e^{3i\sin^{-1}(ax)}(-12\sin^{-1}(ax)^2 + 2i\sin^{-1}(ax) + 1) + 2e^{i\sin^{-1}(ax)}(4\sin^{-1}(ax)^2 - 2i\sin^{-1}(ax) - 3) + e^{5i\sin^{-1}(ax)}(100\sin^{-1}(ax)^2 - 20i\sin^{-1}(ax) - 3)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/ArcSin[a\*x]^(7/2), x]

[Out]  $(9E^{((3I)*\text{ArcSin}[a*x])}*(1 + (2I)*\text{ArcSin}[a*x] - 12*\text{ArcSin}[a*x]^2) + 2E^{(I*\text{ArcSin}[a*x])}*(-3 - (2I)*\text{ArcSin}[a*x] + 4*\text{ArcSin}[a*x]^2) + E^{((5I)*\text{ArcSin}[a*x])}*(-3 - (10I)*\text{ArcSin}[a*x] + 100*\text{ArcSin}[a*x]^2) - 8*\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{ArcSin}[a*x]^2*\text{Gamma}[1/2, (-I)*\text{ArcSin}[a*x]] + (-6 + (4I)*\text{ArcSin}[a*x] + 8*\text{ArcSin}[a*x]^2 + 8E^{(I*\text{ArcSin}[a*x])}*(I*\text{ArcSin}[a*x])^{(5/2)}*\text{Gamma}[1/2, I*\text{ArcSin}[a*x]])/E^{(I*\text{ArcSin}[a*x])} + 108*\text{Sqrt}[3]*\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{ArcSin}[a*x]^2*\text{Gamma}[1/2, (-3I)*\text{ArcSin}[a*x]] - (9*(-1 + (2I)*\text{ArcSin}[a*x] + 12*\text{ArcSin}[a*x]^2 + 12*\text{Sqrt}[3]*E^{((3I)*\text{ArcSin}[a*x])}*(I*\text{ArcSin}[a*x])^{(5/2)}*\text{Gamma}[1/2, (3I)*\text{ArcSin}[a*x]])/E^{((3I)*\text{ArcSin}[a*x])} - 100*\text{Sqrt}[5]*\text{Sqrt}[(-I)*\text{ArcSin}[a*x]]*\text{ArcSin}[a*x]^2*\text{Gamma}[1/2, (-5I)*\text{ArcSin}[a*x]] + (-3 + (10I)*\text{ArcSin}[a*x] + 100*\text{ArcSin}[a*x]^2 + 100*\text{Sqrt}[5]*E^{((5I)*\text{ArcSin}[a*x])}*(I*\text{ArcSin}[a*x])^{(5/2)}*\text{Gamma}[1/2, (5I)*\text{ArcSin}[a*x]])/E^{((5I)*\text{ArcSin}[a*x])})/(240*a^5*\text{ArcSin}[a*x]^{(5/2)})$



**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\arcsin(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(7/2),x, algorithm="giac")

[Out] integrate(x^4/arcsin(a\*x)^(7/2), x)

**maple** [A] time = 0.13, size = 225, normalized size = 0.85

$$\frac{108\sqrt{2} \sqrt{\pi} \sqrt{3} S\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{5}{2}} - 100\sqrt{2} \sqrt{\pi} \sqrt{5} S\left(\frac{\sqrt{2} \sqrt{5} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{5}{2}} - 8\sqrt{2}}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arcsin(a\*x)^(7/2),x)

[Out] 
$$\frac{-1/120/a^5*(108*2^{(1/2)}*Pi^{(1/2)}*3^{(1/2)}*FresnelS(2^{(1/2)}/Pi^{(1/2)}*3^{(1/2)}*\arcsin(a*x)^{(1/2)}*\arcsin(a*x)^{(5/2)}-100*2^{(1/2)}*Pi^{(1/2)}*5^{(1/2)}*FresnelS(2^{(1/2)}/Pi^{(1/2)}*5^{(1/2)}*\arcsin(a*x)^{(1/2)}*\arcsin(a*x)^{(5/2)}-8*2^{(1/2)}*Pi^{(1/2)}*FresnelS(2^{(1/2)}/Pi^{(1/2)}*\arcsin(a*x)^{(1/2)}*\arcsin(a*x)^{(5/2)}-8*\arcsin(a*x)^2*(-a^2*x^2+1)^{(1/2)}+108*\arcsin(a*x)^2*\cos(3*\arcsin(a*x))-100*\arcsin(a*x)^2*\cos(5*\arcsin(a*x))-4*a*x*\arcsin(a*x)+18*\arcsin(a*x)*\sin(3*\arcsin(a*x))-10*\arcsin(a*x)*\sin(5*\arcsin(a*x))+6*(-a^2*x^2+1)^{(1/2)}-9*\cos(3*\arcsin(a*x))+3*\cos(5*\arcsin(a*x)))/\arcsin(a*x)^{(5/2)}}{\dots}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arcsin(a\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\operatorname{asin}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/asin(a\*x)^(7/2),x)

[Out] int(x^4/asin(a\*x)^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asin}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/asin(a\*x)\*\*(7/2),x)

[Out] Integral(x\*\*4/asin(a\*x)\*\*(7/2), x)

$$3.114 \quad \int \frac{x^3}{\sin^{-1}(ax)^{7/2}} dx$$

Optimal. Leaf size=190

$$\frac{32\sqrt{2\pi} C\left(2\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{15a^4} - \frac{16\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^4} - \frac{4x^2}{5a^2 \sin^{-1}(ax)^{3/2}} + \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{2x^3\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} - \frac{16}{5a^2}$$

```
[Out] -4/5*x^2/a^2/arcsin(a*x)^(3/2)+16/15*x^4/arcsin(a*x)^(3/2)-16/15*FresnelC(2*arcsin(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4+32/15*FresnelC(2*2^(1/2)/Pi^(1/2)*arcsin(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-2/5*x^3*(-a^2*x^2+1)^(1/2)/a/arcsin(a*x)^(5/2)-16/5*x*(-a^2*x^2+1)^(1/2)/a^3/arcsin(a*x)^(1/2)+128/15*x^3*(-a^2*x^2+1)^(1/2)/a/arcsin(a*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 12, number of rules / integrand size = 0.417, Rules used = {4633, 4719, 4631, 3304, 3352}

$$\frac{32\sqrt{2\pi} \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{15a^4} - \frac{16\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^4} + \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{2x^3\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} - \frac{16}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSin[a*x]^(7/2), x]
```

```
[Out] (-2*x^3*Sqrt[1 - a^2*x^2])/(5*a*ArcSin[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcSin[a*x]^(3/2)) + (16*x^4)/(15*ArcSin[a*x]^(3/2)) - (16*x*Sqrt[1 - a^2*x^2])/(5*a^3*Sqrt[ArcSin[a*x]]) + (128*x^3*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[ArcSin[a*x]]) + (32*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcSin[a*x]]])/(15*a^4) - (16*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcSin[a*x]])/Sqrt[Pi]])/(15*a^4)
```

#### Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

#### Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

#### Rule 4631

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n*(x_)^m, x_Symbol] := Simp[(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), Sin[x]^(m - 1)*(m - (m + 1)*Sin[x]^2), x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

#### Rule 4633

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n*(x_)^m, x_Symbol] := Simp[(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSin[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcSin[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m,
```

0] && LtQ[n, -2]

### Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n)\*((f\_.)\*(x\_.))^m)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] & & EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sin^{-1}(ax)^{7/2}} dx &= -\frac{2x^3\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} + \frac{6\int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx}{5a} - \frac{1}{5}(8a)\int \frac{x^4}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sin^{-1}(ax)^{3/2}} + \frac{16x^4}{15\sin^{-1}(ax)^{3/2}} - \frac{64}{15}\int \frac{x^3}{\sin^{-1}(ax)^{3/2}} dx + \frac{8\int \frac{x}{\sin^{-1}(ax)}}{5a^2} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sin^{-1}(ax)^{3/2}} + \frac{16x^4}{15\sin^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sin^{-1}(ax)^{3/2}} + \frac{16x^4}{15\sin^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sin^{-1}(ax)^{3/2}} + \frac{16x^4}{15\sin^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} \\ &= -\frac{2x^3\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sin^{-1}(ax)^{3/2}} + \frac{16x^4}{15\sin^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\sin^{-1}(ax)}} + \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} \end{aligned}$$

**Mathematica [C]** time = 1.11, size = 272, normalized size = 1.43

$$-6\sin\left(2\sin^{-1}(ax)\right) + 3\sin\left(4\sin^{-1}(ax)\right) + 4\sin^{-1}(ax)\left(i e^{2i\sin^{-1}(ax)}(-4\sin^{-1}(ax) + i) - 4\sqrt{2}(-i\sin^{-1}(ax))^{3/2}\Gamma\left(\frac{3}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/ArcSin[a\*x]^(7/2), x]

[Out] (4\*ArcSin[a\*x]\*(I\*E^((2\*I)\*ArcSin[a\*x]))\*(I - 4\*ArcSin[a\*x]) - 4\*Sqrt[2]\*((-I)\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (-2\*I)\*ArcSin[a\*x]] + (-1 + (4\*I)\*ArcSin[a\*x] - 4\*Sqrt[2]\*E^((2\*I)\*ArcSin[a\*x]))\*(I\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (2\*I)\*ArcSin[a\*x]])/E^((2\*I)\*ArcSin[a\*x]) - 4\*ArcSin[a\*x]\*(I\*E^((4\*I)\*ArcSin[a\*x]))\*(I - 8\*ArcSin[a\*x]) - 16\*((-I)\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (-4\*I)\*ArcSin[a\*x]] + (-1 + (8\*I)\*ArcSin[a\*x] - 16\*E^((4\*I)\*ArcSin[a\*x]))\*(I\*ArcSin[a\*x])^(3/2)\*Gamma[1/2, (4\*I)\*ArcSin[a\*x]])/E^((4\*I)\*ArcSin[a\*x]) - 6\*Sin[2\*ArcSin[a\*x]] + 3\*Sin[4\*ArcSin[a\*x]]/(60\*a^4\*ArcSin[a\*x]^(5/2))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^(7/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [A] time = 0.09, size = 139, normalized size = 0.73

$$-128\sqrt{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{2}\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)\arcsin(ax)^{\frac{5}{2}}+64\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right)\arcsin(ax)^{\frac{5}{2}}+64\sin$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arcsin(a\*x)^(7/2),x)

[Out]  $-1/60/a^4*(-128*2^{(1/2)}*Pi^{(1/2)}*FresnelC(2*2^{(1/2)}/Pi^{(1/2)}*\arcsin(a*x)^{(1/2)})*\arcsin(a*x)^{(5/2)}+64*Pi^{(1/2)}*FresnelC(2*\arcsin(a*x)^{(1/2)}/Pi^{(1/2)})*\arcsin(a*x)^{(5/2)}+64*\sin(4*\arcsin(a*x))*\arcsin(a*x)^2-32*\sin(2*\arcsin(a*x))*\arcsin(a*x)^2-8*\arcsin(a*x)*\cos(4*\arcsin(a*x))+8*\arcsin(a*x)*\cos(2*\arcsin(a*x))-3*\sin(4*\arcsin(a*x))+6*\sin(2*\arcsin(a*x)))/\arcsin(a*x)^{(5/2)}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arcsin(a\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{asin}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/asin(a\*x)^(7/2),x)

[Out] int(x^3/asin(a\*x)^(7/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/asin(a\*x)\*\*(7/2),x)

[Out] Integral(x\*\*3/asin(a\*x)\*\*(7/2), x)

$$3.115 \quad \int \frac{x^2}{\sin^{-1}(ax)^{7/2}} dx$$

**Optimal.** Leaf size=191

$$\frac{2\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{15a^3} - \frac{6\sqrt{6\pi} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{5a^3} + \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\sin^{-1}(ax)}} - \frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sin^{-1}(ax)^{3/2}} - \frac{1}{15a^3}$$

[Out]  $-8/15*x/a^2/\arcsin(ax)^{(3/2)}+4/5*x^3/\arcsin(ax)^{(3/2)}+2/15*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(ax)^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/a^3-6/5*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*\arcsin(ax)^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/a^3-2/5*x^2*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(ax)^{(5/2)}-16/15*(-a^2*x^2+1)^{(1/2)}/a^3/\arcsin(ax)^{(1/2)}+24/5*x^2*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(ax)^{(1/2)}$

**Rubi [A]** time = 0.35, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {4633, 4719, 4631, 3305, 3351, 4621, 4723}

$$\frac{2\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{15a^3} - \frac{6\sqrt{6\pi} S\left(\sqrt{\frac{6}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{5a^3} + \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\sin^{-1}(ax)}} - \frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\sin^{-1}(ax)}} - \frac{1}{15a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcSin[a\*x]^(7/2), x]

[Out]  $(-2*x^2*\text{Sqrt}[1 - a^2*x^2])/(5*a*\text{ArcSin}[a*x]^{(5/2)}) - (8*x)/(15*a^2*\text{ArcSin}[a*x]^{(3/2)}) + (4*x^3)/(5*\text{ArcSin}[a*x]^{(3/2)}) - (16*\text{Sqrt}[1 - a^2*x^2])/(15*a^3*\text{Sqrt}[\text{ArcSin}[a*x]]) + (24*x^2*\text{Sqrt}[1 - a^2*x^2])/(5*a*\text{Sqrt}[\text{ArcSin}[a*x]]) + (2*\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(15*a^3) - (6*\text{Sqrt}[6*\text{Pi}]*\text{FresnelS}[\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[\text{ArcSin}[a*x]]])/(5*a^3)$

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_), x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rule 4633

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Simp[(x^m*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSin[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*ArcSin[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

#### Rule 4719

```
Int((((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^m*(a + b*ArcSin[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

#### Rule 4723

```
Int(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x]^(2*p + 1), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])
```

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sin^{-1}(ax)^{7/2}} dx &= -\frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} + \frac{4\int \frac{x}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx}{5a} - \frac{1}{5}(6a) \int \frac{x^3}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx \\ &= -\frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^3}{5\sin^{-1}(ax)^{3/2}} - \frac{12}{5} \int \frac{x^2}{\sin^{-1}(ax)^{3/2}} dx + \frac{8\int \frac{x}{\sin^{-1}(ax)^{3/2}} dx}{15} \\ &= -\frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^3}{5\sin^{-1}(ax)^{3/2}} - \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\sin^{-1}(ax)}} + \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\sin^{-1}(ax)}} \\ &= -\frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^3}{5\sin^{-1}(ax)^{3/2}} - \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\sin^{-1}(ax)}} + \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\sin^{-1}(ax)}} \\ &= -\frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^3}{5\sin^{-1}(ax)^{3/2}} - \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\sin^{-1}(ax)}} + \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\sin^{-1}(ax)}} \\ &= -\frac{2x^2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{4x^3}{5\sin^{-1}(ax)^{3/2}} - \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\sin^{-1}(ax)}} + \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\sin^{-1}(ax)}} \end{aligned}$$

**Mathematica [C]** time = 0.34, size = 280, normalized size = 1.47

$$3e^{3i\sin^{-1}(ax)} \left( -12\sin^{-1}(ax)^2 + 2i\sin^{-1}(ax) + 1 \right) + e^{i\sin^{-1}(ax)} \left( 4\sin^{-1}(ax)^2 - 2i\sin^{-1}(ax) - 3 \right) - 4\sqrt{-i\sin^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcSin[a*x]^(7/2), x]
```

```
[Out] (3*E^((3*I)*ArcSin[a*x])*(1 + (2*I)*ArcSin[a*x] - 12*ArcSin[a*x]^2) + E^(I*ArcSin[a*x])*(-3 - (2*I)*ArcSin[a*x] + 4*ArcSin[a*x]^2) - 4*Sqrt[(-I)*ArcSi
```

```
n[a*x]]*ArcSin[a*x]^2*Gamma[1/2, (-I)*ArcSin[a*x]] + (-3 + (2*I)*ArcSin[a*x]
] + 4*ArcSin[a*x]^2 + 4*E^(I*ArcSin[a*x])*(I*ArcSin[a*x])^(5/2)*Gamma[1/2,
I*ArcSin[a*x]])/E^(I*ArcSin[a*x]) + 36*Sqrt[3]*Sqrt[(-I)*ArcSin[a*x]]*ArcSi
n[a*x]^2*Gamma[1/2, (-3*I)*ArcSin[a*x]] - (3*(-1 + (2*I)*ArcSin[a*x] + 12*A
rcSin[a*x]^2 + 12*Sqrt[3]*E^((3*I)*ArcSin[a*x])*(I*ArcSin[a*x])^(5/2)*Gamma
[1/2, (3*I)*ArcSin[a*x]]))/E^((3*I)*ArcSin[a*x]))/(60*a^3*ArcSin[a*x]^(5/2)
)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsin(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\arcsin(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsin(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsin(a*x)^(7/2), x)
```

**maple** [A] time = 0.08, size = 154, normalized size = 0.81

---


$$36\sqrt{2} \sqrt{\pi} \sqrt{3} S\left(\frac{\sqrt{2} \sqrt{3} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{5/2} - 4\sqrt{2} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{5/2} - 4 \arcsin(ax)^2 \sqrt{-a}$$


---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsin(a*x)^(7/2),x)
```

```
[Out] -1/30/a^3*(36*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*ar
csin(a*x)^(1/2))*arcsin(a*x)^(5/2)-4*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(
1/2)*arcsin(a*x)^(1/2))*arcsin(a*x)^(5/2)-4*arcsin(a*x)^2*(-a^2*x^2+1)^(1/2
)+36*arcsin(a*x)^2*cos(3*arcsin(a*x))-2*a*x*arcsin(a*x)+6*arcsin(a*x)*sin(3
*arcsin(a*x))+3*(-a^2*x^2+1)^(1/2)-3*cos(3*arcsin(a*x)))/arcsin(a*x)^(5/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsin(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{asin}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(x^2/asin(a*x)^(7/2),x)
```

```
[Out] int(x^2/asin(a*x)^(7/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^2}{\operatorname{asin}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asin(a*x)**(7/2),x)
```

```
[Out] Integral(x**2/asin(a*x)**(7/2), x)
```

$$3.116 \quad \int \frac{x}{\sin^{-1}(ax)^{7/2}} dx$$

**Optimal.** Leaf size=119

$$-\frac{32\sqrt{\pi} C\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^2} + \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{2x\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{8x^2}{15\sin^{-1}(ax)^{3/2}}$$

[Out]  $-4/15/a^2/\arcsin(a*x)^{(3/2)}+8/15*x^2/\arcsin(a*x)^{(3/2)}-32/15*\text{FresnelC}(2*\arcsin(a*x)^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/a^2-2/5*x*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(5/2)}+32/15*x*(-a^2*x^2+1)^{(1/2)}/a/\arcsin(a*x)^{(1/2)}$

**Rubi [A]** time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4633, 4719, 4631, 3304, 3352, 4641}

$$-\frac{32\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{\sin^{-1}(ax)}}{\sqrt{\pi}}\right)}{15a^2} + \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{2x\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{8x^2}{15\sin^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/ArcSin[a\*x]^(7/2), x]

[Out]  $(-2*x*\text{Sqrt}[1 - a^2*x^2])/(5*a*\text{ArcSin}[a*x]^{(5/2)}) - 4/(15*a^2*\text{ArcSin}[a*x]^{(3/2)}) + (8*x^2)/(15*\text{ArcSin}[a*x]^{(3/2)}) + (32*x*\text{Sqrt}[1 - a^2*x^2])/(15*a*\text{Sqrt}[\text{ArcSin}[a*x]]) - (32*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[\text{ArcSin}[a*x]])/\text{Sqrt}[\text{Pi}]])/(15*a^2)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^2)], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/ (f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^(n\_)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^(n\_)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

#### Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

### Rule 4719

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_)^ (m_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[((f*x)^m*(a + b*ArcSin[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] & EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{x}{\sin^{-1}(ax)^{7/2}} dx &= -\frac{2x\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} + \frac{2\int \frac{1}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx}{5a} - \frac{1}{5}(4a) \int \frac{x^2}{\sqrt{1-a^2x^2}\sin^{-1}(ax)^{5/2}} dx \\ &= -\frac{2x\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{8x^2}{15\sin^{-1}(ax)^{3/2}} - \frac{16}{15} \int \frac{x}{\sin^{-1}(ax)^{3/2}} dx \\ &= -\frac{2x\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{8x^2}{15\sin^{-1}(ax)^{3/2}} + \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{32}{15} \text{Subst}\left(\int \frac{1}{\sqrt{1-u^2}} du, \sin^{-1}(ax)\right) \\ &= -\frac{2x\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{8x^2}{15\sin^{-1}(ax)^{3/2}} + \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{64}{15} \text{Subst}\left(\int \frac{1}{\sqrt{1-u^2}} du, \sin^{-1}(ax)\right) \\ &= -\frac{2x\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sin^{-1}(ax)^{3/2}} + \frac{8x^2}{15\sin^{-1}(ax)^{3/2}} + \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{32\sqrt{\pi} C\left(\frac{2\sqrt{1-a^2x^2}}{\sin^{-1}(ax)}\right)}{15a^2} \end{aligned}$$

**Mathematica [C]** time = 0.37, size = 146, normalized size = 1.23

$$\frac{3 \sin\left(2 \sin^{-1}(ax)\right) + \sin^{-1}(ax) \left(2e^{2i \sin^{-1}(ax)} \left(1 + 4i \sin^{-1}(ax)\right) + 8\sqrt{2} \left(-i \sin^{-1}(ax)\right)^{3/2} \Gamma\left(\frac{1}{2}, -2i \sin^{-1}(ax)\right)\right)}{15a^2 \sin^{-1}(ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcSin[a*x]^(7/2), x]
```

```
[Out] -1/15*(ArcSin[a*x]*(2*E^((2*I)*ArcSin[a*x]))*(1 + (4*I)*ArcSin[a*x]) + 8*Sqrt[2]*((-I)*ArcSin[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcSin[a*x]] + (2 - (8*I)*ArcSin[a*x] + 8*Sqrt[2]*E^((2*I)*ArcSin[a*x]))*(I*ArcSin[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcSin[a*x]])/E^((2*I)*ArcSin[a*x]) + 3*Sin[2*ArcSin[a*x]]/(a^2*ArcSin[a*x]^(5/2))
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsin(a*x)^(7/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\arcsin(ax)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^(7/2),x, algorithm="giac")

[Out] integrate(x/arcsin(a\*x)^(7/2), x)

**maple** [A] time = 0.06, size = 73, normalized size = 0.61

$$\frac{-32\sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) \arcsin(ax)^{\frac{5}{2}} + 16 \sin(2 \arcsin(ax)) \arcsin(ax)^2 - 4 \arcsin(ax) \cos(2 \arcsin(ax))}{15a^2 \arcsin(ax)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arcsin(a\*x)^(7/2),x)

[Out] 1/15/a^2\*(-32\*Pi^(1/2)\*FresnelC(2\*arcsin(a\*x)^(1/2)/Pi^(1/2))\*arcsin(a\*x)^(5/2)+16\*sin(2\*arcsin(a\*x))\*arcsin(a\*x)^2-4\*arcsin(a\*x)\*cos(2\*arcsin(a\*x))-3\*sin(2\*arcsin(a\*x)))/arcsin(a\*x)^(5/2)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arcsin(a\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asin}(ax)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/asin(a\*x)^(7/2),x)

[Out] int(x/asin(a\*x)^(7/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asin}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/asin(a\*x)\*\*(7/2),x)

[Out] Integral(x/asin(a\*x)\*\*(7/2), x)

$$3.117 \quad \int \frac{1}{\sin^{-1}(ax)^{7/2}} dx$$

**Optimal.** Leaf size=105

$$\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} + \frac{8\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{15a} + \frac{4x}{15\sin^{-1}(ax)^{3/2}}$$

[Out] 4/15\*x/arcsin(a\*x)^(3/2)+8/15\*FresnelS(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))\*2^(1/2)\*Pi^(1/2)/a-2/5\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)^(5/2)+8/15\*(-a^2\*x^2+1)^(1/2)/a/arcsin(a\*x)^(1/2)

**Rubi [A]** time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4621, 4719, 4723, 3305, 3351}

$$\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} - \frac{2\sqrt{1-a^2x^2}}{5a\sin^{-1}(ax)^{5/2}} + \frac{8\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}}\sqrt{\sin^{-1}(ax)}\right)}{15a} + \frac{4x}{15\sin^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^(-7/2), x]

[Out] (-2\*Sqrt[1 - a^2\*x^2])/(5\*a\*ArcSin[a\*x]^(5/2)) + (4\*x)/(15\*ArcSin[a\*x]^(3/2)) + (8\*Sqrt[1 - a^2\*x^2])/(15\*a\*Sqrt[ArcSin[a\*x]]) + (8\*Sqrt[2\*Pi]\*FresnelS[Sqrt[2/Pi]\*Sqrt[ArcSin[a\*x]]])/(15\*a)

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^2)], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_), x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*((f\_.)\*(x\_)^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^(m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

#### Rule 4723

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer

Q[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sin^{-1}(ax)^{7/2}} dx &= -\frac{2\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} - \frac{1}{5}(2a) \int \frac{x}{\sqrt{1-a^2x^2} \sin^{-1}(ax)^{5/2}} dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} + \frac{4x}{15 \sin^{-1}(ax)^{3/2}} - \frac{4}{15} \int \frac{1}{\sin^{-1}(ax)^{3/2}} dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} + \frac{4x}{15 \sin^{-1}(ax)^{3/2}} + \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} + \frac{1}{15}(8a) \int \frac{x}{\sqrt{1-a^2x^2} \sqrt{\sin^{-1}(ax)}} dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} + \frac{4x}{15 \sin^{-1}(ax)^{3/2}} + \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} + \frac{8 \operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}} dx, x, \sin^{-1}(ax)\right)}{15a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} + \frac{4x}{15 \sin^{-1}(ax)^{3/2}} + \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} + \frac{16 \operatorname{Subst}\left(\int \sin(x^2) dx, x, \sqrt{\sin^{-1}(ax)}\right)}{15a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{5a \sin^{-1}(ax)^{5/2}} + \frac{4x}{15 \sin^{-1}(ax)^{3/2}} + \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\sin^{-1}(ax)}} + \frac{8\sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}} \sqrt{\sin^{-1}(ax)}\right)}{15a}
\end{aligned}$$

**Mathematica** [C] time = 0.24, size = 143, normalized size = 1.36

$$\frac{2e^{i \sin^{-1}(ax)} \left(4 \sin^{-1}(ax)^2 - 2i \sin^{-1}(ax) - 3\right) - 8\sqrt{-i \sin^{-1}(ax)} \sin^{-1}(ax)^2 \Gamma\left(\frac{1}{2}, -i \sin^{-1}(ax)\right) + e^{-i \sin^{-1}(ax)} \left(8 \sin^{-1}(ax)^2 - 2i \sin^{-1}(ax) - 3\right) + 8\sqrt{i \sin^{-1}(ax)} \sin^{-1}(ax)^2 \Gamma\left(\frac{1}{2}, i \sin^{-1}(ax)\right)}{30a \sin^{-1}(ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[ArcSin[a*x]^(-7/2), x]`

```
[Out] (2*E^(I*ArcSin[a*x])*(-3 - (2*I)*ArcSin[a*x] + 4*ArcSin[a*x]^2) - 8*Sqrt[(-I)*ArcSin[a*x]]*ArcSin[a*x]^2*Gamma[1/2, (-I)*ArcSin[a*x]] + (-6 + (4*I)*ArcSin[a*x] + 8*ArcSin[a*x]^2 + 8*E^(I*ArcSin[a*x])*(I*ArcSin[a*x])^(5/2)*Gamma[1/2, I*ArcSin[a*x]])/E^(I*ArcSin[a*x]))/(30*a*ArcSin[a*x]^(5/2))
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/arcsin(a*x)^(7/2), x, algorithm="fricas")`

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\arcsin(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/arcsin(a*x)^(7/2), x, algorithm="giac")``[Out] integrate(arcsin(a*x)^(-7/2), x)`

**maple** [A] time = 0.06, size = 110, normalized size = 1.05

$$\frac{\sqrt{2} \left( 8 \arcsin(ax)^3 \pi S\left(\frac{\sqrt{2} \sqrt{\arcsin(ax)}}{\sqrt{\pi}}\right) + 4 \arcsin(ax)^{\frac{5}{2}} \sqrt{2} \sqrt{\pi} \sqrt{-a^2x^2+1} + 2 \arcsin(ax)^{\frac{3}{2}} \sqrt{2} \sqrt{\pi} xa - 3 \right)}{15a\sqrt{\pi} \arcsin(ax)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arcsin(a\*x)^(7/2), x)

[Out] 1/15/a\*2^(1/2)/Pi^(1/2)/arcsin(a\*x)^3\*(8\*arcsin(a\*x)^3\*Pi\*FresnelS(2^(1/2)/Pi^(1/2)\*arcsin(a\*x)^(1/2))+4\*arcsin(a\*x)^(5/2)\*2^(1/2)\*Pi^(1/2)\*(-a^2\*x^2+1)^(1/2)+2\*arcsin(a\*x)^(3/2)\*2^(1/2)\*Pi^(1/2)\*x\*a-3\*2^(1/2)\*arcsin(a\*x)^(1/2)\*Pi^(1/2)\*(-a^2\*x^2+1)^(1/2))

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsin(a\*x)^(7/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asin}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/asin(a\*x)^(7/2), x)

[Out] int(1/asin(a\*x)^(7/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/asin(a\*x)\*\*(7/2), x)

[Out] Integral(asin(a\*x)\*\*(-7/2), x)

$$3.118 \quad \int \frac{1}{x \sin^{-1}(ax)^{7/2}} dx$$

**Optimal.** Leaf size=15

$$\text{Int}\left(\frac{1}{x \sin^{-1}(ax)^{7/2}}, x\right)$$

[Out] Unintegrable(1/x/arcsin(a\*x)^(7/2), x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x \sin^{-1}(ax)^{7/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*ArcSin[a\*x]^(7/2)), x]

[Out] Defer[Int][1/(x\*ArcSin[a\*x]^(7/2)), x]

Rubi steps

$$\int \frac{1}{x \sin^{-1}(ax)^{7/2}} dx = \int \frac{1}{x \sin^{-1}(ax)^{7/2}} dx$$

**Mathematica [A]** time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sin^{-1}(ax)^{7/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*ArcSin[a\*x]^(7/2)), x]

[Out] Integrate[1/(x\*ArcSin[a\*x]^(7/2)), x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(7/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(7/2), x, algorithm="giac")

[Out] integrate(1/(x\*arcsin(a\*x)^(7/2)), x)



**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x \arcsin(ax)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsin(a\*x)^(7/2),x)

[Out] int(1/x/arcsin(a\*x)^(7/2),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsin(a\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asin}(ax)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*asin(a\*x)^(7/2)),x)

[Out] int(1/(x\*asin(a\*x)^(7/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asin}^{\frac{7}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/asin(a\*x)\*\*(7/2),x)

[Out] Integral(1/(x\*asin(a\*x)\*\*(7/2)), x)

### 3.119 $\int (bx)^m \sin^{-1}(ax)^4 dx$

Optimal. Leaf size=65

$$\frac{\sin^{-1}(ax)^4 (bx)^{m+1}}{b(m+1)} - \frac{4a \operatorname{Int}\left(\frac{\sin^{-1}(ax)^3 (bx)^{m+1}}{\sqrt{1-a^2x^2}}, x\right)}{b(m+1)}$$

[Out]  $(b*x)^{(1+m)}*\arcsin(a*x)^4/b/(1+m)-4*a*\operatorname{Unintegrable}((b*x)^{(1+m)}*\arcsin(a*x)^3/(-a^2*x^2+1)^{(1/2)},x)/b/(1+m)$

**Rubi [A]** time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (bx)^m \sin^{-1}(ax)^4 dx$$

Verification is Not applicable to the result.

[In] `Int[(b*x)^m*ArcSin[a*x]^4,x]`

[Out]  $((b*x)^{(1+m)}*\operatorname{ArcSin}[a*x]^4)/(b*(1+m)) - (4*a*\operatorname{Defer}[\operatorname{Int}][((b*x)^{(1+m)}*\operatorname{ArcSin}[a*x]^3)/\operatorname{Sqrt}[1 - a^2*x^2], x])/b*(1+m)$

Rubi steps

$$\int (bx)^m \sin^{-1}(ax)^4 dx = \frac{(bx)^{1+m} \sin^{-1}(ax)^4}{b(1+m)} - \frac{(4a) \int \frac{(bx)^{1+m} \sin^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx}{b(1+m)}$$

**Mathematica [A]** time = 1.06, size = 0, normalized size = 0.00

$$\int (bx)^m \sin^{-1}(ax)^4 dx$$

Verification is Not applicable to the result.

[In] `Integrate[(b*x)^m*ArcSin[a*x]^4,x]`

[Out] `Integrate[(b*x)^m*ArcSin[a*x]^4,x]`

**fricas [A]** time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}((bx)^m \arcsin(ax)^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x)^4,x, algorithm="fricas")`

[Out] `integral((b*x)^m*arcsin(a*x)^4,x)`

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x)^4,x, algorithm="giac")`

[Out] `integrate((b*x)^m*arcsin(a*x)^4,x)`

**maple** [A] time = 1.17, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m\*arcsin(a\*x)^4,x)

[Out] int((b\*x)^m\*arcsin(a\*x)^4,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^4,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asin}(ax)^4 (bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^4\*(b\*x)^m,x)

[Out] int(asin(a\*x)^4\*(b\*x)^m, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \operatorname{asin}^4(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)\*\*m\*asin(a\*x)\*\*4,x)

[Out] Integral((b\*x)\*\*m\*asin(a\*x)\*\*4, x)

### 3.120 $\int (bx)^m \sin^{-1}(ax)^3 dx$

Optimal. Leaf size=65

$$\frac{\sin^{-1}(ax)^3 (bx)^{m+1}}{b(m+1)} - \frac{3a \operatorname{Int}\left(\frac{\sin^{-1}(ax)^2 (bx)^{m+1}}{\sqrt{1-a^2x^2}}, x\right)}{b(m+1)}$$

[Out]  $(b*x)^{(1+m)}*\arcsin(a*x)^3/b/(1+m)-3*a*\operatorname{Unintegrable}((b*x)^{(1+m)}*\arcsin(a*x)^2/(-a^2*x^2+1)^{(1/2)},x)/b/(1+m)$

**Rubi [A]** time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (bx)^m \sin^{-1}(ax)^3 dx$$

Verification is Not applicable to the result.

[In] `Int[(b*x)^m*ArcSin[a*x]^3,x]`

[Out]  $((b*x)^{(1+m)}*\operatorname{ArcSin}[a*x]^3)/(b*(1+m)) - (3*a*\operatorname{Defer}[\operatorname{Int}][((b*x)^{(1+m)}*\operatorname{ArcSin}[a*x]^2)/\operatorname{Sqrt}[1 - a^2*x^2], x])/b*(1+m)$

Rubi steps

$$\int (bx)^m \sin^{-1}(ax)^3 dx = \frac{(bx)^{1+m} \sin^{-1}(ax)^3}{b(1+m)} - \frac{(3a) \int \frac{(bx)^{1+m} \sin^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx}{b(1+m)}$$

**Mathematica [A]** time = 0.98, size = 0, normalized size = 0.00

$$\int (bx)^m \sin^{-1}(ax)^3 dx$$

Verification is Not applicable to the result.

[In] `Integrate[(b*x)^m*ArcSin[a*x]^3,x]`

[Out] `Integrate[(b*x)^m*ArcSin[a*x]^3,x]`

**fricas [A]** time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}((bx)^m \arcsin(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x)^3,x, algorithm="fricas")`

[Out] `integral((b*x)^m*arcsin(a*x)^3,x)`

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x)^3,x, algorithm="giac")`

[Out] `integrate((b*x)^m*arcsin(a*x)^3,x)`

**maple** [A] time = 0.81, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m\*arcsin(a\*x)^3,x)

[Out] int((b\*x)^m\*arcsin(a\*x)^3,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^3,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asin}(ax)^3 (bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^3\*(b\*x)^m,x)

[Out] int(asin(a\*x)^3\*(b\*x)^m, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \operatorname{asin}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)\*\*m\*asin(a\*x)\*\*3,x)

[Out] Integral((b\*x)\*\*m\*asin(a\*x)\*\*3, x)

### 3.121 $\int (bx)^m \sin^{-1}(ax)^2 dx$

**Optimal.** Leaf size=150

$$\frac{2a^2(bx)^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; a^2x^2\right)}{b^3(m+1)(m+2)(m+3)} - \frac{2a \sin^{-1}(ax)(bx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right)}{b^2(m+1)(m+2)} + \frac{\sin^{-1}(ax)^2(bx)^{m+1}}{b(m+1)}$$

[Out] (b\*x)^(1+m)\*arcsin(a\*x)^2/b/(1+m)-2\*a\*(b\*x)^(2+m)\*arcsin(a\*x)\*hypergeom([1/2, 1+1/2\*m], [2+1/2\*m], a^2\*x^2)/b^2/(1+m)/(2+m)+2\*a^2\*(b\*x)^(3+m)\*HypergeometricPFQ([1, 3/2+1/2\*m, 3/2+1/2\*m], [2+1/2\*m, 5/2+1/2\*m], a^2\*x^2)/b^3/(3+m)/(m^2+3\*m+2)

**Rubi [A]** time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {4627, 4711}

$$\frac{2a^2(bx)^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; a^2x^2\right)}{b^3(m+1)(m+2)(m+3)} - \frac{2a \sin^{-1}(ax)(bx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right)}{b^2(m+1)(m+2)} + \frac{\sin^{-1}(ax)^2(bx)^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(b\*x)^m\*ArcSin[a\*x]^2,x]

[Out] ((b\*x)^(1+m)\*ArcSin[a\*x]^2)/(b\*(1+m)) - (2\*a\*(b\*x)^(2+m)\*ArcSin[a\*x]\*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, a^2\*x^2])/(b^2\*(1+m)\*(2+m)) + (2\*a^2\*(b\*x)^(3+m)\*HypergeometricPFQ[{1, 3/2+m/2, 3/2+m/2}, {2+m/2, 5/2+m/2}, a^2\*x^2])/(b^3\*(1+m)\*(2+m)\*(3+m))

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4711

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*ArcSin[c\*x])\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/(Sqrt[d]\*f\*(m+1)), x] - Simp[(b\*c\*(f\*x)^(m+2)\*HypergeometricPFQ[{1, 1+m/2, 1+m/2}, {3/2+m/2, 2+m/2}, c^2\*x^2])/(Sqrt[d]\*f^2\*(m+1)\*(m+2)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int (bx)^m \sin^{-1}(ax)^2 dx &= \frac{(bx)^{1+m} \sin^{-1}(ax)^2}{b(1+m)} - \frac{(2a) \int \frac{(bx)^{1+m} \sin^{-1}(ax)}{\sqrt{1-a^2x^2}} dx}{b(1+m)} \\ &= \frac{(bx)^{1+m} \sin^{-1}(ax)^2}{b(1+m)} - \frac{2a(bx)^{2+m} \sin^{-1}(ax) {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{b^2(1+m)(2+m)} + \frac{2a^2(bx)^{3+m} {}_3F_2\left(1, \frac{3+m}{2}, \frac{3+m}{2}; \frac{3+m}{2} + 2, \frac{3+m}{2} + \frac{5}{2}; a^2x^2\right)}{b^3(1+m)(2+m)(3+m)} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 122, normalized size = 0.81

$$\frac{x(bx)^m \left(2a^2x^2 {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; a^2x^2\right) + (m+3) \sin^{-1}(ax) \left((m+2) \sin^{-1}(ax) - 2ax {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right)\right)\right)}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*x)^m\*ArcSin[a\*x]^2,x]

[Out] (x\*(b\*x)^m\*((3 + m)\*ArcSin[a\*x]\*((2 + m)\*ArcSin[a\*x] - 2\*a\*x\*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2\*x^2]) + 2\*a^2\*x^2\*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, a^2\*x^2]))/((1 + m)\*(2 + m)\*(3 + m))

**fricas** [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}((bx)^m \arcsin(ax)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^2,x, algorithm="fricas")

[Out] integral((b\*x)^m\*arcsin(a\*x)^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^2,x, algorithm="giac")

[Out] integrate((b\*x)^m\*arcsin(a\*x)^2, x)

**maple** [F] time = 0.77, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m\*arcsin(a\*x)^2,x)

[Out] int((b\*x)^m\*arcsin(a\*x)^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{asin}(ax)^2 (bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^2\*(b\*x)^m,x)

[Out] int(asin(a\*x)^2\*(b\*x)^m, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \text{asin}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*asin(a*x)**2,x)
```

```
[Out] Integral((b*x)**m*asin(a*x)**2, x)
```



### 3.122 $\int (bx)^m \sin^{-1}(ax) dx$

Optimal. Leaf size=69

$$\frac{\sin^{-1}(ax)(bx)^{m+1}}{b(m+1)} - \frac{a(bx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right)}{b^2(m+1)(m+2)}$$

[Out] (b\*x)^(1+m)\*arcsin(a\*x)/b/(1+m)-a\*(b\*x)^(2+m)\*hypergeom([1/2, 1+1/2\*m], [2+1/2\*m], a^2\*x^2)/b^2/(1+m)/(2+m)

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4627, 364}

$$\frac{\sin^{-1}(ax)(bx)^{m+1}}{b(m+1)} - \frac{a(bx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right)}{b^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(b\*x)^m\*ArcSin[a\*x], x]

[Out] ((b\*x)^(1+m)\*ArcSin[a\*x])/(b\*(1+m)) - (a\*(b\*x)^(2+m)\*Hypergeometric2F1[1[1/2, (2+m)/2, (4+m)/2, a^2\*x^2])/(b^2\*(1+m)\*(2+m))

Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b\*x^n)/a)])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (bx)^m \sin^{-1}(ax) dx &= \frac{(bx)^{1+m} \sin^{-1}(ax)}{b(1+m)} - \frac{a \int \frac{(bx)^{1+m}}{\sqrt{1-a^2x^2}} dx}{b(1+m)} \\ &= \frac{(bx)^{1+m} \sin^{-1}(ax)}{b(1+m)} - \frac{a(bx)^{2+m} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; a^2x^2\right)}{b^2(1+m)(2+m)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.81

$$\frac{x(bx)^m \left( ax {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2x^2\right) - (m+2) \sin^{-1}(ax) \right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*x)^m\*ArcSin[a\*x], x]

[Out]  $-\left(\frac{x(bx)^m(-((2+m)\text{ArcSin}[ax]) + a x \text{Hypergeometric2F1}[1/2, (2+m)/2, (4+m)/2, a^2 x^2])}{(1+m)(2+m)}\right)$

**fricas** [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}((bx)^m \arcsin(ax), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x),x, algorithm="fricas")`

[Out] `integral((b*x)^m*arcsin(a*x), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x),x, algorithm="giac")`

[Out] `integrate((b*x)^m*arcsin(a*x), x)`

**maple** [F] time = 0.80, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x)^m*arcsin(a*x),x)`

[Out] `int((b*x)^m*arcsin(a*x),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^m x x^m \arctan(ax, \sqrt{ax+1} \sqrt{-ax+1}) + \frac{(ab^m m + ab^m) \int \frac{\sqrt{-ax+1} x^m}{\sqrt{ax+1}(ax-1)} dx}{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x),x, algorithm="maxima")`

[Out] `(b^m*x*x^m*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1)) + (a*b^m*m + a*b^m)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m/((a^2*m + a^2)*x^2 - m - 1), x))/(m + 1)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{asin}(ax) (bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)*(b*x)^m,x)`

[Out] `int(asin(a*x)*(b*x)^m, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \text{asin}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)**m*asin(a*x),x)`

[Out] `Integral((b*x)**m*asin(a*x), x)`

$$3.123 \quad \int \frac{(bx)^m}{\sin^{-1}(ax)} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{(bx)^m}{\sin^{-1}(ax)}, x\right)$$

[Out] Unintegrable((b\*x)^m/arcsin(a\*x), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(bx)^m}{\sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^m/ArcSin[a\*x], x]

[Out] Defer[Int] [(b\*x)^m/ArcSin[a\*x], x]

Rubi steps

$$\int \frac{(bx)^m}{\sin^{-1}(ax)} dx = \int \frac{(bx)^m}{\sin^{-1}(ax)} dx$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^m/ArcSin[a\*x], x]

[Out] Integrate[(b\*x)^m/ArcSin[a\*x], x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx)^m}{\arcsin(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x), x, algorithm="fricas")

[Out] integral((b\*x)^m/arcsin(a\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x), x, algorithm="giac")

[Out] integrate((b\*x)^m/arcsin(a\*x), x)

**maple** [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/arcsin(a\*x),x)

[Out] int((b\*x)^m/arcsin(a\*x),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x),x, algorithm="maxima")

[Out] integrate((b\*x)^m/arcsin(a\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{(bx)^m}{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/asin(a\*x),x)

[Out] int((b\*x)^m/asin(a\*x), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)\*\*m/asin(a\*x),x)

[Out] Integral((b\*x)\*\*m/asin(a\*x), x)

$$3.124 \quad \int \frac{(bx)^m}{\sin^{-1}(ax)^2} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{(bx)^m}{\sin^{-1}(ax)^2}, x\right)$$

[Out] Unintegrable((b\*x)^m/arcsin(a\*x)^2, x)

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(bx)^m}{\sin^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^m/ArcSin[a\*x]^2, x]

[Out] Defer[Int] [(b\*x)^m/ArcSin[a\*x]^2, x]

Rubi steps

$$\int \frac{(bx)^m}{\sin^{-1}(ax)^2} dx = \int \frac{(bx)^m}{\sin^{-1}(ax)^2} dx$$

**Mathematica** [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\sin^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^m/ArcSin[a\*x]^2, x]

[Out] Integrate[(b\*x)^m/ArcSin[a\*x]^2, x]

**fricas** [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx)^m}{\arcsin(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x)^2, x, algorithm="fricas")

[Out] integral((b\*x)^m/arcsin(a\*x)^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\arcsin(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x)^2, x, algorithm="giac")

[Out] integrate((b\*x)^m/arcsin(a\*x)^2, x)

**maple** [A] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\arcsin(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/arcsin(a\*x)^2,x)

[Out] int((b\*x)^m/arcsin(a\*x)^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x)^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{(bx)^m}{\operatorname{asin}(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/asin(a\*x)^2,x)

[Out] int((b\*x)^m/asin(a\*x)^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\operatorname{asin}^2(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)\*\*m/asin(a\*x)\*\*2,x)

[Out] Integral((b\*x)\*\*m/asin(a\*x)\*\*2, x)

### 3.125 $\int (bx)^m \sin^{-1}(ax)^{3/2} dx$

**Optimal.** Leaf size=17

$$\text{Int}(\sin^{-1}(ax)^{3/2}(bx)^m, x)$$

[Out] Unintegrable((b\*x)^m\*arcsin(a\*x)^(3/2), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (bx)^m \sin^{-1}(ax)^{3/2} dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^m\*ArcSin[a\*x]^(3/2), x]

[Out] Defer[Int] [(b\*x)^m\*ArcSin[a\*x]^(3/2), x]

Rubi steps

$$\int (bx)^m \sin^{-1}(ax)^{3/2} dx = \int (bx)^m \sin^{-1}(ax)^{3/2} dx$$

**Mathematica [A]** time = 2.69, size = 0, normalized size = 0.00

$$\int (bx)^m \sin^{-1}(ax)^{3/2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^m\*ArcSin[a\*x]^(3/2), x]

[Out] Integrate[(b\*x)^m\*ArcSin[a\*x]^(3/2), x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate((b\*x)^m\*arcsin(a\*x)^(3/2), x)

**maple [A]** time = 0.11, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x)^m*arcsin(a*x)^(3/2),x)`

[Out] `int((b*x)^m*arcsin(a*x)^(3/2),x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \operatorname{asin}(ax)^{3/2} (bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^(3/2)*(b*x)^m,x)`

[Out] `int(asin(a*x)^(3/2)*(b*x)^m, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \operatorname{asin}^{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)**m*asin(a*x)**(3/2),x)`

[Out] `Integral((b*x)**m*asin(a*x)**(3/2), x)`



### 3.126 $\int (bx)^m \sqrt{\sin^{-1}(ax)} dx$

**Optimal.** Leaf size=17

$$\text{Int}\left(\sqrt{\sin^{-1}(ax)}(bx)^m, x\right)$$

[Out] Unintegrable((b\*x)^m\*arcsin(a\*x)^(1/2), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (bx)^m \sqrt{\sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^m\*Sqrt[ArcSin[a\*x]], x]

[Out] Defer[Int][(b\*x)^m\*Sqrt[ArcSin[a\*x]], x]

Rubi steps

$$\int (bx)^m \sqrt{\sin^{-1}(ax)} dx = \int (bx)^m \sqrt{\sin^{-1}(ax)} dx$$

**Mathematica [A]** time = 2.97, size = 0, normalized size = 0.00

$$\int (bx)^m \sqrt{\sin^{-1}(ax)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^m\*Sqrt[ArcSin[a\*x]], x]

[Out] Integrate[(b\*x)^m\*Sqrt[ArcSin[a\*x]], x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] integrate((b\*x)^m\*sqrt(arcsin(a\*x)), x)

**maple [A]** time = 0.09, size = 0, normalized size = 0.00

$$\int (bx)^m \sqrt{\arcsin(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x)^m*arcsin(a*x)^(1/2),x)`

[Out] `int((b*x)^m*arcsin(a*x)^(1/2),x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{\operatorname{asin}(ax)} (bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^(1/2)*(b*x)^m,x)`

[Out] `int(asin(a*x)^(1/2)*(b*x)^m, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \sqrt{\operatorname{asin}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)**m*asin(a*x)**(1/2),x)`

[Out] `Integral((b*x)**m*sqrt(asin(a*x)), x)`

$$3.127 \quad \int \frac{(bx)^m}{\sqrt{\sin^{-1}(ax)}} dx$$

**Optimal.** Leaf size=17

$$\text{Int}\left(\frac{(bx)^m}{\sqrt{\sin^{-1}(ax)}}, x\right)$$

[Out] Unintegrable((b\*x)^m/arcsin(a\*x)^(1/2), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(bx)^m}{\sqrt{\sin^{-1}(ax)}} dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^m/Sqrt[ArcSin[a\*x]], x]

[Out] Defer[Int] [(b\*x)^m/Sqrt[ArcSin[a\*x]], x]

Rubi steps

$$\int \frac{(bx)^m}{\sqrt{\sin^{-1}(ax)}} dx = \int \frac{(bx)^m}{\sqrt{\sin^{-1}(ax)}} dx$$

**Mathematica [A]** time = 2.25, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\sqrt{\sin^{-1}(ax)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^m/Sqrt[ArcSin[a\*x]], x]

[Out] Integrate[(b\*x)^m/Sqrt[ArcSin[a\*x]], x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x)^(1/2), x, algorithm="giac")

[Out] integrate((b\*x)^m/sqrt(arcsin(a\*x)), x)

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/arcsin(a\*x)^(1/2),x)

[Out] int((b\*x)^m/arcsin(a\*x)^(1/2),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(bx)^m}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/asin(a\*x)^(1/2),x)

[Out] int((b\*x)^m/asin(a\*x)^(1/2),x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\sqrt{\arcsin(ax)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)\*\*m/asin(a\*x)\*\*(1/2),x)

[Out] Integral((b\*x)\*\*m/sqrt(asin(a\*x)), x)

$$3.128 \quad \int \frac{(bx)^m}{\sin^{-1}(ax)^{3/2}} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{(bx)^m}{\sin^{-1}(ax)^{3/2}}, x\right)$$

[Out] Unintegrable((b\*x)^m/arcSin(a\*x)^(3/2), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(bx)^m}{\sin^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^m/ArcSin[a\*x]^(3/2), x]

[Out] Defer[Int] [(b\*x)^m/ArcSin[a\*x]^(3/2), x]

Rubi steps

$$\int \frac{(bx)^m}{\sin^{-1}(ax)^{3/2}} dx = \int \frac{(bx)^m}{\sin^{-1}(ax)^{3/2}} dx$$

Mathematica [A] time = 2.28, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\sin^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^m/ArcSin[a\*x]^(3/2), x]

[Out] Integrate[(b\*x)^m/ArcSin[a\*x]^(3/2), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcSin(a\*x)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcSin(a\*x)^(3/2), x, algorithm="giac")

[Out] integrate((b\*x)^m/arcSin(a\*x)^(3/2), x)

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\arcsin(ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/arcsin(a\*x)^(3/2),x)

[Out] int((b\*x)^m/arcsin(a\*x)^(3/2),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m/arcsin(a\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(bx)^m}{\operatorname{asin}(ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x)^m/asin(a\*x)^(3/2),x)

[Out] int((b\*x)^m/asin(a\*x)^(3/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^m}{\operatorname{asin}^{\frac{3}{2}}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)\*\*m/asin(a\*x)\*\*(3/2),x)

[Out] Integral((b\*x)\*\*m/asin(a\*x)\*\*(3/2), x)

### 3.129 $\int (bx)^m \sin^{-1}(ax)^n dx$

**Optimal.** Leaf size=15

$$\text{Int}((bx)^m \sin^{-1}(ax)^n, x)$$

[Out] Unintegrable((b\*x)^m\*arcsin(a\*x)^n,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (bx)^m \sin^{-1}(ax)^n dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^m\*ArcSin[a\*x]^n,x]

[Out] Defer[Int][(b\*x)^m\*ArcSin[a\*x]^n, x]

Rubi steps

$$\int (bx)^m \sin^{-1}(ax)^n dx = \int (bx)^m \sin^{-1}(ax)^n dx$$

**Mathematica [A]** time = 0.85, size = 0, normalized size = 0.00

$$\int (bx)^m \sin^{-1}(ax)^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^m\*ArcSin[a\*x]^n,x]

[Out] Integrate[(b\*x)^m\*ArcSin[a\*x]^n, x]

**fricas [A]** time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}((bx)^m \arcsin(ax)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^n,x, algorithm="fricas")

[Out] integral((b\*x)^m\*arcsin(a\*x)^n, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^m\*arcsin(a\*x)^n,x, algorithm="giac")

[Out] integrate((b\*x)^m\*arcsin(a\*x)^n, x)

**maple [A]** time = 0.98, size = 0, normalized size = 0.00

$$\int (bx)^m \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x)^m*arcsin(a*x)^n,x)`

[Out] `int((b*x)^m*arcsin(a*x)^n,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^m*arcsin(a*x)^n,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \operatorname{asin}(ax)^n (bx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^n*(b*x)^m,x)`

[Out] `int(asin(a*x)^n*(b*x)^m, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \operatorname{asin}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)**m*asin(a*x)**n,x)`

[Out] `Integral((b*x)**m*asin(a*x)**n, x)`



### 3.130 $\int x^3 \sin^{-1}(ax)^n dx$

**Optimal.** Leaf size=167

$$\frac{2^{-n-4} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \Gamma(n+1, -2i \sin^{-1}(ax))}{a^4} + \frac{2^{-2(n+3)} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \Gamma(n+1, -4i \sin^{-1}(ax))}{a^4}$$

```
[Out] -2^(-4-n)*arcsin(a*x)^n*GAMMA(1+n,-2*I*arcsin(a*x))/a^4/((-I*arcsin(a*x))^n)
-2^(-4-n)*arcsin(a*x)^n*GAMMA(1+n,2*I*arcsin(a*x))/a^4/((I*arcsin(a*x))^n)
+arcsin(a*x)^n*GAMMA(1+n,-4*I*arcsin(a*x))/(2^(6+2*n))/a^4/((-I*arcsin(a*x)
)^n)+arcsin(a*x)^n*GAMMA(1+n,4*I*arcsin(a*x))/(2^(6+2*n))/a^4/((I*arcsin(a*x)
)^n)
```

**Rubi [A]** time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4635, 4406, 3308, 2181}

$$\frac{2^{-n-4} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \Gamma(n+1, -2i \sin^{-1}(ax))}{a^4} + \frac{2^{-2(n+3)} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \Gamma(n+1, -4i \sin^{-1}(ax))}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSin[a*x]^n,x]
```

```
[Out] -((2^(-4 - n)*ArcSin[a*x]^n*Gamma[1 + n, (-2*I)*ArcSin[a*x]])/(a^4*((-I)*ArcSin[a*x]^n)) - (2^(-4 - n)*ArcSin[a*x]^n*Gamma[1 + n, (2*I)*ArcSin[a*x]])/(a^4*(I*ArcSin[a*x]^n)) + (ArcSin[a*x]^n*Gamma[1 + n, (-4*I)*ArcSin[a*x]])/(2^(2*(3 + n))*a^4*((-I)*ArcSin[a*x]^n)) + (ArcSin[a*x]^n*Gamma[1 + n, (4*I)*ArcSin[a*x]])/(2^(2*(3 + n))*a^4*(I*ArcSin[a*x]^n))
```

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x]))/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])/d)*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

#### Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

#### Rule 4635

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*(x_)^m, x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

#### Rubi steps

$$\begin{aligned}
\int x^3 \sin^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \cos(x) \sin^3(x) dx, x, \sin^{-1}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{4}x^n \sin(2x) - \frac{1}{8}x^n \sin(4x)\right) dx, x, \sin^{-1}(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int x^n \sin(4x) dx, x, \sin^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int x^n \sin(2x) dx, x, \sin^{-1}(ax)\right)}{4a^4} \\
&= -\frac{i \text{Subst}\left(\int e^{-4ix} x^n dx, x, \sin^{-1}(ax)\right)}{16a^4} + \frac{i \text{Subst}\left(\int e^{4ix} x^n dx, x, \sin^{-1}(ax)\right)}{16a^4} + \frac{i \text{Subst}\left(\int e^{-2ix} x^n dx, x, \sin^{-1}(ax)\right)}{8a^4} \\
&= -\frac{2^{-4-n} (-i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(1+n, -2i \sin^{-1}(ax))}{a^4} - \frac{2^{-4-n} (i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(1+n, 2i \sin^{-1}(ax))}{a^4}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 132, normalized size = 0.79

$$\frac{4^{-n-3} \sin^{-1}(ax)^n (\sin^{-1}(ax)^2)^{-n} \left( -2^{n+2} (-i \sin^{-1}(ax))^n \Gamma(n+1, 2i \sin^{-1}(ax)) + (-i \sin^{-1}(ax))^n \Gamma(n+1, 4i \sin^{-1}(ax)) \right)}{a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcSin[a\*x]^n,x]

[Out] (4^(-3 - n)\*ArcSin[a\*x]^n\*(-(2^(2 + n)\*(I\*ArcSin[a\*x])^n\*Gamma[1 + n, (-2\*I)\*ArcSin[a\*x]]) - 2^(2 + n)\*((-I)\*ArcSin[a\*x])^n\*Gamma[1 + n, (2\*I)\*ArcSin[a\*x]] + (I\*ArcSin[a\*x])^n\*Gamma[1 + n, (-4\*I)\*ArcSin[a\*x]] + ((-I)\*ArcSin[a\*x])^n\*Gamma[1 + n, (4\*I)\*ArcSin[a\*x]]))/(a^4\*(ArcSin[a\*x]^2)^n)

**fricas [F]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}(x^3 \arcsin(ax)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^n,x, algorithm="fricas")

[Out] integral(x^3\*arcsin(a\*x)^n, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^n,x, algorithm="giac")

[Out] integrate(x^3\*arcsin(a\*x)^n, x)

**maple [F]** time = 0.26, size = 0, normalized size = 0.00

$$\int x^3 \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arcsin(a\*x)^n,x)

[Out] int(x^3\*arcsin(a\*x)^n,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arcsin(a\*x)^n,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asin}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*asin(a\*x)^n,x)

[Out] int(x^3\*asin(a\*x)^n, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asin}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*asin(a\*x)\*\*n,x)

[Out] Integral(x\*\*3\*asin(a\*x)\*\*n, x)

### 3.131 $\int x^2 \sin^{-1}(ax)^n dx$

**Optimal.** Leaf size=171

$$\frac{i \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \Gamma(n+1, -i \sin^{-1}(ax))}{8a^3} + \frac{i 3^{-n-1} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \Gamma(n+1, -3i \sin^{-1}(ax))}{8a^3}$$

[Out]  $-1/8 * I * \arcsin(ax)^n * \text{GAMMA}(1+n, -I * \arcsin(ax)) / a^3 / ((-I * \arcsin(ax))^n) + 1/8 * I * \arcsin(ax)^n * \text{GAMMA}(1+n, I * \arcsin(ax)) / a^3 / ((I * \arcsin(ax))^n) + 1/8 * I * 3^{(-1-n)} * \arcsin(ax)^n * \text{GAMMA}(1+n, -3 * I * \arcsin(ax)) / a^3 / ((-I * \arcsin(ax))^n) - 1/8 * I * 3^{(-1-n)} * \arcsin(ax)^n * \text{GAMMA}(1+n, 3 * I * \arcsin(ax)) / a^3 / ((I * \arcsin(ax))^n)$

**Rubi [A]** time = 0.17, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4635, 4406, 3307, 2181}

$$\frac{i \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \text{Gamma}(n+1, -i \sin^{-1}(ax))}{8a^3} + \frac{i 3^{-n-1} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \text{Gamma}(n+1, -3i \sin^{-1}(ax))}{8a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcSin[a\*x]^n,x]

[Out]  $((-I/8) * \text{ArcSin}[a*x]^n * \text{Gamma}[1+n, (-I) * \text{ArcSin}[a*x]]) / (a^3 * ((-I) * \text{ArcSin}[a*x])^n) + ((I/8) * \text{ArcSin}[a*x]^n * \text{Gamma}[1+n, I * \text{ArcSin}[a*x]]) / (a^3 * (I * \text{ArcSin}[a*x])^n) + ((I/8) * 3^{(-1-n)} * \text{ArcSin}[a*x]^n * \text{Gamma}[1+n, (-3*I) * \text{ArcSin}[a*x]]) / (a^3 * ((-I) * \text{ArcSin}[a*x])^n) - ((I/8) * 3^{(-1-n)} * \text{ArcSin}[a*x]^n * \text{Gamma}[1+n, (3*I) * \text{ArcSin}[a*x]]) / (a^3 * (I * \text{ArcSin}[a*x])^n)$

#### Rule 2181

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, -(f\*g\*Log[F])/d]\*(c + d\*x)]/(d\*(-(f\*g\*Log[F])/d)^(IntPart[m] + 1)\*(-(f\*g\*Log[F])\*(c + d\*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3307

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m \* E^(I\*k\*Pi) \* E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 4406

Int[Cos[(a\_) + (b\_)\*(x\_)]^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_)\*Sin[(a\_) + (b\_)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*(x\_)^m\_, x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n \* Sin[x]^m \* Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
\int x^2 \sin^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \cos(x) \sin^2(x) dx, x, \sin^{-1}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{4}x^n \cos(x) - \frac{1}{4}x^n \cos(3x)\right) dx, x, \sin^{-1}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int x^n \cos(x) dx, x, \sin^{-1}(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int x^n \cos(3x) dx, x, \sin^{-1}(ax)\right)}{4a^3} \\
&= \frac{\text{Subst}\left(\int e^{-ix} x^n dx, x, \sin^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int e^{ix} x^n dx, x, \sin^{-1}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int e^{-3ix} x^n dx, x, \sin^{-1}(ax)\right)}{8a^3} \\
&= -\frac{i\left(-i \sin^{-1}(ax)\right)^{-n} \sin^{-1}(ax)^n \Gamma(1+n, -i \sin^{-1}(ax))}{8a^3} + \frac{i\left(i \sin^{-1}(ax)\right)^{-n} \sin^{-1}(ax)^n \Gamma(1+n, i \sin^{-1}(ax))}{8a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 137, normalized size = 0.80

$$\frac{i3^{-n-1} \sin^{-1}(ax)^n \left(\sin^{-1}(ax)^2\right)^{-n} \left(3^{n+1} \left(-i \sin^{-1}(ax)\right)^n \Gamma(n+1, i \sin^{-1}(ax)) - \left(-i \sin^{-1}(ax)\right)^n \Gamma(n+1, 3i \sin^{-1}(ax))\right)}{8a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcSin[a\*x]^n,x]

[Out] ((I/8)\*3^(-1 - n)\*ArcSin[a\*x]^n\*(-(3^(1 + n)\*(I\*ArcSin[a\*x])^n\*Gamma[1 + n, (-I)\*ArcSin[a\*x]]) + 3^(1 + n)\*((-I)\*ArcSin[a\*x])^n\*Gamma[1 + n, I\*ArcSin[a\*x]] + (I\*ArcSin[a\*x])^n\*Gamma[1 + n, (-3\*I)\*ArcSin[a\*x]] - ((-I)\*ArcSin[a\*x])^n\*Gamma[1 + n, (3\*I)\*ArcSin[a\*x]]))/(a^3\*(ArcSin[a\*x]^2)^n)

**fricas [F]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2 \arcsin(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^n,x, algorithm="fricas")

[Out] integral(x^2\*arcsin(a\*x)^n, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arcsin(a\*x)^n,x, algorithm="giac")

[Out] integrate(x^2\*arcsin(a\*x)^n, x)

**maple [F]** time = 0.23, size = 0, normalized size = 0.00

$$\int x^2 \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arcsin(a\*x)^n,x)

[Out] int(x^2\*arcsin(a\*x)^n,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>2</sup>\*arcsin(a\*x)<sup>n</sup>,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asin}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>2</sup>\*asin(a\*x)<sup>n</sup>,x)

[Out] int(x<sup>2</sup>\*asin(a\*x)<sup>n</sup>, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asin}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*asin(a\*x)\*\*n,x)

[Out] Integral(x\*\*2\*asin(a\*x)\*\*n, x)

### 3.132 $\int x \sin^{-1}(ax)^n dx$

**Optimal.** Leaf size=85

$$\frac{2^{-n-3} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \Gamma(n+1, -2i \sin^{-1}(ax))}{a^2} - \frac{2^{-n-3} (i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(n+1, 2i \sin^{-1}(ax))}{a^2}$$

[Out]  $-2^{-(3+n)} \arcsin(ax)^n \text{GAMMA}(1+n, -2i \arcsin(ax)) / a^2 / ((-i \arcsin(ax))^n) - 2^{-(3+n)} \arcsin(ax)^n \text{GAMMA}(1+n, 2i \arcsin(ax)) / a^2 / ((i \arcsin(ax))^n)$

**Rubi [A]** time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4635, 4406, 12, 3308, 2181}

$$\frac{2^{-n-3} \sin^{-1}(ax)^n (-i \sin^{-1}(ax))^{-n} \text{Gamma}(n+1, -2i \sin^{-1}(ax))}{a^2} - \frac{2^{-n-3} (i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \text{Gamma}(n+1, 2i \sin^{-1}(ax))}{a^2}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcSin[a\*x]^n,x]

[Out]  $-((2^{-(3-n)} \text{ArcSin}[a*x]^n \text{Gamma}[1+n, (-2*I) \text{ArcSin}[a*x]]) / (a^2 * ((-I) \text{ArcSin}[a*x])^n)) - (2^{-(3-n)} \text{ArcSin}[a*x]^n \text{Gamma}[1+n, (2*I) \text{ArcSin}[a*x]]) / (a^2 * (I \text{ArcSin}[a*x])^n)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2181

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3308

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

#### Rule 4406

Int[Cos[(a\_) + (b\_)\*(x\_)]^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_)\*Sin[(a\_) + (b\_)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*(x\_)^m\_, x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n \* Sin[x]^m \* Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
\int x \sin^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \cos(x) \sin(x) dx, x, \sin^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{1}{2} x^n \sin(2x) dx, x, \sin^{-1}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int x^n \sin(2x) dx, x, \sin^{-1}(ax)\right)}{2a^2} \\
&= \frac{i \text{Subst}\left(\int e^{-2ix} x^n dx, x, \sin^{-1}(ax)\right)}{4a^2} - \frac{i \text{Subst}\left(\int e^{2ix} x^n dx, x, \sin^{-1}(ax)\right)}{4a^2} \\
&= -\frac{2^{-3-n} (-i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(1+n, -2i \sin^{-1}(ax))}{a^2} - \frac{2^{-3-n} (i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(1+n, 2i \sin^{-1}(ax))}{a^2}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 75, normalized size = 0.88

$$\frac{2^{-n-3} \sin^{-1}(ax)^n (\sin^{-1}(ax)^2)^{-n} \left( (-i \sin^{-1}(ax))^n \Gamma(n+1, 2i \sin^{-1}(ax)) + (i \sin^{-1}(ax))^n \Gamma(n+1, -2i \sin^{-1}(ax)) \right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcSin[a\*x]^n,x]

[Out] -((2^(-3-n)\*ArcSin[a\*x]^n\*((I\*ArcSin[a\*x])^n\*Gamma[1+n,(-2\*I)\*ArcSin[a\*x]])+((-I)\*ArcSin[a\*x])^n\*Gamma[1+n,(2\*I)\*ArcSin[a\*x]]))/(a^2\*(ArcSin[a\*x]^2)^n)

**fricas** [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}(x \arcsin(ax)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^n,x, algorithm="fricas")

[Out] integral(x\*arcsin(a\*x)^n, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arcsin(a\*x)^n,x, algorithm="giac")

[Out] integrate(x\*arcsin(a\*x)^n, x)

**maple** [C] time = 0.17, size = 138, normalized size = 1.62

$$\frac{\sqrt{\pi} \left( \frac{2 \arcsin(ax)^{1+n} \sin(2 \arcsin(ax))}{\sqrt{\pi} (2+n)} - \frac{2^{\frac{1}{2}-n} \sqrt{\arcsin(ax)} \text{LommelS1}\left(n+\frac{3}{2}, \frac{3}{2}, 2 \arcsin(ax)\right) \sin(2 \arcsin(ax))}{\sqrt{\pi} (2+n)} - \frac{3 \cdot 2^{-\frac{3}{2}-n} \left(\frac{4}{3} + \frac{2n}{3}\right) (2 \arcsin(ax) \cos(2 \arcsin(ax)))^n}{\sqrt{\pi} (2+n)} \right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arcsin(a\*x)^n,x)



```
[Out] 1/4*Pi^(1/2)/a^2*(2/Pi^(1/2)/(2+n)*arcsin(a*x)^(1+n)*sin(2*arcsin(a*x))-2^(1/2-n)/Pi^(1/2)/(2+n)*arcsin(a*x)^(1/2)*LommelS1(n+3/2,3/2,2*arcsin(a*x))*sin(2*arcsin(a*x))-3*2^(-3/2-n)/Pi^(1/2)/(2+n)/arcsin(a*x)^(1/2)*(4/3+2/3*n)*(2*arcsin(a*x)*cos(2*arcsin(a*x))-sin(2*arcsin(a*x)))*LommelS1(n+1/2,1/2,2*arcsin(a*x))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
      expt: undefined: 0 to a negative exponent.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{asin}(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asin(a*x)^n,x)
```

```
[Out] int(x*asin(a*x)^n, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asin}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(a*x)**n,x)
```

```
[Out] Integral(x*asin(a*x)**n, x)
```

### 3.133 $\int \sin^{-1}(ax)^n dx$

**Optimal.** Leaf size=79

$$\frac{i(i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(n+1, i \sin^{-1}(ax))}{2a} - \frac{i(-i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(n+1, -i \sin^{-1}(ax))}{2a}$$

[Out]  $-1/2 * I * \arcsin(a*x)^n * \text{GAMMA}(1+n, -I * \arcsin(a*x)) / a / ((-I * \arcsin(a*x))^n) + 1/2 * I * \arcsin(a*x)^n * \text{GAMMA}(1+n, I * \arcsin(a*x)) / a / ((I * \arcsin(a*x))^n)$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4623, 3307, 2181}

$$\frac{i(i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \text{Gamma}(n+1, i \sin^{-1}(ax))}{2a} - \frac{i(-i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \text{Gamma}(n+1, -i \sin^{-1}(ax))}{2a}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[a\*x]^n, x]

[Out]  $((-I/2) * \text{ArcSin}[a*x]^n * \text{Gamma}[1+n, (-I) * \text{ArcSin}[a*x]]) / (a * ((-I) * \text{ArcSin}[a*x])^n) + ((I/2) * \text{ArcSin}[a*x]^n * \text{Gamma}[1+n, I * \text{ArcSin}[a*x]]) / (a * (I * \text{ArcSin}[a*x])^n)$

#### Rule 2181

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3307

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m \* E^(I\*k\*Pi) \* E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 4623

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_), x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n \* Cos[a/b - x/b], x], x, a + b \* ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rubi steps

$$\begin{aligned} \int \sin^{-1}(ax)^n dx &= \frac{\text{Subst}\left(\int x^n \cos(x) dx, x, \sin^{-1}(ax)\right)}{a} \\ &= \frac{\text{Subst}\left(\int e^{-ix} x^n dx, x, \sin^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int e^{ix} x^n dx, x, \sin^{-1}(ax)\right)}{2a} \\ &= -\frac{i(-i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(1+n, -i \sin^{-1}(ax))}{2a} + \frac{i(i \sin^{-1}(ax))^{-n} \sin^{-1}(ax)^n \Gamma(1+n, i \sin^{-1}(ax))}{2a} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 73, normalized size = 0.92

$$\frac{i \sin^{-1}(ax)^n (\sin^{-1}(ax)^2)^{-n} \left( (-i \sin^{-1}(ax))^n \Gamma(n+1, i \sin^{-1}(ax)) - (i \sin^{-1}(ax))^n \Gamma(n+1, -i \sin^{-1}(ax)) \right)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[a\*x]^n,x]

[Out] ((I/2)\*ArcSin[a\*x]^n\*(-((I\*ArcSin[a\*x])^n\*Gamma[1+n,(-I)\*ArcSin[a\*x]])+((-I)\*ArcSin[a\*x])^n\*Gamma[1+n,I\*ArcSin[a\*x]]))/(a\*(ArcSin[a\*x]^2)^n)

**fricas [F]** time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}(\arcsin(ax)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n,x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^n, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n,x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^n, x)

**maple [C]** time = 0.11, size = 240, normalized size = 3.04

$$\frac{2^n \sqrt{\pi} \left( \frac{2^{-1-n} \arcsin(ax)^n (6+2n)ax}{\sqrt{\pi} (1+n)(3+n)} + \frac{\arcsin(ax)^n 2^{-n} \sqrt{-a^2x^2+1} (a^2x^2 \arcsin(ax) - \arcsin(ax) + ax \sqrt{-a^2x^2+1})}{\sqrt{\pi} (1+n)(a^2x^2-1)} + \frac{2^{-n} \sqrt{\arcsin(ax)} n \text{Lommel}}{\sqrt{\pi}} \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^n,x)

[Out] 2^n\*Pi^(1/2)/a\*(2^(-1-n)/Pi^(1/2)/(1+n)\*arcsin(a\*x)^n\*(6+2\*n)/(3+n)\*a\*x+1/Pi^(1/2)/(1+n)\*arcsin(a\*x)^n\*2^(-n)\*(-a^2\*x^2+1)^(1/2)/(a^2\*x^2-1)\*(a^2\*x^2\*arcsin(a\*x)-arcsin(a\*x)+a\*x\*(-a^2\*x^2+1)^(1/2))+2^(-n)/Pi^(1/2)/(1+n)\*arcsin(a\*x)^(1/2)\*n\*LommelS1(n+1/2,3/2,arcsin(a\*x))\*a\*x-2^(-n)/Pi^(1/2)/(1+n)/arcsin(a\*x)^(1/2)\*(-a^2\*x^2+1)^(1/2)/(a^2\*x^2-1)\*(a^2\*x^2\*arcsin(a\*x)-arcsin(a\*x)+a\*x\*(-a^2\*x^2+1)^(1/2))\*LommelS1(n+3/2,1/2,arcsin(a\*x)))

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(a*x)^n,x)
```

```
[Out] int(asin(a*x)^n, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \operatorname{asin}^n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(a*x)**n,x)
```

```
[Out] Integral(asin(a*x)**n, x)
```

$$3.134 \quad \int \frac{\sin^{-1}(ax)^n}{x} dx$$

**Optimal.** Leaf size=13

$$\text{Int}\left(\frac{\sin^{-1}(ax)^n}{x}, x\right)$$

[Out] Unintegrable(arcsin(a\*x)^n/x, x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sin^{-1}(ax)^n}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcSin[a\*x]^n/x, x]

[Out] Defer[Int][ArcSin[a\*x]^n/x, x]

Rubi steps

$$\int \frac{\sin^{-1}(ax)^n}{x} dx = \int \frac{\sin^{-1}(ax)^n}{x} dx$$

**Mathematica [A]** time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(ax)^n}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcSin[a\*x]^n/x, x]

[Out] Integrate[ArcSin[a\*x]^n/x, x]

**fricas [A]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arcsin(ax)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/x, x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^n/x, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/x, x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^n/x, x)

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^n/x,x)

[Out] int(arcsin(a\*x)^n/x,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/x,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\asin(ax)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^n/x,x)

[Out] int(asin(a\*x)^n/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\asin^n(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*n/x,x)

[Out] Integral(asin(a\*x)\*\*n/x, x)

$$3.135 \quad \int \frac{\sin^{-1}(ax)^n}{x^2} dx$$

**Optimal.** Leaf size=13

$$\text{Int}\left(\frac{\sin^{-1}(ax)^n}{x^2}, x\right)$$

[Out] Unintegrable(arcsin(a\*x)^n/x^2, x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sin^{-1}(ax)^n}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[ArcSin[a\*x]^n/x^2, x]

[Out] Defer[Int][ArcSin[a\*x]^n/x^2, x]

Rubi steps

$$\int \frac{\sin^{-1}(ax)^n}{x^2} dx = \int \frac{\sin^{-1}(ax)^n}{x^2} dx$$

**Mathematica [A]** time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(ax)^n}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcSin[a\*x]^n/x^2, x]

[Out] Integrate[ArcSin[a\*x]^n/x^2, x]

**fricas [A]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\arcsin(ax)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/x^2, x, algorithm="fricas")

[Out] integral(arcsin(a\*x)^n/x^2, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/x^2, x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^n/x^2, x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^n/x^2,x)

[Out] int(arcsin(a\*x)^n/x^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/x^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\operatorname{asin}(ax)^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^n/x^2,x)

[Out] int(asin(a\*x)^n/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^n(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*n/x\*\*2,x)

[Out] Integral(asin(a\*x)\*\*n/x\*\*2, x)



### 3.136 $\int (bx)^{3/2} \sin^{-1}(ax)^n dx$

**Optimal.** Leaf size=17

$$\text{Int}((bx)^{3/2} \sin^{-1}(ax)^n, x)$$

[Out] Unintegrable((b\*x)^(3/2)\*arcsin(a\*x)^n,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (bx)^{3/2} \sin^{-1}(ax)^n dx$$

Verification is Not applicable to the result.

[In] Int[(b\*x)^(3/2)\*ArcSin[a\*x]^n,x]

[Out] Defer[Int][(b\*x)^(3/2)\*ArcSin[a\*x]^n, x]

Rubi steps

$$\int (bx)^{3/2} \sin^{-1}(ax)^n dx = \int (bx)^{3/2} \sin^{-1}(ax)^n dx$$

**Mathematica [A]** time = 3.58, size = 0, normalized size = 0.00

$$\int (bx)^{3/2} \sin^{-1}(ax)^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(b\*x)^(3/2)\*ArcSin[a\*x]^n,x]

[Out] Integrate[(b\*x)^(3/2)\*ArcSin[a\*x]^n, x]

**fricas [A]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx} bx \arcsin(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^(3/2)\*arcsin(a\*x)^n,x, algorithm="fricas")

[Out] integral(sqrt(b\*x)\*b\*x\*arcsin(a\*x)^n, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^{\frac{3}{2}} \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^(3/2)\*arcsin(a\*x)^n,x, algorithm="giac")

[Out] integrate((b\*x)^(3/2)\*arcsin(a\*x)^n, x)

**maple [A]** time = 0.13, size = 0, normalized size = 0.00

$$\int (bx)^{\frac{3}{2}} \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x)^(3/2)*arcsin(a*x)^n,x)`

[Out] `int((b*x)^(3/2)*arcsin(a*x)^n,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^(3/2)*arcsin(a*x)^n,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \operatorname{asin}(ax)^n (bx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^n*(b*x)^(3/2),x)`

[Out] `int(asin(a*x)^n*(b*x)^(3/2), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)**(3/2)*asin(a*x)**n,x)`

[Out] Timed out

### 3.137 $\int \sqrt{bx} \sin^{-1}(ax)^n dx$

Optimal. Leaf size=17

$$\text{Int}\left(\sqrt{bx} \sin^{-1}(ax)^n, x\right)$$

[Out] Unintegrable((b\*x)^(1/2)\*arcsin(a\*x)^n, x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \sqrt{bx} \sin^{-1}(ax)^n dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[b\*x]\*ArcSin[a\*x]^n, x]

[Out] Defer[Int][Sqrt[b\*x]\*ArcSin[a\*x]^n, x]

Rubi steps

$$\int \sqrt{bx} \sin^{-1}(ax)^n dx = \int \sqrt{bx} \sin^{-1}(ax)^n dx$$

Mathematica [A] time = 4.28, size = 0, normalized size = 0.00

$$\int \sqrt{bx} \sin^{-1}(ax)^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[b\*x]\*ArcSin[a\*x]^n, x]

[Out] Integrate[Sqrt[b\*x]\*ArcSin[a\*x]^n, x]

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx} \arcsin(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^(1/2)\*arcsin(a\*x)^n, x, algorithm="fricas")

[Out] integral(sqrt(b\*x)\*arcsin(a\*x)^n, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx} \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x)^(1/2)\*arcsin(a\*x)^n, x, algorithm="giac")

[Out] integrate(sqrt(b\*x)\*arcsin(a\*x)^n, x)

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{bx} \arcsin(ax)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x)^(1/2)*arcsin(a*x)^n,x)`

[Out] `int((b*x)^(1/2)*arcsin(a*x)^n,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)^(1/2)*arcsin(a*x)^n,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \operatorname{asin}(ax)^n \sqrt{bx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(a*x)^n*(b*x)^(1/2),x)`

[Out] `int(asin(a*x)^n*(b*x)^(1/2), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx} \operatorname{asin}^n(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x)**(1/2)*asin(a*x)**n,x)`

[Out] `Integral(sqrt(b*x)*asin(a*x)**n, x)`

$$3.138 \quad \int \frac{\sin^{-1}(ax)^n}{\sqrt{bx}} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\sin^{-1}(ax)^n}{\sqrt{bx}}, x\right)$$

[Out] Unintegrable(arcsin(a\*x)^n/(b\*x)^(1/2), x)

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sin^{-1}(ax)^n}{\sqrt{bx}} dx$$

Verification is Not applicable to the result.

[In] Int[ArcSin[a\*x]^n/Sqrt[b\*x], x]

[Out] Defer[Int][ArcSin[a\*x]^n/Sqrt[b\*x], x]

Rubi steps

$$\int \frac{\sin^{-1}(ax)^n}{\sqrt{bx}} dx = \int \frac{\sin^{-1}(ax)^n}{\sqrt{bx}} dx$$

**Mathematica** [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(ax)^n}{\sqrt{bx}} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcSin[a\*x]^n/Sqrt[b\*x], x]

[Out] Integrate[ArcSin[a\*x]^n/Sqrt[b\*x], x]

**fricas** [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx} \arcsin(ax)^n}{bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/(b\*x)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b\*x)\*arcsin(a\*x)^n/(b\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{\sqrt{bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/(b\*x)^(1/2), x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^n/sqrt(b\*x), x)

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{\sqrt{bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^n/(b\*x)^(1/2),x)

[Out] int(arcsin(a\*x)^n/(b\*x)^(1/2),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/(b\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{asin}(ax)^n}{\sqrt{bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^n/(b\*x)^(1/2),x)

[Out] int(asin(a\*x)^n/(b\*x)^(1/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^n(ax)}{\sqrt{bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*n/(b\*x)\*\*(1/2),x)

[Out] Integral(asin(a\*x)\*\*n/sqrt(b\*x), x)

$$3.139 \quad \int \frac{\sin^{-1}(ax)^n}{(bx)^{3/2}} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\sin^{-1}(ax)^n}{(bx)^{3/2}}, x\right)$$

[Out] Unintegrable(arcsin(a\*x)^n/(b\*x)^(3/2), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sin^{-1}(ax)^n}{(bx)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[ArcSin[a\*x]^n/(b\*x)^(3/2), x]

[Out] Defer[Int][ArcSin[a\*x]^n/(b\*x)^(3/2), x]

Rubi steps

$$\int \frac{\sin^{-1}(ax)^n}{(bx)^{3/2}} dx = \int \frac{\sin^{-1}(ax)^n}{(bx)^{3/2}} dx$$

Mathematica [A] time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{\sin^{-1}(ax)^n}{(bx)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcSin[a\*x]^n/(b\*x)^(3/2), x]

[Out] Integrate[ArcSin[a\*x]^n/(b\*x)^(3/2), x]

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx} \arcsin(ax)^n}{b^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/(b\*x)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(b\*x)\*arcsin(a\*x)^n/(b^2\*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{(bx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/(b\*x)^(3/2), x, algorithm="giac")

[Out] integrate(arcsin(a\*x)^n/(b\*x)^(3/2), x)

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(ax)^n}{(bx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(a\*x)^n/(b\*x)^(3/2), x)

[Out] int(arcsin(a\*x)^n/(b\*x)^(3/2), x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(a\*x)^n/(b\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{asin}(ax)^n}{(bx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(a\*x)^n/(b\*x)^(3/2), x)

[Out] int(asin(a\*x)^n/(b\*x)^(3/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^n(ax)}{(bx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(a\*x)\*\*n/(b\*x)\*\*(3/2), x)

[Out] Integral(asin(a\*x)\*\*n/(b\*x)\*\*(3/2), x)



### 3.140 $\int x^3 (a + b \sin^{-1}(cx)) dx$

**Optimal.** Leaf size=76

$$\frac{1}{4}x^4 (a + b \sin^{-1}(cx)) - \frac{3b \sin^{-1}(cx)}{32c^4} + \frac{bx^3 \sqrt{1-c^2x^2}}{16c} + \frac{3bx \sqrt{1-c^2x^2}}{32c^3}$$

[Out]  $-3/32*b*\arcsin(c*x)/c^4+1/4*x^4*(a+b*\arcsin(c*x))+3/32*b*x*(-c^2*x^2+1)^(1/2)/c^3+1/16*b*x^3*(-c^2*x^2+1)^(1/2)/c$

**Rubi [A]** time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4627, 321, 216}

$$\frac{1}{4}x^4 (a + b \sin^{-1}(cx)) + \frac{bx^3 \sqrt{1-c^2x^2}}{16c} + \frac{3bx \sqrt{1-c^2x^2}}{32c^3} - \frac{3b \sin^{-1}(cx)}{32c^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcSin[c\*x]),x]

[Out]  $(3*b*x*\text{Sqrt}[1 - c^2*x^2])/(32*c^3) + (b*x^3*\text{Sqrt}[1 - c^2*x^2])/(16*c) - (3*b*\text{ArcSin}[c*x])/(32*c^4) + (x^4*(a + b*\text{ArcSin}[c*x]))/4$

Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int x^3 (a + b \sin^{-1}(cx)) dx &= \frac{1}{4}x^4 (a + b \sin^{-1}(cx)) - \frac{1}{4}(bc) \int \frac{x^4}{\sqrt{1-c^2x^2}} dx \\ &= \frac{bx^3 \sqrt{1-c^2x^2}}{16c} + \frac{1}{4}x^4 (a + b \sin^{-1}(cx)) - \frac{(3b) \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{16c} \\ &= \frac{3bx \sqrt{1-c^2x^2}}{32c^3} + \frac{bx^3 \sqrt{1-c^2x^2}}{16c} + \frac{1}{4}x^4 (a + b \sin^{-1}(cx)) - \frac{(3b) \int \frac{1}{\sqrt{1-c^2x^2}} dx}{32c^3} \\ &= \frac{3bx \sqrt{1-c^2x^2}}{32c^3} + \frac{bx^3 \sqrt{1-c^2x^2}}{16c} - \frac{3b \sin^{-1}(cx)}{32c^4} + \frac{1}{4}x^4 (a + b \sin^{-1}(cx)) \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 81, normalized size = 1.07

$$\frac{ax^4}{4} - \frac{3b \sin^{-1}(cx)}{32c^4} + \frac{bx^3 \sqrt{1-c^2x^2}}{16c} + \frac{3bx \sqrt{1-c^2x^2}}{32c^3} + \frac{1}{4} bx^4 \sin^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcSin[c\*x]),x]

[Out] (a\*x^4)/4 + (3\*b\*x\*Sqrt[1 - c^2\*x^2])/(32\*c^3) + (b\*x^3\*Sqrt[1 - c^2\*x^2])/(16\*c) - (3\*b\*ArcSin[c\*x])/(32\*c^4) + (b\*x^4\*ArcSin[c\*x])/4

**fricas** [A] time = 0.43, size = 61, normalized size = 0.80

$$\frac{8ac^4x^4 + (8bc^4x^4 - 3b) \arcsin(cx) + (2bc^3x^3 + 3bcx) \sqrt{-c^2x^2 + 1}}{32c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsin(c\*x)),x, algorithm="fricas")

[Out] 1/32\*(8\*a\*c^4\*x^4 + (8\*b\*c^4\*x^4 - 3\*b)\*arcsin(c\*x) + (2\*b\*c^3\*x^3 + 3\*b\*c\*x)\*sqrt(-c^2\*x^2 + 1))/c^4

**giac** [A] time = 0.30, size = 95, normalized size = 1.25

$$\frac{1}{4} ax^4 - \frac{(-c^2x^2 + 1)^{\frac{3}{2}} bx}{16c^3} + \frac{(c^2x^2 - 1)^2 b \arcsin(cx)}{4c^4} + \frac{5 \sqrt{-c^2x^2 + 1} bx}{32c^3} + \frac{(c^2x^2 - 1) b \arcsin(cx)}{2c^4} + \frac{5 b \arcsin(cx)}{32c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsin(c\*x)),x, algorithm="giac")

[Out] 1/4\*a\*x^4 - 1/16\*(-c^2\*x^2 + 1)^(3/2)\*b\*x/c^3 + 1/4\*(c^2\*x^2 - 1)^2\*b\*arcsin(c\*x)/c^4 + 5/32\*sqrt(-c^2\*x^2 + 1)\*b\*x/c^3 + 1/2\*(c^2\*x^2 - 1)\*b\*arcsin(c\*x)/c^4 + 5/32\*b\*arcsin(c\*x)/c^4

**maple** [A] time = 0.01, size = 72, normalized size = 0.95

$$\frac{\frac{c^4 x^4 a}{4} + b \left( \frac{c^4 x^4 \arcsin(cx)}{4} + \frac{c^3 x^3 \sqrt{-c^2 x^2 + 1}}{16} + \frac{3cx \sqrt{-c^2 x^2 + 1}}{32} - \frac{3 \arcsin(cx)}{32} \right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arcsin(c\*x)),x)

[Out] 1/c^4\*(1/4\*c^4\*x^4\*a+b\*(1/4\*c^4\*x^4\*arcsin(c\*x)+1/16\*c^3\*x^3\*(-c^2\*x^2+1)^(1/2)+3/32\*c\*x\*(-c^2\*x^2+1)^(1/2)-3/32\*arcsin(c\*x)))

**maxima** [A] time = 0.45, size = 70, normalized size = 0.92

$$\frac{1}{4} ax^4 + \frac{1}{32} \left( 8x^4 \arcsin(cx) + \left( \frac{2 \sqrt{-c^2x^2 + 1} x^3}{c^2} + \frac{3 \sqrt{-c^2x^2 + 1} x}{c^4} - \frac{3 \arcsin(cx)}{c^5} \right) c \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] 1/4\*a\*x^4 + 1/32\*(8\*x^4\*arcsin(c\*x) + (2\*sqrt(-c^2\*x^2 + 1)\*x^3/c^2 + 3\*sqrt(-c^2\*x^2 + 1)\*x/c^4 - 3\*arcsin(c\*x)/c^5)\*c)\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (a + b \operatorname{asin}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b*asin(c*x)),x)`

[Out] `int(x^3*(a + b*asin(c*x)), x)`

**sympy** [A] time = 1.16, size = 80, normalized size = 1.05

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{asin}(cx)}{4} + \frac{bx^3 \sqrt{-c^2x^2+1}}{16c} + \frac{3bx \sqrt{-c^2x^2+1}}{32c^3} - \frac{3b \operatorname{asin}(cx)}{32c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*asin(c*x)),x)`

[Out] `Piecewise((a*x**4/4 + b*x**4*asin(c*x)/4 + b*x**3*sqrt(-c**2*x**2 + 1)/(16*c) + 3*b*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 3*b*asin(c*x)/(32*c**4), Ne(c, 0)), (a*x**4/4, True))`

### 3.141 $\int x^2 (a + b \sin^{-1}(cx)) dx$

**Optimal.** Leaf size=60

$$\frac{1}{3}x^3(a + b \sin^{-1}(cx)) - \frac{b(1 - c^2x^2)^{3/2}}{9c^3} + \frac{b\sqrt{1 - c^2x^2}}{3c^3}$$

[Out]  $-1/9*b*(-c^2*x^2+1)^{(3/2)}/c^3+1/3*x^3*(a+b*\arcsin(c*x))+1/3*b*(-c^2*x^2+1)^{(1/2)}/c^3$

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4627, 266, 43}

$$\frac{1}{3}x^3(a + b \sin^{-1}(cx)) - \frac{b(1 - c^2x^2)^{3/2}}{9c^3} + \frac{b\sqrt{1 - c^2x^2}}{3c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcSin[c\*x]),x]

[Out] (b\*Sqrt[1 - c^2\*x^2])/(3\*c^3) - (b\*(1 - c^2\*x^2)^(3/2))/(9\*c^3) + (x^3\*(a + b\*ArcSin[c\*x]))/3

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \sin^{-1}(cx)) dx &= \frac{1}{3}x^3(a + b \sin^{-1}(cx)) - \frac{1}{3}(bc) \int \frac{x^3}{\sqrt{1 - c^2x^2}} dx \\ &= \frac{1}{3}x^3(a + b \sin^{-1}(cx)) - \frac{1}{6}(bc) \text{Subst} \left( \int \frac{x}{\sqrt{1 - c^2x}} dx, x, x^2 \right) \\ &= \frac{1}{3}x^3(a + b \sin^{-1}(cx)) - \frac{1}{6}(bc) \text{Subst} \left( \int \left( \frac{1}{c^2\sqrt{1 - c^2x}} - \frac{\sqrt{1 - c^2x}}{c^2} \right) dx, x, x^2 \right) \\ &= \frac{b\sqrt{1 - c^2x^2}}{3c^3} - \frac{b(1 - c^2x^2)^{3/2}}{9c^3} + \frac{1}{3}x^3(a + b \sin^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 49, normalized size = 0.82

$$\frac{1}{9} \left( 3ax^3 + \frac{b\sqrt{1-c^2x^2}(c^2x^2+2)}{c^3} + 3bx^3 \sin^{-1}(cx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcSin[c\*x]),x]

[Out] (3\*a\*x^3 + (b\*Sqrt[1 - c^2\*x^2]\*(2 + c^2\*x^2))/c^3 + 3\*b\*x^3\*ArcSin[c\*x])/9

**fricas [A]** time = 0.47, size = 53, normalized size = 0.88

$$\frac{3bc^3x^3 \arcsin(cx) + 3ac^3x^3 + (bc^2x^2 + 2b)\sqrt{-c^2x^2 + 1}}{9c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x)),x, algorithm="fricas")

[Out] 1/9\*(3\*b\*c^3\*x^3\*arcsin(c\*x) + 3\*a\*c^3\*x^3 + (b\*c^2\*x^2 + 2\*b)\*sqrt(-c^2\*x^2 + 1))/c^3

**giac [A]** time = 0.35, size = 74, normalized size = 1.23

$$\frac{1}{3}ax^3 + \frac{(c^2x^2 - 1)bx \arcsin(cx)}{3c^2} + \frac{bx \arcsin(cx)}{3c^2} - \frac{(-c^2x^2 + 1)^{\frac{3}{2}}b}{9c^3} + \frac{\sqrt{-c^2x^2 + 1}b}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x)),x, algorithm="giac")

[Out] 1/3\*a\*x^3 + 1/3\*(c^2\*x^2 - 1)\*b\*x\*arcsin(c\*x)/c^2 + 1/3\*b\*x\*arcsin(c\*x)/c^2 - 1/9\*(-c^2\*x^2 + 1)^(3/2)\*b/c^3 + 1/3\*sqrt(-c^2\*x^2 + 1)\*b/c^3

**maple [A]** time = 0.00, size = 64, normalized size = 1.07

$$\frac{\frac{c^3x^3a}{3} + b \left( \frac{c^3x^3 \arcsin(cx)}{3} + \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} + \frac{2\sqrt{-c^2x^2+1}}{9} \right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsin(c\*x)),x)

[Out] 1/c^3\*(1/3\*c^3\*x^3\*a+b\*(1/3\*c^3\*x^3\*arcsin(c\*x)+1/9\*c^2\*x^2\*(-c^2\*x^2+1)^(1/2)+2/9\*(-c^2\*x^2+1)^(1/2)))

**maxima [A]** time = 0.45, size = 59, normalized size = 0.98

$$\frac{1}{3}ax^3 + \frac{1}{9} \left( 3x^3 \arcsin(cx) + c \left( \frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 1/9\*(3\*x^3\*arcsin(c\*x) + c\*(sqrt(-c^2\*x^2 + 1)\*x^2/c^2 + 2\*sqrt(-c^2\*x^2 + 1)/c^4))\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 (a + b \operatorname{asin}(cx)) dx = \begin{cases} b \left( \frac{\sqrt{\frac{1}{c^2} - x^2} \left( \frac{2}{c^2} + x^2 \right)}{9} + \frac{x^3 \operatorname{asin}(cx)}{3} \right) + \frac{ax^3}{3} & \text{if } 0 < c \\ \int x^2 (a + b \operatorname{asin}(cx)) dx & \text{if } -0 < c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*asin(c*x)),x)`

[Out] `piecewise(0 < c, b*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 + (x^3*asin(c*x))/3) + (a*x^3)/3, ~0 < c, int(x^2*(a + b*asin(c*x)), x))`

**sympy** [A] time = 0.49, size = 65, normalized size = 1.08

$$\int x^2 (a + b \operatorname{asin}(cx)) dx = \begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{asin}(cx)}{3} + \frac{bx^2 \sqrt{-c^2x^2+1}}{9c} + \frac{2b \sqrt{-c^2x^2+1}}{9c^3} & \text{for } c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asin(c*x)),x)`

[Out] `Piecewise((a*x**3/3 + b*x**3*asin(c*x)/3 + b*x**2*sqrt(-c**2*x**2 + 1)/(9*c) + 2*b*sqrt(-c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*x**3/3, True))`

### 3.142 $\int x (a + b \sin^{-1}(cx)) dx$

**Optimal.** Leaf size=51

$$\frac{1}{2}x^2 (a + b \sin^{-1}(cx)) + \frac{bx\sqrt{1-c^2x^2}}{4c} - \frac{b \sin^{-1}(cx)}{4c^2}$$

[Out]  $-1/4*b*\arcsin(c*x)/c^2+1/2*x^2*(a+b*\arcsin(c*x))+1/4*b*x*(-c^2*x^2+1)^(1/2)/c$

**Rubi [A]** time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4627, 321, 216}

$$\frac{1}{2}x^2 (a + b \sin^{-1}(cx)) + \frac{bx\sqrt{1-c^2x^2}}{4c} - \frac{b \sin^{-1}(cx)}{4c^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcSin[c\*x]),x]

[Out]  $(b*x*\text{Sqrt}[1 - c^2*x^2])/(4*c) - (b*\text{ArcSin}[c*x])/(4*c^2) + (x^2*(a + b*\text{ArcSin}[c*x]))/2$

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x (a + b \sin^{-1}(cx)) dx &= \frac{1}{2}x^2 (a + b \sin^{-1}(cx)) - \frac{1}{2}(bc) \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \\ &= \frac{bx\sqrt{1-c^2x^2}}{4c} + \frac{1}{2}x^2 (a + b \sin^{-1}(cx)) - \frac{b \int \frac{1}{\sqrt{1-c^2x^2}} dx}{4c} \\ &= \frac{bx\sqrt{1-c^2x^2}}{4c} - \frac{b \sin^{-1}(cx)}{4c^2} + \frac{1}{2}x^2 (a + b \sin^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 56, normalized size = 1.10

$$\frac{ax^2}{2} + \frac{bx\sqrt{1-c^2x^2}}{4c} - \frac{b \sin^{-1}(cx)}{4c^2} + \frac{1}{2}bx^2 \sin^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcSin[c\*x]),x]

[Out] (a\*x^2)/2 + (b\*x\*sqrt[1 - c^2\*x^2])/(4\*c) - (b\*ArcSin[c\*x])/(4\*c^2) + (b\*x^2\*ArcSin[c\*x])/2

**fricas** [A] time = 0.45, size = 49, normalized size = 0.96

$$\frac{2ac^2x^2 + \sqrt{-c^2x^2 + 1}bcx + (2bc^2x^2 - b)\arcsin(cx)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x)),x, algorithm="fricas")

[Out] 1/4\*(2\*a\*c^2\*x^2 + sqrt(-c^2\*x^2 + 1)\*b\*c\*x + (2\*b\*c^2\*x^2 - b)\*arcsin(c\*x))/c^2

**giac** [A] time = 0.52, size = 64, normalized size = 1.25

$$\frac{\sqrt{-c^2x^2 + 1}bx}{4c} + \frac{(c^2x^2 - 1)b\arcsin(cx)}{2c^2} + \frac{(c^2x^2 - 1)a}{2c^2} + \frac{b\arcsin(cx)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x)),x, algorithm="giac")

[Out] 1/4\*sqrt(-c^2\*x^2 + 1)\*b\*x/c + 1/2\*(c^2\*x^2 - 1)\*b\*arcsin(c\*x)/c^2 + 1/2\*(c^2\*x^2 - 1)\*a/c^2 + 1/4\*b\*arcsin(c\*x)/c^2

**maple** [A] time = 0.01, size = 52, normalized size = 1.02

$$\frac{\frac{c^2x^2a}{2} + b\left(\frac{c^2x^2\arcsin(cx)}{2} + \frac{cx\sqrt{-c^2x^2+1}}{4} - \frac{\arcsin(cx)}{4}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsin(c\*x)),x)

[Out] 1/c^2\*(1/2\*c^2\*x^2\*a+b\*(1/2\*c^2\*x^2\*arcsin(c\*x)+1/4\*c\*x\*(-c^2\*x^2+1)^(1/2)-1/4\*arcsin(c\*x)))

**maxima** [A] time = 0.43, size = 49, normalized size = 0.96

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(2x^2\arcsin(cx) + c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] 1/2\*a\*x^2 + 1/4\*(2\*x^2\*arcsin(c\*x) + c\*(sqrt(-c^2\*x^2 + 1)\*x/c^2 - arcsin(c\*x)/c^3))\*b

**mupad** [B] time = 0.15, size = 45, normalized size = 0.88

$$\frac{ax^2}{2} + \frac{b\left(\frac{\arcsin(cx)(2c^2x^2-1)}{4} + \frac{cx\sqrt{1-c^2x^2}}{4}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(x*(a + b*asin(c*x)),x)
```

```
[Out] (a*x^2)/2 + (b*((asin(c*x)*(2*c^2*x^2 - 1))/4 + (c*x*(1 - c^2*x^2)^(1/2))/4
))/c^2
```

**sympy [A]** time = 0.26, size = 54, normalized size = 1.06

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{asin}(cx)}{2} + \frac{bx\sqrt{-c^2x^2+1}}{4c} - \frac{b \operatorname{asin}(cx)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asin(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*asin(c*x)/2 + b*x*sqrt(-c**2*x**2 + 1)/(4*c) -
b*asin(c*x)/(4*c**2), Ne(c, 0)), (a*x**2/2, True))
```

### 3.143 $\int (a + b \sin^{-1}(cx)) dx$

**Optimal.** Leaf size=30

$$ax + \frac{b\sqrt{1-c^2x^2}}{c} + bx \sin^{-1}(cx)$$

[Out] a\*x+b\*x\*arcsin(c\*x)+b\*(-c^2\*x^2+1)^(1/2)/c

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4619, 261}

$$ax + \frac{b\sqrt{1-c^2x^2}}{c} + bx \sin^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcSin[c\*x], x]

[Out] a\*x + (b\*Sqrt[1 - c^2\*x^2])/c + b\*x\*ArcSin[c\*x]

Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int (a + b \sin^{-1}(cx)) dx &= ax + b \int \sin^{-1}(cx) dx \\ &= ax + bx \sin^{-1}(cx) - (bc) \int \frac{x}{\sqrt{1-c^2x^2}} dx \\ &= ax + \frac{b\sqrt{1-c^2x^2}}{c} + bx \sin^{-1}(cx) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$ax + \frac{b\sqrt{1-c^2x^2}}{c} + bx \sin^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcSin[c\*x], x]

[Out] a\*x + (b\*Sqrt[1 - c^2\*x^2])/c + b\*x\*ArcSin[c\*x]

**fricas [A]** time = 0.45, size = 31, normalized size = 1.03

$$\frac{bcx \arcsin(cx) + acx + \sqrt{-c^2x^2 + 1} b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arcsin(c\*x),x, algorithm="fricas")

[Out] (b\*c\*x\*arcsin(c\*x) + a\*c\*x + sqrt(-c^2\*x^2 + 1)\*b)/c

**giac** [A] time = 0.46, size = 29, normalized size = 0.97

$$ax + \frac{\left(cx \arcsin(cx) + \sqrt{-c^2x^2 + 1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arcsin(c\*x),x, algorithm="giac")

[Out] a\*x + (c\*x\*arcsin(c\*x) + sqrt(-c^2\*x^2 + 1))\*b/c

**maple** [A] time = 0.00, size = 30, normalized size = 1.00

$$ax + \frac{b\left(cx \arcsin(cx) + \sqrt{-c^2x^2 + 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arcsin(c\*x),x)

[Out] a\*x+b/c\*(c\*x\*arcsin(c\*x)+(-c^2\*x^2+1)^(1/2))

**maxima** [A] time = 0.43, size = 29, normalized size = 0.97

$$ax + \frac{\left(cx \arcsin(cx) + \sqrt{-c^2x^2 + 1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arcsin(c\*x),x, algorithm="maxima")

[Out] a\*x + (c\*x\*arcsin(c\*x) + sqrt(-c^2\*x^2 + 1))\*b/c

**mupad** [B] time = 0.17, size = 28, normalized size = 0.93

$$ax + \frac{b\sqrt{1-c^2x^2}}{c} + bx \operatorname{asin}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b\*asin(c\*x),x)

[Out] a\*x + (b\*(1 - c^2\*x^2)^(1/2))/c + b\*x\*asin(c\*x)

**sympy** [A] time = 0.14, size = 26, normalized size = 0.87

$$ax + b \begin{cases} x \operatorname{asin}(cx) + \frac{\sqrt{-c^2x^2+1}}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*asin(c\*x),x)

[Out] a\*x + b\*Piecewise((x\*asin(c\*x) + sqrt(-c\*\*2\*x\*\*2 + 1)/c, Ne(c, 0)), (0, True))

$$3.144 \quad \int \frac{a+b \sin^{-1}(cx)}{x} dx$$

**Optimal.** Leaf size=63

$$-\frac{i(a+b \sin^{-1}(cx))^2}{2b} + \log(1 - e^{2i \sin^{-1}(cx)})(a+b \sin^{-1}(cx)) - \frac{1}{2}ib \operatorname{Li}_2(e^{2i \sin^{-1}(cx)})$$

[Out]  $-1/2*I*(a+b*\arcsin(c*x))^2/b+(a+b*\arcsin(c*x))*\ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-1/2*I*b*\operatorname{polylog}(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))$

**Rubi [A]** time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {4625, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}ib \operatorname{PolyLog}(2, e^{2i \sin^{-1}(cx)}) - \frac{i(a+b \sin^{-1}(cx))^2}{2b} + \log(1 - e^{2i \sin^{-1}(cx)})(a+b \sin^{-1}(cx))$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSin}[c*x])/x, x]$

[Out]  $((-I/2)*(a + b*\operatorname{ArcSin}[c*x])^2)/b + (a + b*\operatorname{ArcSin}[c*x])* \operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcSin}[c*x])}] - (I/2)*b*\operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcSin}[c*x])}]$

#### Rule 2190

$\operatorname{Int}[(((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_)*((c_)+(d_)*(x_))^{(m_))}})/((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_))}}), x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^m*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n)/a)]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n)/a)], x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_)+(b_)*((F_)^{((e_)*((c_)+(d_)*(x_)))^{(n_))}}], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a+b*x]/x, x], x, (F^{(e*(c+d*x))})^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_)+(e_)*(x_))^{(n_)}]/(x_), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /;$   $\operatorname{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \operatorname{EqQ}[c*d, 1]$

#### Rule 3717

$\operatorname{Int}[((c_)+(d_)*(x_))^{(m_)}*\tan[(e_)+\operatorname{Pi}*(k_)+(f_)*(x_)], x\_Symbol] \rightarrow \operatorname{Simp}[(I*(c+d*x)^{(m+1)})/(d*(m+1)), x] - \operatorname{Dist}[2*I, \operatorname{Int}[(c+d*x)^m*\operatorname{E}^{(2*I*k*\operatorname{Pi})*\operatorname{E}^{(2*I*(e+f*x))}}/(1+\operatorname{E}^{(2*I*k*\operatorname{Pi})*\operatorname{E}^{(2*I*(e+f*x))}}), x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \operatorname{IntegerQ}[4*k] \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 4625

$\operatorname{Int}[(a_)+\operatorname{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}]/(x_), x\_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[(a+b*x)^n/\operatorname{Tan}[x], x], x, \operatorname{ArcSin}[c*x]] /;$   $\operatorname{FreeQ}\{a, b, c\}, x\} \ \&\& \ \operatorname{IGtQ}[n, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sin^{-1}(cx)}{x} dx &= \text{Subst} \left( \int (a + bx) \cot(x) dx, x, \sin^{-1}(cx) \right) \\
&= -\frac{i(a + b \sin^{-1}(cx))^2}{2b} - 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)}{1 - e^{2ix}} dx, x, \sin^{-1}(cx) \right) \\
&= -\frac{i(a + b \sin^{-1}(cx))^2}{2b} + (a + b \sin^{-1}(cx)) \log(1 - e^{2i \sin^{-1}(cx)}) - b \text{Subst} \left( \int \log(1 - e^{2ix}) dx, x, \sin^{-1}(cx) \right) \\
&= -\frac{i(a + b \sin^{-1}(cx))^2}{2b} + (a + b \sin^{-1}(cx)) \log(1 - e^{2i \sin^{-1}(cx)}) + \frac{1}{2}(ib) \text{Subst} \left( \int \frac{\log(1 - e^{2ix})}{x} dx, x, \sin^{-1}(cx) \right) \\
&= -\frac{i(a + b \sin^{-1}(cx))^2}{2b} + (a + b \sin^{-1}(cx)) \log(1 - e^{2i \sin^{-1}(cx)}) - \frac{1}{2}ib \text{Li}_2(e^{2i \sin^{-1}(cx)})
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 52, normalized size = 0.83

$$a \log(x) - \frac{1}{2}ib \left( \sin^{-1}(cx)^2 + \text{Li}_2(e^{2i \sin^{-1}(cx)}) \right) + b \sin^{-1}(cx) \log(1 - e^{2i \sin^{-1}(cx)})$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSin[c\*x])/x,x]

[Out] b\*ArcSin[c\*x]\*Log[1 - E^((2\*I)\*ArcSin[c\*x])] + a\*Log[x] - (I/2)\*b\*(ArcSin[c\*x]^2 + PolyLog[2, E^((2\*I)\*ArcSin[c\*x])])

**fricas [F]** time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b \arcsin(cx) + a}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x,x, algorithm="fricas")

[Out] integral((b\*arcsin(c\*x) + a)/x, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \arcsin(cx) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x,x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)/x, x)

**maple [A]** time = 0.05, size = 122, normalized size = 1.94

$$a \ln(cx) - \frac{ib \arcsin(cx)^2}{2} + b \arcsin(cx) \ln \left( 1 + icx + \sqrt{-c^2x^2 + 1} \right) + b \arcsin(cx) \ln \left( 1 - icx - \sqrt{-c^2x^2 + 1} \right) - ib \text{Li}_2(e^{2i \arcsin(cx)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))/x,x)

[Out] a\*ln(c\*x) - 1/2\*I\*b\*arcsin(c\*x)^2 + b\*arcsin(c\*x)\*ln(1 + I\*c\*x + (-c^2\*x^2 + 1)^(1/2)) + b\*arcsin(c\*x)\*ln(1 - I\*c\*x - (-c^2\*x^2 + 1)^(1/2)) - I\*b\*polylog(2, I\*c\*x + (-c^2\*x^2 + 1)^(1/2)) - I\*b\*polylog(2, -I\*c\*x - (-c^2\*x^2 + 1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)}{x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x,x, algorithm="maxima")

[Out] b\*integrate(arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))/x, x) + a\*log(x)

**mupad** [B] time = 0.15, size = 48, normalized size = 0.76

$$a \ln(x) - \frac{b \operatorname{polylog}\left(2, e^{\operatorname{asin}(cx)2i}\right) 1i}{2} - \frac{b \operatorname{asin}(cx)^2 1i}{2} + b \ln\left(1 - e^{\operatorname{asin}(cx)2i}\right) \operatorname{asin}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))/x,x)

[Out] a\*log(x) - (b\*polylog(2, exp(asin(c\*x)\*2i))\*1i)/2 - (b\*asin(c\*x)^2\*1i)/2 + b\*log(1 - exp(asin(c\*x)\*2i))\*asin(c\*x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asin}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))/x,x)

[Out] Integral((a + b\*asin(c\*x))/x, x)

$$3.145 \quad \int \frac{a+b \sin^{-1}(cx)}{x^2} dx$$

**Optimal.** Leaf size=33

$$-\frac{a+b \sin^{-1}(cx)}{x} - bc \tanh^{-1}\left(\sqrt{1-c^2x^2}\right)$$

[Out]  $(-a-b*\arcsin(c*x))/x-b*c*\operatorname{arctanh}((-c^2*x^2+1)^{(1/2)})$

**Rubi [A]** time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4627, 266, 63, 208}

$$-\frac{a+b \sin^{-1}(cx)}{x} - bc \tanh^{-1}\left(\sqrt{1-c^2x^2}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSin}[c*x])/x^2, x]$

[Out]  $-(a + b*\operatorname{ArcSin}[c*x])/x - b*c*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2*x^2]]$

#### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

#### Rule 266

$\operatorname{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x\_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, m, n, p\}, x] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

#### Rule 4627

$\operatorname{Int}[(a_. + \operatorname{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)*((d_.)*(x_.))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcSin}[c*x])^n/(d*(m+1)), x] - \operatorname{Dist}[(b*c*n)/(d*(m+1)), \operatorname{Int}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcSin}[c*x])^{(n-1)}/\operatorname{Sqrt}[1 - c^2*x^2], x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sin^{-1}(cx)}{x^2} dx &= -\frac{a + b \sin^{-1}(cx)}{x} + (bc) \int \frac{1}{x\sqrt{1 - c^2x^2}} dx \\
&= -\frac{a + b \sin^{-1}(cx)}{x} + \frac{1}{2}(bc) \text{Subst} \left( \int \frac{1}{x\sqrt{1 - c^2x}} dx, x, x^2 \right) \\
&= -\frac{a + b \sin^{-1}(cx)}{x} - \frac{b \text{Subst} \left( \int \frac{1}{\frac{1}{c^2} - \frac{x^2}{c^2}} dx, x, \sqrt{1 - c^2x^2} \right)}{c} \\
&= -\frac{a + b \sin^{-1}(cx)}{x} - bc \tanh^{-1} \left( \sqrt{1 - c^2x^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 36, normalized size = 1.09

$$-\frac{a}{x} - bc \tanh^{-1} \left( \sqrt{1 - c^2x^2} \right) - \frac{b \sin^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])/x^2,x]

[Out] -(a/x) - (b\*ArcSin[c\*x])/x - b\*c\*ArcTanh[Sqrt[1 - c^2\*x^2]]

**fricas [A]** time = 0.56, size = 55, normalized size = 1.67

$$\frac{bcx \log \left( \sqrt{-c^2x^2 + 1} + 1 \right) - bcx \log \left( \sqrt{-c^2x^2 + 1} - 1 \right) + 2b \arcsin(cx) + 2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^2,x, algorithm="fricas")

[Out] -1/2\*(b\*c\*x\*log(sqrt(-c^2\*x^2 + 1) + 1) - b\*c\*x\*log(sqrt(-c^2\*x^2 + 1) - 1) + 2\*b\*arcsin(c\*x) + 2\*a)/x

**giac [B]** time = 0.37, size = 325, normalized size = 9.85

$$\frac{\sqrt{-c^2x^2 + 1} bc^2x \arcsin(cx)}{2 \left( \sqrt{-c^2x^2 + 1} + 1 \right)^2} - \frac{bc^2x \arcsin(cx)}{2 \left( \sqrt{-c^2x^2 + 1} + 1 \right)^2} - \frac{\sqrt{-c^2x^2 + 1} ac^2x}{2 \left( \sqrt{-c^2x^2 + 1} + 1 \right)^2} + \frac{\sqrt{-c^2x^2 + 1} bc \log(|c||x|)}{\sqrt{-c^2x^2 + 1} + 1} - \frac{\sqrt{-c^2x^2 + 1}}{\sqrt{-c^2x^2 + 1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^2,x, algorithm="giac")

[Out] -1/2\*sqrt(-c^2\*x^2 + 1)\*b\*c^2\*x\*arcsin(c\*x)/(sqrt(-c^2\*x^2 + 1) + 1)^2 - 1/2\*b\*c^2\*x\*arcsin(c\*x)/(sqrt(-c^2\*x^2 + 1) + 1)^2 - 1/2\*sqrt(-c^2\*x^2 + 1)\*a\*c^2\*x/(sqrt(-c^2\*x^2 + 1) + 1)^2 + sqrt(-c^2\*x^2 + 1)\*b\*c\*log(abs(c)\*abs(x))/(sqrt(-c^2\*x^2 + 1) + 1) - sqrt(-c^2\*x^2 + 1)\*b\*c\*log(sqrt(-c^2\*x^2 + 1) + 1)/(sqrt(-c^2\*x^2 + 1) + 1) - 1/2\*a\*c^2\*x/(sqrt(-c^2\*x^2 + 1) + 1)^2 + b\*c\*log(abs(c)\*abs(x))/(sqrt(-c^2\*x^2 + 1) + 1) - b\*c\*log(sqrt(-c^2\*x^2 + 1) + 1)/(sqrt(-c^2\*x^2 + 1) + 1) - 1/2\*sqrt(-c^2\*x^2 + 1)\*b\*arcsin(c\*x)/x - 1/2\*b\*arcsin(c\*x)/x - 1/2\*sqrt(-c^2\*x^2 + 1)\*a/x - 1/2\*a/x

**maple [A]** time = 0.00, size = 43, normalized size = 1.30

$$c \left( -\frac{a}{cx} + b \left( -\frac{\arcsin(cx)}{cx} - \operatorname{arctanh} \left( \frac{1}{\sqrt{-c^2x^2 + 1}} \right) \right) \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsin(c*x))/x^2,x)`

[Out] `c*(-a/c/x+b*(-1/c/x*arcsin(c*x)-arctanh(1/(-c^2*x^2+1)^(1/2))))`

**maxima** [A] time = 0.42, size = 47, normalized size = 1.42

$$-\left(c \log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\arcsin(cx)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

[Out] `-(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b - a/x`

**mupad** [B] time = 0.13, size = 34, normalized size = 1.03

$$-\frac{a}{x} - \frac{b \operatorname{asin}(cx)}{x} - bc \operatorname{atanh}\left(\frac{1}{\sqrt{1-c^2x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))/x^2,x)`

[Out] `-a/x - (b*asin(c*x))/x - b*c*atanh(1/(1 - c^2*x^2)^(1/2))`

**sympy** [A] time = 1.71, size = 39, normalized size = 1.18

$$-\frac{a}{x} + bc \left( \begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) - \frac{b \operatorname{asin}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asin(c*x))/x**2,x)`

[Out] `-a/x + b*c*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*asin(c*x)/x`

$$3.146 \quad \int \frac{a+b \sin^{-1}(cx)}{x^3} dx$$

**Optimal.** Leaf size=39

$$-\frac{a+b \sin^{-1}(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x}$$

[Out] 1/2\*(-a-b\*arcsin(c\*x))/x^2-1/2\*b\*c\*(-c^2\*x^2+1)^(1/2)/x

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {4627, 264}

$$-\frac{a+b \sin^{-1}(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])/x^3,x]

[Out] -(b\*c\*Sqrt[1 - c^2\*x^2])/(2\*x) - (a + b\*ArcSin[c\*x])/(2\*x^2)

**Rule 264**

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

**Rule 4627**

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{a+b \sin^{-1}(cx)}{x^3} dx &= -\frac{a+b \sin^{-1}(cx)}{2x^2} + \frac{1}{2}(bc) \int \frac{1}{x^2\sqrt{1-c^2x^2}} dx \\ &= -\frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{a+b \sin^{-1}(cx)}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 44, normalized size = 1.13

$$-\frac{a}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{b \sin^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])/x^3,x]

[Out] -1/2\*a/x^2 - (b\*c\*Sqrt[1 - c^2\*x^2])/(2\*x) - (b\*ArcSin[c\*x])/(2\*x^2)

**fricas [A]** time = 0.54, size = 35, normalized size = 0.90

$$-\frac{\sqrt{-c^2x^2+1}bcx - ax^2 + b \arcsin(cx) + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^3,x, algorithm="fricas")

[Out]  $-1/2*(\sqrt{-c^2*x^2 + 1})*b*c*x - a*x^2 + b*arcsin(c*x) + a)/x^2$

**giac** [B] time = 0.29, size = 163, normalized size = 4.18

$$\frac{bc^4x^2 \arcsin(cx)}{8(\sqrt{-c^2x^2+1}+1)^2} - \frac{ac^4x^2}{8(\sqrt{-c^2x^2+1}+1)^2} + \frac{bc^3x}{4(\sqrt{-c^2x^2+1}+1)} - \frac{1}{4}bc^2 \arcsin(cx) - \frac{1}{4}ac^2 - \frac{bc(\sqrt{-c^2x^2+1})}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^3,x, algorithm="giac")

[Out]  $-1/8*b*c^4*x^2*arcsin(c*x)/(\sqrt{-c^2*x^2 + 1} + 1)^2 - 1/8*a*c^4*x^2/(\sqrt{-c^2*x^2 + 1} + 1)^2 + 1/4*b*c^3*x/(\sqrt{-c^2*x^2 + 1} + 1) - 1/4*b*c^2*arcsin(c*x) - 1/4*a*c^2 - 1/4*b*c*(\sqrt{-c^2*x^2 + 1} + 1)/x - 1/8*b*(\sqrt{-c^2*x^2 + 1} + 1)^2*arcsin(c*x)/x^2 - 1/8*a*(\sqrt{-c^2*x^2 + 1} + 1)^2/x^2$

**maple** [A] time = 0.01, size = 50, normalized size = 1.28

$$c^2 \left( -\frac{a}{2c^2x^2} + b \left( -\frac{\arcsin(cx)}{2c^2x^2} - \frac{\sqrt{-c^2x^2+1}}{2cx} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))/x^3,x)

[Out]  $c^2*(-1/2*a/c^2/x^2+b*(-1/2*arcsin(c*x)/c^2/x^2-1/2/c/x*(\sqrt{-c^2*x^2+1})^{(1/2)}))$

**maxima** [A] time = 0.47, size = 36, normalized size = 0.92

$$-\frac{1}{2}b \left( \frac{\sqrt{-c^2x^2+1}c}{x} + \frac{\arcsin(cx)}{x^2} \right) - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^3,x, algorithm="maxima")

[Out]  $-1/2*b*(\sqrt{-c^2*x^2 + 1})*c/x + arcsin(c*x)/x^2) - 1/2*a/x^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{asin}(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))/x^3,x)

[Out] int((a + b\*asin(c\*x))/x^3, x)

**sympy** [A] time = 1.45, size = 61, normalized size = 1.56

$$-\frac{a}{2x^2} + \frac{bc \left( \begin{cases} -\frac{i\sqrt{c^2x^2-1}}{x} & \text{for } |c^2x^2| > 1 \\ -\frac{\sqrt{-c^2x^2+1}}{x} & \text{otherwise} \end{cases} \right)}{2} - \frac{b \operatorname{asin}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asin(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) + b*c*Piecewise((-I*sqrt(c**2*x**2 - 1)/x, Abs(c**2*x**2) > 1),  
(-sqrt(-c**2*x**2 + 1)/x, True))/2 - b*asin(c*x)/(2*x**2)
```

$$3.147 \quad \int \frac{a+b \sin^{-1}(cx)}{x^4} dx$$

**Optimal.** Leaf size=62

$$-\frac{a+b \sin^{-1}(cx)}{3x^3} - \frac{bc\sqrt{1-c^2x^2}}{6x^2} - \frac{1}{6}bc^3 \tanh^{-1}\left(\sqrt{1-c^2x^2}\right)$$

[Out] 1/3\*(-a-b\*arcsin(c\*x))/x^3-1/6\*b\*c^3\*arctanh((-c^2\*x^2+1)^(1/2))-1/6\*b\*c\*(-c^2\*x^2+1)^(1/2)/x^2

**Rubi [A]** time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {4627, 266, 51, 63, 208}

$$-\frac{a+b \sin^{-1}(cx)}{3x^3} - \frac{bc\sqrt{1-c^2x^2}}{6x^2} - \frac{1}{6}bc^3 \tanh^{-1}\left(\sqrt{1-c^2x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])/x^4, x]

[Out] -(b\*c\*Sqrt[1 - c^2\*x^2])/(6\*x^2) - (a + b\*ArcSin[c\*x])/(3\*x^3) - (b\*c^3\*ArcTanh[Sqrt[1 - c^2\*x^2]])/6

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] ] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^p\_, x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sin^{-1}(cx)}{x^4} dx &= -\frac{a + b \sin^{-1}(cx)}{3x^3} + \frac{1}{3}(bc) \int \frac{1}{x^3 \sqrt{1 - c^2 x^2}} dx \\
&= -\frac{a + b \sin^{-1}(cx)}{3x^3} + \frac{1}{6}(bc) \operatorname{Subst} \left( \int \frac{1}{x^2 \sqrt{1 - c^2 x}} dx, x, x^2 \right) \\
&= -\frac{bc \sqrt{1 - c^2 x^2}}{6x^2} - \frac{a + b \sin^{-1}(cx)}{3x^3} + \frac{1}{12} (bc^3) \operatorname{Subst} \left( \int \frac{1}{x \sqrt{1 - c^2 x}} dx, x, x^2 \right) \\
&= -\frac{bc \sqrt{1 - c^2 x^2}}{6x^2} - \frac{a + b \sin^{-1}(cx)}{3x^3} - \frac{1}{6} (bc) \operatorname{Subst} \left( \int \frac{1}{\frac{1}{c^2} - \frac{x^2}{c^2}} dx, x, \sqrt{1 - c^2 x^2} \right) \\
&= -\frac{bc \sqrt{1 - c^2 x^2}}{6x^2} - \frac{a + b \sin^{-1}(cx)}{3x^3} - \frac{1}{6} bc^3 \tanh^{-1} \left( \sqrt{1 - c^2 x^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 67, normalized size = 1.08

$$-\frac{a}{3x^3} - \frac{bc\sqrt{1-c^2x^2}}{6x^2} - \frac{1}{6}bc^3 \tanh^{-1}\left(\sqrt{1-c^2x^2}\right) - \frac{b \sin^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])/x^4, x]

[Out] -1/3\*a/x^3 - (b\*c\*Sqrt[1 - c^2\*x^2])/(6\*x^2) - (b\*ArcSin[c\*x])/(3\*x^3) - (b\*c^3\*ArcTanh[Sqrt[1 - c^2\*x^2]])/6

**fricas [A]** time = 0.57, size = 80, normalized size = 1.29

$$\frac{bc^3x^3 \log\left(\sqrt{-c^2x^2+1}+1\right) - bc^3x^3 \log\left(\sqrt{-c^2x^2+1}-1\right) + 2\sqrt{-c^2x^2+1}bcx + 4b \arcsin(cx) + 4a}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^4, x, algorithm="fricas")

[Out] -1/12\*(b\*c^3\*x^3\*log(sqrt(-c^2\*x^2 + 1) + 1) - b\*c^3\*x^3\*log(sqrt(-c^2\*x^2 + 1) - 1) + 2\*sqrt(-c^2\*x^2 + 1)\*b\*c\*x + 4\*b\*arcsin(c\*x) + 4\*a)/x^3

**giac [B]** time = 0.92, size = 284, normalized size = 4.58

$$\frac{bc^6x^3 \arcsin(cx)}{24\left(\sqrt{-c^2x^2+1}+1\right)^3} - \frac{ac^6x^3}{24\left(\sqrt{-c^2x^2+1}+1\right)^3} + \frac{bc^5x^2}{24\left(\sqrt{-c^2x^2+1}+1\right)^2} - \frac{bc^4x \arcsin(cx)}{8\left(\sqrt{-c^2x^2+1}+1\right)} - \frac{ac^4x}{8\left(\sqrt{-c^2x^2+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^4, x, algorithm="giac")

[Out] -1/24\*b\*c^6\*x^3\*arcsin(c\*x)/(sqrt(-c^2\*x^2 + 1) + 1)^3 - 1/24\*a\*c^6\*x^3/(sqrt(-c^2\*x^2 + 1) + 1)^3 + 1/24\*b\*c^5\*x^2/(sqrt(-c^2\*x^2 + 1) + 1)^2 - 1/8\*b\*c^4\*x\*arcsin(c\*x)/(sqrt(-c^2\*x^2 + 1) + 1) - 1/8\*a\*c^4\*x/(sqrt(-c^2\*x^2 + 1) + 1) + 1/6\*b\*c^3\*log(abs(c)\*abs(x)) - 1/6\*b\*c^3\*log(sqrt(-c^2\*x^2 + 1) + 1) - 1/8\*b\*c^2\*(sqrt(-c^2\*x^2 + 1) + 1)\*arcsin(c\*x)/x - 1/8\*a\*c^2\*(sqrt(-c^2\*x^2 + 1) + 1)/x - 1/24\*b\*c\*(sqrt(-c^2\*x^2 + 1) + 1)^2/x^2 - 1/24\*b\*(sqrt(-c^2\*x^2 + 1) + 1)^3\*arcsin(c\*x)/x^3 - 1/24\*a\*(sqrt(-c^2\*x^2 + 1) + 1)^3/x^3

**maple** [A] time = 0.00, size = 65, normalized size = 1.05

$$c^3 \left( -\frac{a}{3c^3x^3} + b \left( -\frac{\arcsin(cx)}{3c^3x^3} - \frac{\sqrt{-c^2x^2+1}}{6c^2x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))/x^4,x)

[Out]  $c^3*(-1/3*a/c^3/x^3+b*(-1/3*arcsin(c*x)/c^3/x^3-1/6/c^2/x^2*(-c^2*x^2+1)^{(1/2)}-1/6*arctanh(1/(-c^2*x^2+1)^{(1/2)})))$

**maxima** [A] time = 0.44, size = 69, normalized size = 1.11

$$-\frac{1}{6} \left( \left( c^2 \log \left( \frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-c^2x^2+1}}{x^2} \right) c + \frac{2\arcsin(cx)}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/x^4,x, algorithm="maxima")

[Out]  $-1/6*((c^2*\log(2*\sqrt{-c^2*x^2+1}/\operatorname{abs}(x)+2/\operatorname{abs}(x))+\sqrt{-c^2*x^2+1}/x^2)*c+2*\arcsin(c*x)/x^3)*b-1/3*a/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asin}(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))/x^4,x)

[Out] int((a + b\*asin(c\*x))/x^4, x)

**sympy** [A] time = 2.61, size = 119, normalized size = 1.92

$$-\frac{a}{3x^3} + \frac{bc \left( \begin{array}{l} \left( -\frac{c^2 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} - \frac{c\sqrt{-1+\frac{1}{c^2x^2}}}{2x} \right) \text{ for } \frac{1}{|c^2x^2|} > 1 \\ \left( \frac{ic^2 \operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic}{2x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{i}{2cx^3\sqrt{1-\frac{1}{c^2x^2}}} \right) \text{ otherwise} \end{array} \right)}{3} - \frac{b \operatorname{asin}(cx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))/x\*\*4,x)

[Out]  $-a/(3*x**3) + b*c*\operatorname{Piecewise}((-c**2*\operatorname{acosh}(1/(c*x))/2 - c*\sqrt{-1 + 1/(c**2*x**2)})/(2*x), 1/\operatorname{Abs}(c**2*x**2) > 1), (I*c**2*\operatorname{asin}(1/(c*x))/2 - I*c/(2*x*\sqrt{1 - 1/(c**2*x**2)}) + I/(2*c*x**3*\sqrt{1 - 1/(c**2*x**2)}), True))/3 - b*a \operatorname{sin}(c*x)/(3*x**3)$

### 3.148 $\int x^2 (a + b \sin^{-1}(cx))^2 dx$

**Optimal.** Leaf size=102

$$\frac{2bx^2\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))}{9c} + \frac{4b\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))}{9c^3} + \frac{1}{3}x^3(a+b\sin^{-1}(cx))^2 - \frac{4b^2x}{9c^2} - \frac{2}{27}b^2x^3$$

[Out]  $-4/9*b^2*x/c^2-2/27*b^2*x^3+1/3*x^3*(a+b*\arcsin(c*x))^2+4/9*b*(a+b*\arcsin(c*x))*(-c^2*x^2+1)^{(1/2)}/c^3+2/9*b*x^2*(a+b*\arcsin(c*x))*(-c^2*x^2+1)^{(1/2)}/c$

**Rubi [A]** time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {4627, 4707, 4677, 8, 30}

$$\frac{2bx^2\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))}{9c} + \frac{4b\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))}{9c^3} + \frac{1}{3}x^3(a+b\sin^{-1}(cx))^2 - \frac{4b^2x}{9c^2} - \frac{2}{27}b^2x^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcSin[c\*x])^2,x]

[Out]  $(-4*b^2*x)/(9*c^2) - (2*b^2*x^3)/27 + (4*b*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(9*c^3) + (2*b*x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(9*c) + (x^3*(a + b*ArcSin[c*x])^2)/3$

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4677

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4707

Int[((a\_) + ArcSin[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)/sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n]/sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*sqrt[1 - c^2\*x^2])/(c\*m\*sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0]



&& GtQ[m, 1] && IntegerQ[m]

### Rubi steps

$$\begin{aligned} \int x^2 (a + b \sin^{-1}(cx))^2 dx &= \frac{1}{3} x^3 (a + b \sin^{-1}(cx))^2 - \frac{1}{3} (2bc) \int \frac{x^3 (a + b \sin^{-1}(cx))}{\sqrt{1 - c^2 x^2}} dx \\ &= \frac{2bx^2 \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))}{9c} + \frac{1}{3} x^3 (a + b \sin^{-1}(cx))^2 - \frac{1}{9} (2b^2) \int x^2 dx - \dots \\ &= -\frac{2}{27} b^2 x^3 + \frac{4b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))}{9c^3} + \frac{2bx^2 \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))}{9c} + \dots \\ &= -\frac{4b^2 x}{9c^2} - \frac{2b^2 x^3}{27} + \frac{4b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))}{9c^3} + \frac{2bx^2 \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))}{9c} \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 95, normalized size = 0.93

$$\frac{1}{3} \left( x^3 (a + b \sin^{-1}(cx))^2 - \frac{2b \left( -3c^2 x^2 \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx)) - 6 \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx)) + bc^3 x^3 + 6b \right)}{9c^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcSin[c\*x])^2,x]

[Out] (x^3\*(a + b\*ArcSin[c\*x])^2 - (2\*b\*(6\*b\*c\*x + b\*c^3\*x^3 - 6\*Sqrt[1 - c^2\*x^2])\*(a + b\*ArcSin[c\*x]) - 3\*c^2\*x^2\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])))/(9\*c^3)/3

**fricas [A]** time = 0.45, size = 111, normalized size = 1.09

$$\frac{9b^2c^3x^3 \arcsin(cx)^2 + 18abc^3x^3 \arcsin(cx) + (9a^2 - 2b^2)c^3x^3 - 12b^2cx + 6(abc^2x^2 + 2ab + (b^2c^2x^2 + 2b^2))}{27c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] 1/27\*(9\*b^2\*c^3\*x^3\*arcsin(c\*x)^2 + 18\*a\*b\*c^3\*x^3\*arcsin(c\*x) + (9\*a^2 - 2\*b^2)\*c^3\*x^3 - 12\*b^2\*c\*x + 6\*(a\*b\*c^2\*x^2 + 2\*a\*b + (b^2\*c^2\*x^2 + 2\*b^2))\*arcsin(c\*x))\*sqrt(-c^2\*x^2 + 1))/c^3

**giac [B]** time = 0.39, size = 194, normalized size = 1.90

$$\frac{1}{3} a^2 x^3 + \frac{(c^2 x^2 - 1) b^2 x \arcsin(cx)^2}{3c^2} + \frac{2(c^2 x^2 - 1) abx \arcsin(cx)}{3c^2} + \frac{b^2 x \arcsin(cx)^2}{3c^2} - \frac{2(c^2 x^2 - 1) b^2 x}{27c^2} + \frac{2 abx \arcsin(cx)}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] 1/3\*a^2\*x^3 + 1/3\*(c^2\*x^2 - 1)\*b^2\*x\*arcsin(c\*x)^2/c^2 + 2/3\*(c^2\*x^2 - 1)\*a\*b\*x\*arcsin(c\*x)/c^2 + 1/3\*b^2\*x\*arcsin(c\*x)^2/c^2 - 2/27\*(c^2\*x^2 - 1)\*b^2\*x/c^2 + 2/3\*a\*b\*x\*arcsin(c\*x)/c^2 - 2/9\*(-c^2\*x^2 + 1)^(3/2)\*b^2\*arcsin(c\*x)/c^3 - 14/27\*b^2\*x/c^2 - 2/9\*(-c^2\*x^2 + 1)^(3/2)\*a\*b/c^3 + 2/3\*sqrt(-c^2\*x^2 + 1)\*b^2\*arcsin(c\*x)/c^3 + 2/3\*sqrt(-c^2\*x^2 + 1)\*a\*b/c^3

**maple [A]** time = 0.03, size = 126, normalized size = 1.24

$$\frac{\frac{a^2c^3x^3}{3} + b^2 \left( \frac{c^3x^3 \arcsin(cx)^2}{3} + \frac{2 \arcsin(cx)(c^2x^2+2)\sqrt{-c^2x^2+1}}{9} - \frac{2c^3x^3}{27} - \frac{4cx}{9} \right) + 2ab \left( \frac{c^3x^3 \arcsin(cx)}{3} + \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} + \frac{2\sqrt{-c^2x^2+1}}{9} \right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsin(c\*x))^2,x)

[Out] 1/c^3\*(1/3\*a^2\*c^3\*x^3+b^2\*(1/3\*c^3\*x^3\*arcsin(c\*x)^2+2/9\*arcsin(c\*x)\*(c^2\*x^2+2)\*(-c^2\*x^2+1)^(1/2)-2/27\*c^3\*x^3-4/9\*c\*x)+2\*a\*b\*(1/3\*c^3\*x^3\*arcsin(c\*x)+1/9\*c^2\*x^2\*(-c^2\*x^2+1)^(1/2)+2/9\*(-c^2\*x^2+1)^(1/2))

**maxima [A]** time = 0.46, size = 142, normalized size = 1.39

$$\frac{1}{3} b^2 x^3 \arcsin(cx)^2 + \frac{1}{3} a^2 x^3 + \frac{2}{9} \left( 3 x^3 \arcsin(cx) + c \left( \frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4} \right) \right) ab + \frac{2}{27} \left( 3c \left( \frac{\sqrt{-c^2x^2+1}x^2}{c^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] 1/3\*b^2\*x^3\*arcsin(c\*x)^2 + 1/3\*a^2\*x^3 + 2/9\*(3\*x^3\*arcsin(c\*x) + c\*(sqrt(-c^2\*x^2 + 1)\*x^2/c^2 + 2\*sqrt(-c^2\*x^2 + 1)/c^4))\*a\*b + 2/27\*(3\*c\*(sqrt(-c^2\*x^2 + 1)\*x^2/c^2 + 2\*sqrt(-c^2\*x^2 + 1)/c^4)\*arcsin(c\*x) - (c^2\*x^3 + 6\*x)/c^2)\*b^2

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + b \operatorname{asin}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*asin(c\*x))^2,x)

[Out] int(x^2\*(a + b\*asin(c\*x))^2, x)

**sympy [A]** time = 1.06, size = 170, normalized size = 1.67

$$\left\{ \begin{array}{l} \frac{a^2x^3}{3} + \frac{2abx^3 \operatorname{asin}(cx)}{3} + \frac{2abx^2\sqrt{-c^2x^2+1}}{9c} + \frac{4ab\sqrt{-c^2x^2+1}}{9c^3} + \frac{b^2x^3 \operatorname{asin}^2(cx)}{3} - \frac{2b^2x^3}{27} + \frac{2b^2x^2\sqrt{-c^2x^2+1} \operatorname{asin}(cx)}{9c} - \frac{4b^2x}{9c^2} + \frac{4b^2\sqrt{-c^2x^2+1}}{9c} \\ \frac{a^2x^3}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*asin(c\*x))\*\*2,x)

[Out] Piecewise((a\*\*2\*x\*\*3/3 + 2\*a\*b\*x\*\*3\*asin(c\*x)/3 + 2\*a\*b\*x\*\*2\*sqrt(-c\*\*2\*x\*\*2 + 1)/(9\*c) + 4\*a\*b\*sqrt(-c\*\*2\*x\*\*2 + 1)/(9\*c\*\*3) + b\*\*2\*x\*\*3\*asin(c\*x)\*\*2/3 - 2\*b\*\*2\*x\*\*3/27 + 2\*b\*\*2\*x\*\*2\*sqrt(-c\*\*2\*x\*\*2 + 1)\*asin(c\*x)/(9\*c) - 4\*b\*\*2\*x/(9\*c\*\*2) + 4\*b\*\*2\*sqrt(-c\*\*2\*x\*\*2 + 1)\*asin(c\*x)/(9\*c\*\*3), Ne(c, 0)), (a\*\*2\*x\*\*3/3, True))

### 3.149 $\int x \left( a + b \sin^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=76

$$\frac{bx\sqrt{1-c^2x^2} (a + b \sin^{-1}(cx))}{2c} - \frac{(a + b \sin^{-1}(cx))^2}{4c^2} + \frac{1}{2}x^2 (a + b \sin^{-1}(cx))^2 - \frac{1}{4}b^2x^2$$

[Out]  $-1/4*b^2*x^2-1/4*(a+b*\arcsin(c*x))^2/c^2+1/2*x^2*(a+b*\arcsin(c*x))^2+1/2*b*x*(a+b*\arcsin(c*x))*(-c^2*x^2+1)^(1/2)/c$

**Rubi [A]** time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4627, 4707, 4641, 30}

$$\frac{bx\sqrt{1-c^2x^2} (a + b \sin^{-1}(cx))}{2c} - \frac{(a + b \sin^{-1}(cx))^2}{4c^2} + \frac{1}{2}x^2 (a + b \sin^{-1}(cx))^2 - \frac{1}{4}b^2x^2$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcSin[c\*x])^2,x]

[Out]  $-(b^2*x^2)/4 + (b*x*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c) - (a + b*ArcSin[c*x])^2/(4*c^2) + (x^2*(a + b*ArcSin[c*x])^2)/2$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((f\_)\*(x\_))^(m\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[((f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n)/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int x(a + b \sin^{-1}(cx))^2 dx &= \frac{1}{2}x^2(a + b \sin^{-1}(cx))^2 - (bc) \int \frac{x^2(a + b \sin^{-1}(cx))}{\sqrt{1 - c^2x^2}} dx \\ &= \frac{bx\sqrt{1 - c^2x^2}(a + b \sin^{-1}(cx))}{2c} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^2 - \frac{1}{2}b^2 \int x dx - \frac{b \int \frac{a+b \sin^{-1}(cx)}{\sqrt{1 - c^2x^2}} dx}{2c} \\ &= -\frac{1}{4}b^2x^2 + \frac{bx\sqrt{1 - c^2x^2}(a + b \sin^{-1}(cx))}{2c} - \frac{(a + b \sin^{-1}(cx))^2}{4c^2} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^2 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 73, normalized size = 0.96

$$\frac{-2c^2x^2(a + b \sin^{-1}(cx))^2 - 2bcx\sqrt{1 - c^2x^2}(a + b \sin^{-1}(cx)) + (a + b \sin^{-1}(cx))^2 + b^2c^2x^2}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcSin[c\*x])^2,x]

[Out] -1/4\*(b^2\*c^2\*x^2 - 2\*b\*c\*x\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x]) + (a + b\*ArcSin[c\*x])^2 - 2\*c^2\*x^2\*(a + b\*ArcSin[c\*x])^2)/c^2

**fricas [A]** time = 1.09, size = 99, normalized size = 1.30

$$\frac{(2a^2 - b^2)c^2x^2 + (2b^2c^2x^2 - b^2) \arcsin(cx)^2 + 2(2abc^2x^2 - ab) \arcsin(cx) + 2(b^2cx \arcsin(cx) + abcx)\sqrt{-c^2x^2 + 1}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] 1/4\*((2\*a^2 - b^2)\*c^2\*x^2 + (2\*b^2\*c^2\*x^2 - b^2)\*arcsin(c\*x)^2 + 2\*(2\*a\*b\*c^2\*x^2 - a\*b)\*arcsin(c\*x) + 2\*(b^2\*c\*x\*arcsin(c\*x) + a\*b\*c\*x)\*sqrt(-c^2\*x^2 + 1))/c^2

**giac [B]** time = 0.26, size = 155, normalized size = 2.04

$$\frac{\sqrt{-c^2x^2 + 1} b^2 x \arcsin(cx)}{2c} + \frac{(c^2x^2 - 1)b^2 \arcsin(cx)^2}{2c^2} + \frac{\sqrt{-c^2x^2 + 1} abx}{2c} + \frac{(c^2x^2 - 1)ab \arcsin(cx)}{c^2} + \frac{b^2 \arcsin(cx)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] 1/2\*sqrt(-c^2\*x^2 + 1)\*b^2\*x\*arcsin(c\*x)/c + 1/2\*(c^2\*x^2 - 1)\*b^2\*arcsin(c\*x)^2/c^2 + 1/2\*sqrt(-c^2\*x^2 + 1)\*a\*b\*x/c + (c^2\*x^2 - 1)\*a\*b\*arcsin(c\*x)/c^2 + 1/4\*b^2\*arcsin(c\*x)^2/c^2 + 1/2\*(c^2\*x^2 - 1)\*a^2/c^2 - 1/4\*(c^2\*x^2 - 1)\*b^2/c^2 + 1/2\*a\*b\*arcsin(c\*x)/c^2 - 1/8\*b^2/c^2

**maple [A]** time = 0.04, size = 120, normalized size = 1.58

$$\frac{\frac{a^2c^2x^2}{2} + b^2 \left( \frac{(c^2x^2-1) \arcsin(cx)^2}{2} + \frac{\arcsin(cx)(cx\sqrt{-c^2x^2+1} + \arcsin(cx))}{2} - \frac{\arcsin(cx)^2}{4} - \frac{c^2x^2}{4} \right) + 2ab \left( \frac{c^2x^2 \arcsin(cx)}{2} + \frac{cx\sqrt{-c^2x^2+1}}{4} \right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsin(c\*x))^2,x)

[Out]  $1/c^2*(1/2*a^2*c^2*x^2+b^2*(1/2*(c^2*x^2-1)*\arcsin(c*x))^2+1/2*\arcsin(c*x)*(c*x*(-c^2*x^2+1)^{(1/2)}+\arcsin(c*x))-1/4*\arcsin(c*x)^2-1/4*c^2*x^2)+2*a*b*(1/2*c^2*x^2*\arcsin(c*x)+1/4*c*x*(-c^2*x^2+1)^{(1/2)}-1/4*\arcsin(c*x))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2x^2 + \frac{1}{2}\left(2x^2 \arcsin(cx) + c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3}\right)\right)ab + \frac{1}{2}\left(x^2 \arctan\left(cx, \sqrt{cx+1}\sqrt{-cx+1}\right)^2 + 2c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out]  $1/2*a^2*x^2 + 1/2*(2*x^2*\arcsin(c*x) + c*(\sqrt{-c^2*x^2 + 1}*x/c^2 - \arcsin(c*x)/c^3))*a*b + 1/2*(x^2*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1})^2 + 2*c*\int(\sqrt{c*x + 1}*\sqrt{-c*x + 1}*x^2*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1}))/(\sqrt{-c*x^2 - 1}), x)*b^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x(a + b \operatorname{asin}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asin(c\*x))^2,x)

[Out] int(x\*(a + b\*asin(c\*x))^2, x)

**sympy** [A] time = 0.55, size = 126, normalized size = 1.66

$$\begin{cases} \frac{a^2x^2}{2} + abx^2 \operatorname{asin}(cx) + \frac{abx\sqrt{-c^2x^2+1}}{2c} - \frac{ab \operatorname{asin}(cx)}{2c^2} + \frac{b^2x^2 \operatorname{asin}^2(cx)}{2} - \frac{b^2x^2}{4} + \frac{b^2x\sqrt{-c^2x^2+1} \operatorname{asin}(cx)}{2c} - \frac{b^2 \operatorname{asin}^2(cx)}{4c^2} & \text{for } c \neq 0 \\ \frac{a^2x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asin(c\*x))\*\*2,x)

[Out] Piecewise((a\*\*2\*x\*\*2/2 + a\*b\*x\*\*2\*asin(c\*x) + a\*b\*x\*sqrt(-c\*\*2\*x\*\*2 + 1)/(2\*c) - a\*b\*asin(c\*x)/(2\*c\*\*2) + b\*\*2\*x\*\*2\*asin(c\*x)\*\*2/2 - b\*\*2\*x\*\*2/4 + b\*\*2\*x\*sqrt(-c\*\*2\*x\*\*2 + 1)\*asin(c\*x)/(2\*c) - b\*\*2\*asin(c\*x)\*\*2/(4\*c\*\*2), Ne(c, 0)), (a\*\*2\*x\*\*2/2, True))

### 3.150 $\int (a + b \sin^{-1}(cx))^2 dx$

**Optimal.** Leaf size=47

$$\frac{2b\sqrt{1-c^2x^2} (a + b \sin^{-1}(cx))}{c} + x(a + b \sin^{-1}(cx))^2 - 2b^2x$$

[Out]  $-2*b^2*x + x*(a + b*\arcsin(c*x))^2 + 2*b*(a + b*\arcsin(c*x))*(-c^2*x^2 + 1)^{(1/2)}/c$

**Rubi [A]** time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4619, 4677, 8}

$$\frac{2b\sqrt{1-c^2x^2} (a + b \sin^{-1}(cx))}{c} + x(a + b \sin^{-1}(cx))^2 - 2b^2x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^2, x]

[Out]  $-2*b^2*x + (2*b*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x]))/c + x*(a + b*\text{ArcSin}[c*x])^2$

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^n\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int (a + b \sin^{-1}(cx))^2 dx &= x(a + b \sin^{-1}(cx))^2 - (2bc) \int \frac{x(a + b \sin^{-1}(cx))}{\sqrt{1 - c^2x^2}} dx \\ &= \frac{2b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))}{c} + x(a + b \sin^{-1}(cx))^2 - (2b^2) \int 1 dx \\ &= -2b^2x + \frac{2b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))}{c} + x(a + b \sin^{-1}(cx))^2 \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 47, normalized size = 1.00

$$\frac{2b\sqrt{1-c^2x^2} (a + b \sin^{-1}(cx))}{c} + x(a + b \sin^{-1}(cx))^2 - 2b^2x$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^2,x]

[Out]  $-2*b^2*x + (2*b*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x]))/c + x*(a + b*\text{ArcSin}[c*x])^2$

**fricas** [A] time = 0.40, size = 65, normalized size = 1.38

$$\frac{b^2cx \arcsin(cx)^2 + 2abcx \arcsin(cx) + (a^2 - 2b^2)cx + 2\sqrt{-c^2x^2 + 1}(b^2 \arcsin(cx) + ab)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out]  $(b^2*c*x*\arcsin(c*x)^2 + 2*a*b*c*x*\arcsin(c*x) + (a^2 - 2*b^2)*c*x + 2*\text{sqrt}(-c^2*x^2 + 1)*(b^2*\arcsin(c*x) + a*b))/c$

**giac** [A] time = 0.40, size = 75, normalized size = 1.60

$$b^2x \arcsin(cx)^2 + 2abx \arcsin(cx) + a^2x - 2b^2x + \frac{2\sqrt{-c^2x^2 + 1}b^2 \arcsin(cx)}{c} + \frac{2\sqrt{-c^2x^2 + 1}ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out]  $b^2*x*\arcsin(c*x)^2 + 2*a*b*x*\arcsin(c*x) + a^2*x - 2*b^2*x + 2*\text{sqrt}(-c^2*x^2 + 1)*b^2*\arcsin(c*x)/c + 2*\text{sqrt}(-c^2*x^2 + 1)*a*b/c$

**maple** [A] time = 0.04, size = 72, normalized size = 1.53

$$\frac{cx a^2 + b^2 \left( cx \arcsin(cx)^2 - 2cx + 2 \arcsin(cx) \sqrt{-c^2x^2 + 1} \right) + 2ab \left( cx \arcsin(cx) + \sqrt{-c^2x^2 + 1} \right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^2,x)

[Out]  $1/c*(c*x*a^2+b^2*(c*x*\arcsin(c*x)^2-2*c*x+2*\arcsin(c*x)*(-c^2*x^2+1)^(1/2))+2*a*b*(c*x*\arcsin(c*x)+(-c^2*x^2+1)^(1/2)))$

**maxima** [A] time = 0.47, size = 72, normalized size = 1.53

$$b^2x \arcsin(cx)^2 - 2b^2 \left( x - \frac{\sqrt{-c^2x^2 + 1} \arcsin(cx)}{c} \right) + a^2x + \frac{2 \left( cx \arcsin(cx) + \sqrt{-c^2x^2 + 1} \right) ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out]  $b^2*x*\arcsin(c*x)^2 - 2*b^2*(x - \text{sqrt}(-c^2*x^2 + 1)*\arcsin(c*x)/c) + a^2*x + 2*(c*x*\arcsin(c*x) + \text{sqrt}(-c^2*x^2 + 1))*a*b/c$

**mupad** [B] time = 0.35, size = 142, normalized size = 3.02

$$\begin{cases} b^2 \left( x \left( \arcsin(cx)^2 - 2 \right) + 2 \arcsin(cx) \sqrt{\frac{1}{c^2} - x^2} \right) + a^2x + \frac{2ab \left( \sqrt{1-c^2x^2} + cx \arcsin(cx) \right)}{c} & \text{if } 0 < c \\ a^2x + b^2x \left( \arcsin(cx)^2 - 2 \right) + \frac{2b^2 \arcsin(cx) \sqrt{1-c^2x^2}}{c} + \frac{2ab \left( \sqrt{1-c^2x^2} + cx \arcsin(cx) \right)}{c} & \text{if } -0 < c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(c*x))^2,x)
```

```
[Out] piecewise(0 < c, b^2*(x*(asin(c*x)^2 - 2) + 2*asin(c*x)*(1/c^2 - x^2)^(1/2)
) + a^2*x + (2*a*b*((- c^2*x^2 + 1)^(1/2) + c*x*asin(c*x)))/c, ~0 < c, a^2*
x + b^2*x*(asin(c*x)^2 - 2) + (2*b^2*asin(c*x)*(- c^2*x^2 + 1)^(1/2))/c + (
2*a*b*((- c^2*x^2 + 1)^(1/2) + c*x*asin(c*x)))/c)
```

**sympy** [A] time = 0.27, size = 82, normalized size = 1.74

$$\begin{cases} a^2x + 2abx \operatorname{asin}(cx) + \frac{2ab\sqrt{-c^2x^2+1}}{c} + b^2x \operatorname{asin}^2(cx) - 2b^2x + \frac{2b^2\sqrt{-c^2x^2+1} \operatorname{asin}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asin(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asin(c*x) + 2*a*b*sqrt(-c**2*x**2 + 1)/c + b**2
*x*asin(c*x)**2 - 2*b**2*x + 2*b**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/c, Ne(c,
0)), (a**2*x, True))
```



$$3.151 \quad \int \frac{(a+b \sin^{-1}(cx))^2}{x} dx$$

**Optimal.** Leaf size=90

$$-ib\text{Li}_2(e^{2i \sin^{-1}(cx)})(a+b \sin^{-1}(cx)) - \frac{i(a+b \sin^{-1}(cx))^3}{3b} + \log(1 - e^{2i \sin^{-1}(cx)})(a+b \sin^{-1}(cx))^2 + \frac{1}{2}b^2\text{Li}_3(e^{2i \sin^{-1}(cx)})$$

[Out]  $-1/3*I*(a+b*\arcsin(c*x))^3/b+(a+b*\arcsin(c*x))^2*\ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*b*(a+b*\arcsin(c*x))*\text{polylog}(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*b^2*\text{polylog}(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)$

**Rubi [A]** time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {4625, 3717, 2190, 2531, 2282, 6589}

$$-ib\text{PolyLog}(2, e^{2i \sin^{-1}(cx)})(a+b \sin^{-1}(cx)) + \frac{1}{2}b^2\text{PolyLog}(3, e^{2i \sin^{-1}(cx)}) - \frac{i(a+b \sin^{-1}(cx))^3}{3b} + \log(1 - e^{2i \sin^{-1}(cx)})$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^2/x, x]

[Out]  $((-I/3)*(a + b*\text{ArcSin}[c*x])^3)/b + (a + b*\text{ArcSin}[c*x])^2*\text{Log}[1 - E^{((2*I)*\text{ArcSin}[c*x])}] - I*b*(a + b*\text{ArcSin}[c*x])* \text{PolyLog}[2, E^{((2*I)*\text{ArcSin}[c*x])}] + (b^2*\text{PolyLog}[3, E^{((2*I)*\text{ArcSin}[c*x])}])/2$

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^(c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 3717

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \sin^{-1}(cx))^2}{x} dx &= \text{Subst} \left( \int (a + bx)^2 \cot(x) dx, x, \sin^{-1}(cx) \right) \\ &= -\frac{i(a + b \sin^{-1}(cx))^3}{3b} - 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)^2}{1 - e^{2ix}} dx, x, \sin^{-1}(cx) \right) \\ &= -\frac{i(a + b \sin^{-1}(cx))^3}{3b} + (a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)}) - (2b) \text{Subst} \left( \int (a + bx) \right) \\ &= -\frac{i(a + b \sin^{-1}(cx))^3}{3b} + (a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)}) - ib(a + b \sin^{-1}(cx)) \text{Li}_2(e^{-2i \sin^{-1}(cx)}) \\ &= -\frac{i(a + b \sin^{-1}(cx))^3}{3b} + (a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)}) - ib(a + b \sin^{-1}(cx)) \text{Li}_2(e^{-2i \sin^{-1}(cx)}) \\ &= -\frac{i(a + b \sin^{-1}(cx))^3}{3b} + (a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)}) - ib(a + b \sin^{-1}(cx)) \text{Li}_2(e^{-2i \sin^{-1}(cx)}) \end{aligned}$$

**Mathematica** [A] time = 0.15, size = 143, normalized size = 1.59

$$a^2 \log(cx) + 2ab \left( \sin^{-1}(cx) \log(1 - e^{2i \sin^{-1}(cx)}) - \frac{1}{2} i (\sin^{-1}(cx))^2 + \text{Li}_2(e^{2i \sin^{-1}(cx)}) \right) + b^2 \left( i \sin^{-1}(cx) \text{Li}_2(e^{-2i \sin^{-1}(cx)}) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSin[c\*x])^2/x,x]

[Out] a^2\*Log[c\*x] + 2\*a\*b\*(ArcSin[c\*x]\*Log[1 - E^((2\*I)\*ArcSin[c\*x])] - (I/2)\*(ArcSin[c\*x]^2 + PolyLog[2, E^((2\*I)\*ArcSin[c\*x])])) + b^2\*((-1/24\*I)\*Pi^3 + (I/3)\*ArcSin[c\*x]^3 + ArcSin[c\*x]^2\*Log[1 - E^((-2\*I)\*ArcSin[c\*x])] + I\*ArcSin[c\*x]\*PolyLog[2, E^((-2\*I)\*ArcSin[c\*x])] + PolyLog[3, E^((-2\*I)\*ArcSin[c\*x])])/2)

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/x,x, algorithm="fricas")

[Out] integral((b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2)/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/x,x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^2/x, x)

**maple** [B] time = 0.05, size = 319, normalized size = 3.54

$$a^2 \ln(cx) - \frac{ib^2 \arcsin(cx)^3}{3} + b^2 \arcsin(cx)^2 \ln\left(1 + icx + \sqrt{-c^2x^2 + 1}\right) - 2ib^2 \arcsin(cx) \operatorname{polylog}\left(2, -icx - \sqrt{-c^2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^2/x,x)

[Out] a^2\*ln(c\*x)-1/3\*I\*b^2\*arcsin(c\*x)^3+b^2\*arcsin(c\*x)^2\*ln(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))-2\*I\*b^2\*arcsin(c\*x)\*polylog(2,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+2\*b^2\*polylog(3,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+b^2\*arcsin(c\*x)^2\*ln(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))-2\*I\*b^2\*arcsin(c\*x)\*polylog(2,I\*c\*x+(-c^2\*x^2+1)^(1/2))+2\*b^2\*polylog(3,I\*c\*x+(-c^2\*x^2+1)^(1/2))-I\*a\*b\*arcsin(c\*x)^2-2\*I\*a\*b\*polylog(2,I\*c\*x+(-c^2\*x^2+1)^(1/2))-2\*I\*a\*b\*polylog(2,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+2\*a\*b\*arcsin(c\*x)\*ln(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))+2\*a\*b\*arcsin(c\*x)\*ln(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \log(x) + \int \frac{b^2 \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^2 + 2ab \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/x,x, algorithm="maxima")

[Out] a^2\*log(x) + integrate((b^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*a\*b\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asin}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^2/x,x)

[Out] int((a + b\*asin(c\*x))^2/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*2/x,x)

[Out] Integral((a + b\*asin(c\*x))\*\*2/x, x)

$$3.152 \quad \int \frac{(a+b \sin^{-1}(cx))^2}{x^2} dx$$

**Optimal.** Leaf size=81

$$-\frac{(a+b \sin^{-1}(cx))^2}{x} - 4bc \tanh^{-1}(e^{i \sin^{-1}(cx)})(a+b \sin^{-1}(cx)) + 2ib^2 c \operatorname{Li}_2(-e^{i \sin^{-1}(cx)}) - 2ib^2 c \operatorname{Li}_2(e^{i \sin^{-1}(cx)})$$

[Out]  $-(a+b \operatorname{arcsin}(c*x))^2/x - 4*b*c*(a+b \operatorname{arcsin}(c*x))*\operatorname{arctanh}(I*c*x+(-c^2*x^2+1)^(1/2))+2*I*b^2*c*\operatorname{polylog}(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*b^2*c*\operatorname{polylog}(2,I*c*x+(-c^2*x^2+1)^(1/2))$

**Rubi [A]** time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {4627, 4709, 4183, 2279, 2391}

$$2ib^2 c \operatorname{PolyLog}(2, -e^{i \sin^{-1}(cx)}) - 2ib^2 c \operatorname{PolyLog}(2, e^{i \sin^{-1}(cx)}) - \frac{(a+b \sin^{-1}(cx))^2}{x} - 4bc \tanh^{-1}(e^{i \sin^{-1}(cx)})(a+b \sin^{-1}(cx))$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcSin[c*x])^2/x^2, x]`

[Out]  $-\left((a + b \operatorname{ArcSin}[c*x])^2/x\right) - 4*b*c*(a + b \operatorname{ArcSin}[c*x])* \operatorname{ArcTanh}[E^{(I \operatorname{ArcSin}[c*x])}] + (2*I)*b^2*c*\operatorname{PolyLog}[2, -E^{(I \operatorname{ArcSin}[c*x])}] - (2*I)*b^2*c*\operatorname{PolyLog}[2, E^{(I \operatorname{ArcSin}[c*x])}]$

#### Rule 2279

`Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))]^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

#### Rule 2391

`Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

#### Rule 4183

`Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

#### Rule 4627

`Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_)*(x_))^(m_), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*ArcSin[c*x])^n)/(d*(m+1)), x] - Dist[(b*c*n)/(d*(m+1)), Int[((d*x)^(m+1)*(a + b*ArcSin[c*x])^(n-1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

#### Rule 4709

`Int[(((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(x_)^(m_))/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Dist[1/(c^(m+1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]`

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sin^{-1}(cx))^2}{x^2} dx &= -\frac{(a + b \sin^{-1}(cx))^2}{x} + (2bc) \int \frac{a + b \sin^{-1}(cx)}{x\sqrt{1 - c^2x^2}} dx \\
&= -\frac{(a + b \sin^{-1}(cx))^2}{x} + (2bc) \text{Subst} \left( \int (a + bx) \csc(x) dx, x, \sin^{-1}(cx) \right) \\
&= -\frac{(a + b \sin^{-1}(cx))^2}{x} - 4bc (a + b \sin^{-1}(cx)) \tanh^{-1} (e^{i \sin^{-1}(cx)}) - (2b^2c) \text{Subst} \left( \int \frac{1}{1 - c^2x^2} dx, x, \sin^{-1}(cx) \right) \\
&= -\frac{(a + b \sin^{-1}(cx))^2}{x} - 4bc (a + b \sin^{-1}(cx)) \tanh^{-1} (e^{i \sin^{-1}(cx)}) + (2ib^2c) \text{Subst} \left( \int \frac{1}{1 - c^2x^2} dx, x, \sin^{-1}(cx) \right) \\
&= -\frac{(a + b \sin^{-1}(cx))^2}{x} - 4bc (a + b \sin^{-1}(cx)) \tanh^{-1} (e^{i \sin^{-1}(cx)}) + 2ib^2c \text{Li}_2 (-e^{i \sin^{-1}(cx)})
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 126, normalized size = 1.56

$$\frac{a^2 + 2ab \left( cx \tanh^{-1} \left( \sqrt{1 - c^2x^2} \right) + \sin^{-1}(cx) \right) - ib^2 \left( 2cx \text{Li}_2 \left( -e^{i \sin^{-1}(cx)} \right) - 2cx \text{Li}_2 \left( e^{i \sin^{-1}(cx)} \right) + i \sin^{-1}(cx) \right)}{x}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSin[c\*x])^2/x^2, x]

[Out] -((a^2 + 2\*a\*b\*(ArcSin[c\*x] + c\*x\*ArcTanh[Sqrt[1 - c^2\*x^2]]) - I\*b^2\*(I\*ArcSin[c\*x]\*(ArcSin[c\*x] + 2\*c\*x\*(-Log[1 - E^(I\*ArcSin[c\*x]])] + Log[1 + E^(I\*ArcSin[c\*x]]))) + 2\*c\*x\*PolyLog[2, -E^(I\*ArcSin[c\*x])] - 2\*c\*x\*PolyLog[2, E^(I\*ArcSin[c\*x])]))/x

**fricas [F]** time = 0.54, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/x^2, x, algorithm="fricas")

[Out] integral((b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2)/x^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/x^2, x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^2/x^2, x)

**maple [A]** time = 0.03, size = 171, normalized size = 2.11

$$-\frac{a^2}{x} - \frac{b^2 \arcsin(cx)^2}{x} + 2cb^2 \arcsin(cx) \ln \left( 1 - icx - \sqrt{-c^2x^2 + 1} \right) - 2cb^2 \arcsin(cx) \ln \left( 1 + icx + \sqrt{-c^2x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^2/x^2,x)

[Out]  $-a^2/x - b^2/x * \arcsin(cx)^2 + 2 * c * b^2 * \arcsin(cx) * \ln(1 - I * c * x - (-c^2 * x^2 + 1)^{1/2}) - 2 * c * b^2 * \arcsin(cx) * \ln(1 + I * c * x + (-c^2 * x^2 + 1)^{1/2}) + 2 * I * c * b^2 * \operatorname{dilog}(1 + I * c * x + (-c^2 * x^2 + 1)^{1/2}) - 2 * I * c * b^2 * \operatorname{dilog}(1 - I * c * x - (-c^2 * x^2 + 1)^{1/2}) - 2 * a * b / x * \arcsin(cx) - 2 * c * a * b * \operatorname{arctanh}(1 / (-c^2 * x^2 + 1)^{1/2})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2 \left( c \log \left( \frac{2 \sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) ab - \frac{\left( 2 cx \int \frac{\sqrt{-cx+1} \arctan(cx, \sqrt{cx+1} \sqrt{-cx+1})}{\sqrt{cx+1} (cx-1)x} dx + \arctan(cx, \sqrt{cx+1}) \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/x^2,x, algorithm="maxima")

[Out]  $-2 * (c * \log(2 * \sqrt{-c^2 * x^2 + 1} / \operatorname{abs}(x) + 2 / \operatorname{abs}(x)) + \arcsin(cx) / x) * a * b - (2 * c * x * \operatorname{integrate}(\sqrt{cx + 1} * \sqrt{-cx + 1} * \operatorname{arctan2}(cx, \sqrt{cx + 1} * \sqrt{-cx + 1}) / (c^2 * x^3 - x), x) + \operatorname{arctan2}(cx, \sqrt{cx + 1} * \sqrt{-cx + 1}))^2 * b^2 / x - a^2 / x$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asin}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^2/x^2,x)

[Out] int((a + b\*asin(c\*x))^2/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*2/x\*\*2,x)

[Out] Integral((a + b\*asin(c\*x))\*\*2/x\*\*2, x)

### 3.153 $\int x^2 (a + b \sin^{-1}(cx))^3 dx$

**Optimal.** Leaf size=178

$$-\frac{4ab^2x}{3c^2} - \frac{2}{9}b^2x^3(a + b \sin^{-1}(cx)) + \frac{bx^2\sqrt{1-c^2x^2}(a + b \sin^{-1}(cx))^2}{3c} + \frac{2b\sqrt{1-c^2x^2}(a + b \sin^{-1}(cx))^2}{3c^3} + \frac{1}{3}x^3(a + b \sin^{-1}(cx))^3$$

[Out]  $-4/3*a*b^2*x/c^2 + 2/27*b^3*(-c^2*x^2+1)^{(3/2)}/c^3 - 4/3*b^3*x*\arcsin(c*x)/c^2 - 2/9*b^2*x^3*(a+b*\arcsin(c*x)) + 1/3*x^3*(a+b*\arcsin(c*x))^3 - 14/9*b^3*(-c^2*x^2+1)^{(1/2)}/c^3 + 2/3*b*(a+b*\arcsin(c*x))^2*(-c^2*x^2+1)^{(1/2)}/c^3 + 1/3*b*x^2*(a+b*\arcsin(c*x))^2*(-c^2*x^2+1)^{(1/2)}/c$

**Rubi [A]** time = 0.30, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4627, 4707, 4677, 4619, 261, 266, 43}

$$-\frac{4ab^2x}{3c^2} - \frac{2}{9}b^2x^3(a + b \sin^{-1}(cx)) + \frac{bx^2\sqrt{1-c^2x^2}(a + b \sin^{-1}(cx))^2}{3c} + \frac{2b\sqrt{1-c^2x^2}(a + b \sin^{-1}(cx))^2}{3c^3} + \frac{1}{3}x^3(a + b \sin^{-1}(cx))^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcSin[c\*x])^3,x]

[Out]  $(-4*a*b^2*x)/(3*c^2) - (14*b^3*\text{Sqrt}[1 - c^2*x^2])/(9*c^3) + (2*b^3*(1 - c^2*x^2)^{(3/2)})/(27*c^3) - (4*b^3*x*\text{ArcSin}[c*x])/(3*c^2) - (2*b^2*x^3*(a + b*\text{ArcSin}[c*x]))/9 + (2*b*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^2)/(3*c^3) + (b*x^2*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^2)/(3*c) + (x^3*(a + b*\text{ArcSin}[c*x])^3)/3$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x]

$*x^2], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

### Rule 4677

$\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)]^{(n_.)}*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p + 1)}*(a + b*\text{ArcSin}[c*x])^n]/(2*e*(p + 1)), x] + \text{Dist}[(b*n*d^{\text{IntPart}[p]}*(d + e*x^2)^{\text{FracPart}[p]})/(2*c*(p + 1)*(1 - c^2*x^2)^{\text{FracPart}[p]}), \text{Int}[(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

### Rule 4707

$\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)]^{(n_.)}*((f_.)*(x_.))^{(m_.)}/\text{Sqrt}[d_. + (e_.)*(x_.)^2], x\_Symbol] \rightarrow \text{Simp}[(f*(f*x)^{(m - 1)}*\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcSin}[c*x])^n)/(e*m), x] + (\text{Dist}[(f^2*(m - 1))/(c^2*m), \text{Int}[(f*x)^{(m - 2)}*(a + b*\text{ArcSin}[c*x])^n]/\text{Sqrt}[d + e*x^2], x], x] + \text{Dist}[(b*f*n*\text{Sqrt}[1 - c^2*x^2])/(c*m*\text{Sqrt}[d + e*x^2]), \text{Int}[(f*x)^{(m - 1)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{IntegerQ}[m]$

### Rubi steps

$$\begin{aligned} \int x^2 (a + b \sin^{-1}(cx))^3 dx &= \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^3 - (bc) \int \frac{x^3 (a + b \sin^{-1}(cx))^2}{\sqrt{1 - c^2x^2}} dx \\ &= \frac{bx^2\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{3c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^3 - \frac{1}{3}(2b^2) \int x^2 (a + b \sin^{-1}(cx))^2 dx \\ &= -\frac{2}{9}b^2x^3 (a + b \sin^{-1}(cx)) + \frac{2b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{3c^3} + \frac{bx^2\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))}{3c} \\ &= -\frac{4ab^2x}{3c^2} - \frac{2}{9}b^2x^3 (a + b \sin^{-1}(cx)) + \frac{2b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{3c^3} + \frac{bx^2\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))}{3c} \\ &= -\frac{4ab^2x}{3c^2} - \frac{4b^3x \sin^{-1}(cx)}{3c^2} - \frac{2}{9}b^2x^3 (a + b \sin^{-1}(cx)) + \frac{2b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))}{3c^3} \\ &= -\frac{4ab^2x}{3c^2} - \frac{14b^3\sqrt{1 - c^2x^2}}{9c^3} + \frac{2b^3(1 - c^2x^2)^{3/2}}{27c^3} - \frac{4b^3x \sin^{-1}(cx)}{3c^2} - \frac{2}{9}b^2x^3 (a + b \sin^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.38, size = 163, normalized size = 0.92

$$\frac{1}{27} \left( \frac{b \left( 9c^2x^2\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2 + 18 \left( \sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2 - 2b \left( acx + b\sqrt{1 - c^2x^2} + bcx \sin^{-1}(cx) \right) \right) \right)}{c^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcSin[c\*x])^3,x]

[Out] (9\*x^3\*(a + b\*ArcSin[c\*x])^3 + (b\*(9\*c^2\*x^2\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^2 - 2\*b\*(b\*Sqrt[1 - c^2\*x^2]\*(2 + c^2\*x^2) + 3\*c^3\*x^3\*(a + b\*ArcSin[c\*x])) + 18\*(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^2 - 2\*b\*(a\*c\*x + b\*Sqrt[1 - c^2\*x^2] + b\*c\*x\*ArcSin[c\*x]))))/c^3)/27



**fricas** [A] time = 1.77, size = 194, normalized size = 1.09

$$9b^3c^3x^3 \arcsin(cx)^3 + 27ab^2c^3x^3 \arcsin(cx)^2 + 3(3a^3 - 2ab^2)c^3x^3 - 36ab^2cx + 3((9a^2b - 2b^3)c^3x^3 - 12b^3c^3x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out]  $\frac{1}{27}(9b^3c^3x^3 \arcsin(cx)^3 + 27a^2b^2c^3x^3 \arcsin(cx)^2 + 3(3a^3 - 2a^2b^2)c^3x^3 - 36a^2b^2cx + 3((9a^2b - 2b^3)c^3x^3 - 12b^3c^3x^3) \arcsin(cx) + ((9a^2b - 2b^3)c^2x^2 + 18a^2b^2 - 40b^3 + 9(b^3c^2x^2 + 2b^3) \arcsin(cx)^2 + 18(a^2b^2c^2x^2 + 2a^2b^2) \arcsin(cx)) \sqrt{-c^2x^2 + 1})/c^3$

**giac** [B] time = 0.59, size = 368, normalized size = 2.07

$$\frac{1}{3}a^3x^3 + \frac{(c^2x^2 - 1)b^3x \arcsin(cx)^3}{3c^2} + \frac{(c^2x^2 - 1)ab^2x \arcsin(cx)^2}{c^2} + \frac{b^3x \arcsin(cx)^3}{3c^2} + \frac{(c^2x^2 - 1)a^2bx \arcsin(cx)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out]  $\frac{1}{3}a^3x^3 + \frac{1}{3}(c^2x^2 - 1)b^3x \arcsin(cx)^3/c^2 + (c^2x^2 - 1)a^2b^2x \arcsin(cx)^2/c^2 + \frac{1}{3}b^3x \arcsin(cx)^3/c^2 + (c^2x^2 - 1)a^2b^2x \arcsin(cx)/c^2 - \frac{2}{9}(c^2x^2 - 1)b^3x \arcsin(cx)/c^2 + a^2b^2x \arcsin(cx)^2/c^2 - \frac{1}{3}(-c^2x^2 + 1)^{(3/2)}b^3 \arcsin(cx)^2/c^3 - \frac{2}{9}(c^2x^2 - 1)a^2b^2x/c^2 + a^2b^2x \arcsin(cx)/c^2 - \frac{14}{9}b^3x \arcsin(cx)/c^2 - \frac{2}{3}(-c^2x^2 + 1)^{(3/2)}a^2b^2 \arcsin(cx)/c^3 + \sqrt{-c^2x^2 + 1}b^3 \arcsin(cx)^2/c^3 - \frac{14}{9}a^2b^2x/c^2 - \frac{1}{3}(-c^2x^2 + 1)^{(3/2)}a^2b/c^3 + \frac{2}{27}(-c^2x^2 + 1)^{(3/2)}b^3/c^3 + 2\sqrt{-c^2x^2 + 1}a^2b^2 \arcsin(cx)/c^3 + \sqrt{-c^2x^2 + 1}a^2b/c^3 - \frac{14}{9}\sqrt{-c^2x^2 + 1}b^3/c^3$

**maple** [A] time = 0.03, size = 235, normalized size = 1.32

$$\frac{a^3c^3x^3}{3} + b^3 \left( \frac{c^3x^3 \arcsin(cx)^3}{3} + \frac{\arcsin(cx)^2(c^2x^2+2)\sqrt{-c^2x^2+1}}{3} - \frac{4\sqrt{-c^2x^2+1}}{3} - \frac{4cx \arcsin(cx)}{3} - \frac{2c^3x^3 \arcsin(cx)}{9} - \frac{2(c^2x^2+2)\sqrt{-c^2x^2+1}}{27} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsin(c\*x))^3,x)

[Out]  $\frac{1}{c^3}(\frac{1}{3}a^3c^3x^3 + b^3(\frac{1}{3}c^3x^3 \arcsin(cx)^3 + \frac{1}{3} \arcsin(cx)^2(c^2x^2+2)\sqrt{-c^2x^2+1} - \frac{4\sqrt{-c^2x^2+1}}{3} - \frac{4cx \arcsin(cx)}{3} - \frac{2c^3x^3 \arcsin(cx)}{9} - \frac{2(c^2x^2+2)\sqrt{-c^2x^2+1}}{27})) + \frac{2}{9}(-c^2x^2+1)^{(1/2)})$

**maxima** [A] time = 0.48, size = 273, normalized size = 1.53

$$\frac{1}{3}b^3x^3 \arcsin(cx)^3 + ab^2x^3 \arcsin(cx)^2 + \frac{1}{3}a^3x^3 + \frac{1}{3} \left( 3x^3 \arcsin(cx) + c \left( \frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4} \right) \right) a^2b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

```
[Out] 1/3*b^3*x^3*arcsin(c*x)^3 + a*b^2*x^3*arcsin(c*x)^2 + 1/3*a^3*x^3 + 1/3*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*a^2*b + 2/9*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcsin(c*x) - (c^2*x^3 + 6*x)/c^2)*a*b^2 + 1/27*(9*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcsin(c*x)^2 - 2*c*((sqrt(-c^2*x^2 + 1)*x^2 + 20*sqrt(-c^2*x^2 + 1)/c^2)/c^2 + 3*(c^2*x^3 + 6*x)*arcsin(c*x)/c^3))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + b \operatorname{asin}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(c*x))^3,x)
```

```
[Out] int(x^2*(a + b*asin(c*x))^3, x)
```

sympy [A] time = 2.49, size = 328, normalized size = 1.84

$$\left\{ \begin{array}{l} \frac{a^3 x^3}{3} + a^2 b x^3 \operatorname{asin}(cx) + \frac{a^2 b x^2 \sqrt{-c^2 x^2 + 1}}{3c} + \frac{2a^2 b \sqrt{-c^2 x^2 + 1}}{3c^3} + a b^2 x^3 \operatorname{asin}^2(cx) - \frac{2ab^2 x^3}{9} + \frac{2ab^2 x^2 \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx)}{3c} - \frac{4ab^2 x}{3c^2} \\ \frac{a^3 x^3}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asin(c*x))**3,x)
```

```
[Out] Piecewise((a**3*x**3/3 + a**2*b*x**3*asin(c*x) + a**2*b*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*a**2*b*sqrt(-c**2*x**2 + 1)/(3*c**3) + a*b**2*x**3*asin(c*x)**2 - 2*a*b**2*x**3/9 + 2*a*b**2*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(3*c) - 4*a*b**2*x/(3*c**2) + 4*a*b**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(3*c**3) + b**3*x**3*asin(c*x)**3/3 - 2*b**3*x**3*asin(c*x)/9 + b**3*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)**2/(3*c) - 2*b**3*x**2*sqrt(-c**2*x**2 + 1)/(27*c) - 4*b**3*x*asin(c*x)/(3*c**2) + 2*b**3*sqrt(-c**2*x**2 + 1)*asin(c*x)**2/(3*c**3) - 40*b**3*sqrt(-c**2*x**2 + 1)/(27*c**3), Ne(c, 0)), (a**3*x**3/3, True))
```

### 3.154 $\int x \left( a + b \sin^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=125

$$-\frac{3}{4}b^2x^2(a+b\sin^{-1}(cx))+\frac{3bx\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))^2}{4c}-\frac{(a+b\sin^{-1}(cx))^3}{4c^2}+\frac{1}{2}x^2(a+b\sin^{-1}(cx))^3-\frac{3b^3x}{4}$$

[Out]  $3/8*b^3*\arcsin(c*x)/c^2-3/4*b^2*x^2*(a+b*\arcsin(c*x))-1/4*(a+b*\arcsin(c*x))^3/c^2+1/2*x^2*(a+b*\arcsin(c*x))^3-3/8*b^3*x*(-c^2*x^2+1)^(1/2)/c+3/4*b*x*(a+b*\arcsin(c*x))^2*(-c^2*x^2+1)^(1/2)/c$

**Rubi [A]** time = 0.20, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {4627, 4707, 4641, 321, 216}

$$-\frac{3}{4}b^2x^2(a+b\sin^{-1}(cx))+\frac{3bx\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))^2}{4c}-\frac{(a+b\sin^{-1}(cx))^3}{4c^2}+\frac{1}{2}x^2(a+b\sin^{-1}(cx))^3-\frac{3b^3x}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcSin[c\*x])^3,x]

[Out]  $(-3*b^3*x*\text{Sqrt}[1 - c^2*x^2])/(8*c) + (3*b^3*\text{ArcSin}[c*x])/(8*c^2) - (3*b^2*x^2*(a + b*\text{ArcSin}[c*x]))/4 + (3*b*x*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^2)/(4*c) - (a + b*\text{ArcSin}[c*x])^3/(4*c^2) + (x^2*(a + b*\text{ArcSin}[c*x])^3)/2$

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

#### Rule 4707

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_)^(m\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[((f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n)/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1),

$x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{IntegerQ}[m]$

### Rubi steps

$$\begin{aligned} \int x (a + b \sin^{-1}(cx))^3 dx &= \frac{1}{2}x^2 (a + b \sin^{-1}(cx))^3 - \frac{1}{2}(3bc) \int \frac{x^2 (a + b \sin^{-1}(cx))^2}{\sqrt{1 - c^2x^2}} dx \\ &= \frac{3bx\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{4c} + \frac{1}{2}x^2 (a + b \sin^{-1}(cx))^3 - \frac{1}{2}(3b^2) \int x (a + b \sin^{-1}(cx))^2 dx \\ &= -\frac{3}{4}b^2x^2 (a + b \sin^{-1}(cx)) + \frac{3bx\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{4c} - \frac{(a + b \sin^{-1}(cx))^3}{4c^2} + \frac{(a + b \sin^{-1}(cx))^2}{4c} \\ &= -\frac{3b^3x\sqrt{1 - c^2x^2}}{8c} - \frac{3}{4}b^2x^2 (a + b \sin^{-1}(cx)) + \frac{3bx\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{4c} - \frac{(a + b \sin^{-1}(cx))^3}{4c^2} \\ &= -\frac{3b^3x\sqrt{1 - c^2x^2}}{8c} + \frac{3b^3 \sin^{-1}(cx)}{8c^2} - \frac{3}{4}b^2x^2 (a + b \sin^{-1}(cx)) + \frac{3bx\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{4c} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 114, normalized size = 0.91

$$\frac{-3b^2 \left( cx \left( 2acx + b\sqrt{1 - c^2x^2} \right) + b \left( 2c^2x^2 - 1 \right) \sin^{-1}(cx) \right) + 4c^2x^2 (a + b \sin^{-1}(cx))^3 + 6bcx\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{8c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcSin[c\*x])^3,x]

[Out] (6\*b\*c\*x\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^2 - 2\*(a + b\*ArcSin[c\*x])^3 + 4\*c^2\*x^2\*(a + b\*ArcSin[c\*x])^3 - 3\*b^2\*(c\*x\*(2\*a\*c\*x + b\*Sqrt[1 - c^2\*x^2]) + b\*(-1 + 2\*c^2\*x^2)\*ArcSin[c\*x]))/(8\*c^2)

**fricas [A]** time = 0.55, size = 169, normalized size = 1.35

$$\frac{2(2a^3 - 3ab^2)c^2x^2 + 2(2b^3c^2x^2 - b^3)\arcsin(cx)^3 + 6(2ab^2c^2x^2 - ab^2)\arcsin(cx)^2 + 3(2(2a^2b - b^3)c^2x^2 - 2ab^2)\arcsin(cx) + (2a^2b - b^3)c^2x^2}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out] 1/8\*(2\*(2\*a^3 - 3\*a\*b^2)\*c^2\*x^2 + 2\*(2\*b^3\*c^2\*x^2 - b^3)\*arcsin(c\*x)^3 + 6\*(2\*a\*b^2\*c^2\*x^2 - a\*b^2)\*arcsin(c\*x)^2 + 3\*(2\*(2\*a^2\*b - b^3)\*c^2\*x^2 - 2\*a^2\*b + b^3)\*arcsin(c\*x) + 3\*(2\*b^3\*c\*x\*arcsin(c\*x)^2 + 4\*a\*b^2\*c\*x\*arcsin(c\*x) + (2\*a^2\*b - b^3)\*c\*x)\*sqrt(-c^2\*x^2 + 1))/c^2

**giac [B]** time = 0.42, size = 285, normalized size = 2.28

$$\frac{3\sqrt{-c^2x^2 + 1}b^3x\arcsin(cx)^2}{4c} + \frac{(c^2x^2 - 1)b^3\arcsin(cx)^3}{2c^2} + \frac{3\sqrt{-c^2x^2 + 1}ab^2x\arcsin(cx)}{2c} + \frac{3(c^2x^2 - 1)ab^2\arcsin(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out] 3/4\*sqrt(-c^2\*x^2 + 1)\*b^3\*x\*arcsin(c\*x)^2/c + 1/2\*(c^2\*x^2 - 1)\*b^3\*arcsin(c\*x)^3/c^2 + 3/2\*sqrt(-c^2\*x^2 + 1)\*a\*b^2\*x\*arcsin(c\*x)/c + 3/2\*(c^2\*x^2 - 1)\*a\*b^2\*arcsin(c\*x)^2/c^2

$$1) * a * b^2 * \arcsin(c * x)^2 / c^2 + 1/4 * b^3 * \arcsin(c * x)^3 / c^2 + 3/4 * \sqrt{-c^2 * x^2 + 1} * a^2 * b * x / c - 3/8 * \sqrt{-c^2 * x^2 + 1} * b^3 * x / c + 3/2 * (c^2 * x^2 - 1) * a^2 * b * \arcsin(c * x) / c^2 - 3/4 * (c^2 * x^2 - 1) * b^3 * \arcsin(c * x) / c^2 + 3/4 * a * b^2 * \arcsin(c * x)^2 / c^2 + 1/2 * (c^2 * x^2 - 1) * a^3 / c^2 - 3/4 * (c^2 * x^2 - 1) * a * b^2 / c^2 + 3/4 * a^2 * b * \arcsin(c * x) / c^2 - 3/8 * b^3 * \arcsin(c * x) / c^2 - 3/8 * a * b^2 / c^2$$

**maple** [A] time = 0.04, size = 219, normalized size = 1.75

$$\frac{c^2 x^2 a^3}{2} + b^3 \left( \frac{(c^2 x^2 - 1) \arcsin(cx)^3}{2} + \frac{3 \arcsin(cx)^2 (cx \sqrt{-c^2 x^2 + 1} + \arcsin(cx))}{4} - \frac{3(c^2 x^2 - 1) \arcsin(cx)}{4} - \frac{3cx \sqrt{-c^2 x^2 + 1}}{8} - \frac{3 \arcsin(cx)}{8} - a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsin(c\*x))^3,x)

[Out] 1/c^2\*(1/2\*c^2\*x^2\*a^3+b^3\*(1/2\*(c^2\*x^2-1)\*arcsin(c\*x)^3+3/4\*arcsin(c\*x)^2\*(c\*x\*(-c^2\*x^2+1)^(1/2)+arcsin(c\*x))-3/4\*(c^2\*x^2-1)\*arcsin(c\*x)-3/8\*c\*x\*(-c^2\*x^2+1)^(1/2)-3/8\*arcsin(c\*x)-1/2\*arcsin(c\*x)^3)+3\*a\*b^2\*(1/2\*(c^2\*x^2-1)\*arcsin(c\*x)^2+1/2\*arcsin(c\*x)\*(c\*x\*(-c^2\*x^2+1)^(1/2)+arcsin(c\*x))-1/4\*arcsin(c\*x)^2-1/4\*c^2\*x^2)+3\*a^2\*b\*(1/2\*c^2\*x^2\*arcsin(c\*x)+1/4\*c\*x\*(-c^2\*x^2+1)^(1/2)-1/4\*arcsin(c\*x)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b^3 x^2 \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^3 + \frac{1}{2} a^3 x^2 + \frac{3}{4} \left( 2x^2 \arcsin(cx) + c \left( \frac{\sqrt{-c^2 x^2 + 1} x}{c^2} - \frac{\arcsin(cx)}{c^3} \right) \right) a^2 b + \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out] 1/2\*b^3\*x^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^3 + 1/2\*a^3\*x^2 + 3/4\*(2\*x^2\*arcsin(c\*x) + c\*(sqrt(-c^2\*x^2 + 1)\*x/c^2 - arcsin(c\*x)/c^3))\*a^2\*b + integrate(3/2\*(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b^3\*c\*x^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*(a\*b^2\*c^2\*x^3 - a\*b^2\*x)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2)/(c^2\*x^2 - 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x (a + b \operatorname{asin}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asin(c\*x))^3,x)

[Out] int(x\*(a + b\*asin(c\*x))^3, x)

**sympy** [A] time = 1.30, size = 264, normalized size = 2.11

$$\left\{ \begin{array}{l} \frac{a^3 x^2}{2} + \frac{3a^2 b x^2 \operatorname{asin}(cx)}{2} + \frac{3a^2 b x \sqrt{-c^2 x^2 + 1}}{4c} - \frac{3a^2 b \operatorname{asin}(cx)}{4c^2} + \frac{3ab^2 x^2 \operatorname{asin}^2(cx)}{2} - \frac{3ab^2 x^2}{4} + \frac{3ab^2 x \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx)}{2c} - \frac{3ab^2 \operatorname{asin}^2(cx)}{4c^2} \\ \frac{a^3 x^2}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asin(c\*x))\*\*3,x)

[Out] Piecewise((a\*\*3\*x\*\*2/2 + 3\*a\*\*2\*b\*x\*\*2\*asin(c\*x)/2 + 3\*a\*\*2\*b\*x\*sqrt(-c\*\*2\*x\*\*2 + 1)/(4\*c) - 3\*a\*\*2\*b\*asin(c\*x)/(4\*c\*\*2) + 3\*a\*b\*\*2\*x\*\*2\*asin(c\*x)\*\*2/

```

2 - 3*a*b**2*x**2/4 + 3*a*b**2*x*sqrt(-c**2*x**2 + 1)*asin(c*x)/(2*c) - 3*a
*b**2*asin(c*x)**2/(4*c**2) + b**3*x**2*asin(c*x)**3/2 - 3*b**3*x**2*asin(c
*x)/4 + 3*b**3*x*sqrt(-c**2*x**2 + 1)*asin(c*x)**2/(4*c) - 3*b**3*x*sqrt(-c
**2*x**2 + 1)/(8*c) - b**3*asin(c*x)**3/(4*c**2) + 3*b**3*asin(c*x)/(8*c**2
), Ne(c, 0)), (a**3*x**2/2, True))

```

### 3.155 $\int (a + b \sin^{-1}(cx))^3 dx$

**Optimal.** Leaf size=82

$$-6ab^2x + \frac{3b\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))^2}{c} + x(a+b\sin^{-1}(cx))^3 - \frac{6b^3\sqrt{1-c^2x^2}}{c} - 6b^3x\sin^{-1}(cx)$$

[Out]  $-6*a*b^2*x - 6*b^3*x*\arcsin(c*x) + x*(a+b*\arcsin(c*x))^3 - 6*b^3*(-c^2*x^2+1)^(1/2)/c + 3*b*(a+b*\arcsin(c*x))^2*(-c^2*x^2+1)^(1/2)/c$

**Rubi [A]** time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {4619, 4677, 261}

$$-6ab^2x + \frac{3b\sqrt{1-c^2x^2}(a+b\sin^{-1}(cx))^2}{c} + x(a+b\sin^{-1}(cx))^3 - \frac{6b^3\sqrt{1-c^2x^2}}{c} - 6b^3x\sin^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^3, x]

[Out]  $-6*a*b^2*x - (6*b^3*\text{Sqrt}[1 - c^2*x^2])/c - 6*b^3*x*\text{ArcSin}[c*x] + (3*b*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^2)/c + x*(a + b*\text{ArcSin}[c*x])^3$

#### Rule 261

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rule 4619

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_), x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4677

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int (a + b \sin^{-1}(cx))^3 dx &= x(a + b \sin^{-1}(cx))^3 - (3bc) \int \frac{x(a + b \sin^{-1}(cx))^2}{\sqrt{1 - c^2x^2}} dx \\
&= \frac{3b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{c} + x(a + b \sin^{-1}(cx))^3 - (6b^2) \int (a + b \sin^{-1}(cx)) dx \\
&= -6ab^2x + \frac{3b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{c} + x(a + b \sin^{-1}(cx))^3 - (6b^3) \int \sin^{-1}(cx) dx \\
&= -6ab^2x - 6b^3x \sin^{-1}(cx) + \frac{3b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{c} + x(a + b \sin^{-1}(cx))^3 + (6b^3) \arcsin(cx) \\
&= -6ab^2x - \frac{6b^3\sqrt{1 - c^2x^2}}{c} - 6b^3x \sin^{-1}(cx) + \frac{3b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2}{c} + x(a + b \sin^{-1}(cx))^3 + 6b^3 \arcsin(cx)
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 77, normalized size = 0.94

$$\frac{3b \left( \sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^2 - 2b \left( acx + b\sqrt{1 - c^2x^2} + bcx \sin^{-1}(cx) \right) \right)}{c} + x(a + b \sin^{-1}(cx))^3$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^3,x]

[Out] x\*(a + b\*ArcSin[c\*x])^3 + (3\*b\*(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^2 - 2\*b\*(a\*c\*x + b\*Sqrt[1 - c^2\*x^2] + b\*c\*x\*ArcSin[c\*x])))/c

**fricas [A]** time = 2.03, size = 108, normalized size = 1.32

$$\frac{b^3cx \arcsin(cx)^3 + 3ab^2cx \arcsin(cx)^2 + 3(a^2b - 2b^3)cx \arcsin(cx) + (a^3 - 6ab^2)cx + 3(b^3 \arcsin(cx)^2 + 2ab^3 \arcsin(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out] (b^3\*c\*x\*arcsin(c\*x)^3 + 3\*a\*b^2\*c\*x\*arcsin(c\*x)^2 + 3\*(a^2\*b - 2\*b^3)\*c\*x\*arcsin(c\*x) + (a^3 - 6\*a\*b^2)\*c\*x + 3\*(b^3\*arcsin(c\*x)^2 + 2\*a\*b^2\*arcsin(c\*x) + a^2\*b - 2\*b^3)\*sqrt(-c^2\*x^2 + 1))/c

**giac [A]** time = 0.63, size = 150, normalized size = 1.83

$$b^3x \arcsin(cx)^3 + 3ab^2x \arcsin(cx)^2 + 3a^2bx \arcsin(cx) - 6b^3x \arcsin(cx) + \frac{3\sqrt{-c^2x^2 + 1} b^3 \arcsin(cx)^2}{c} + a^3x - 6ab^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out] b^3\*x\*arcsin(c\*x)^3 + 3\*a\*b^2\*x\*arcsin(c\*x)^2 + 3\*a^2\*b\*x\*arcsin(c\*x) - 6\*b^3\*x\*arcsin(c\*x) + 3\*sqrt(-c^2\*x^2 + 1)\*b^3\*arcsin(c\*x)^2/c + a^3\*x - 6\*a\*b^3\*x + 6\*sqrt(-c^2\*x^2 + 1)\*a\*b^2\*arcsin(c\*x)/c + 3\*sqrt(-c^2\*x^2 + 1)\*a^2\*b/c - 6\*sqrt(-c^2\*x^2 + 1)\*b^3/c

**maple [A]** time = 0.04, size = 132, normalized size = 1.61

$$\frac{cx a^3 + b^3 \left( cx \arcsin(cx)^3 + 3 \arcsin(cx)^2 \sqrt{-c^2x^2 + 1} - 6\sqrt{-c^2x^2 + 1} - 6cx \arcsin(cx) \right) + 3a b^2 \left( cx \arcsin(cx)^2 \right)}{c}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^3,x)

[Out]  $1/c*(c*x*a^3+b^3*(c*x*arcsin(c*x))^3+3*arcsin(c*x)^2*(-c^2*x^2+1)^{(1/2)}-6*(-c^2*x^2+1)^{(1/2)}-6*c*x*arcsin(c*x))+3*a*b^2*(c*x*arcsin(c*x)^2-2*c*x+2*arcsin(c*x)*(-c^2*x^2+1)^{(1/2}))+3*a^2*b*(c*x*arcsin(c*x)+(-c^2*x^2+1)^{(1/2}))$

**maxima** [A] time = 0.44, size = 141, normalized size = 1.72

$$b^3 x \arcsin(cx)^3 + 3 ab^2 x \arcsin(cx)^2 + 3 \left( \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{c} - \frac{2 \left( cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1} \right)}{c} \right) b^3 - 6 ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out]  $b^3*x*arcsin(c*x)^3 + 3*a*b^2*x*arcsin(c*x)^2 + 3*(sqrt(-c^2*x^2 + 1)*arcsin(c*x)^2/c - 2*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))/c)*b^3 - 6*a*b^2*(x - sqrt(-c^2*x^2 + 1)*arcsin(c*x)/c) + a^3*x + 3*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*a^2*b/c$

**mupad** [B] time = 0.41, size = 242, normalized size = 2.95

$$\left\{ \begin{array}{l} a^3 x - b^3 \left( x \left( 6 \arcsin(cx) - \arcsin(cx)^3 \right) - \sqrt{\frac{1}{c^2} - x^2} \left( 3 \arcsin(cx)^2 - 6 \right) \right) + 3 ab^2 \left( x \left( \arcsin(cx)^2 - 2 \right) + 2 \arcsin(cx) \right) \\ a^3 x + \frac{3 a^2 b \left( \sqrt{1 - c^2 x^2} + c x \arcsin(cx) \right)}{c} + 3 ab^2 x \left( \arcsin(cx)^2 - 2 \right) + b^3 x \arcsin(cx) \left( \arcsin(cx)^2 - 6 \right) + \frac{3 b^3 \sqrt{1 - c^2 x^2}}{c} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^3,x)

[Out]  $piecewise(0 < c, a^3*x - b^3*(x*(6*asin(c*x) - asin(c*x)^3) - (1/c^2 - x^2)^{(1/2)}*(3*asin(c*x)^2 - 6)) + 3*a*b^2*(x*(asin(c*x)^2 - 2) + 2*asin(c*x)*(1/c^2 - x^2)^{(1/2)}) + (3*a^2*b*((-c^2*x^2 + 1)^{(1/2)} + c*x*asin(c*x)))/c, ~ 0 < c, a^3*x + (3*a^2*b*((-c^2*x^2 + 1)^{(1/2)} + c*x*asin(c*x)))/c + 3*a*b^2*x*(asin(c*x)^2 - 2) + b^3*x*asin(c*x)*(asin(c*x)^2 - 6) + (3*b^3*(-c^2*x^2 + 1)^{(1/2)}*(asin(c*x)^2 - 2))/c + (6*a*b^2*asin(c*x)*(-c^2*x^2 + 1)^{(1/2}))/c)$

**sympy** [A] time = 0.60, size = 160, normalized size = 1.95

$$\left\{ \begin{array}{l} a^3 x + 3 a^2 b x \arcsin(cx) + \frac{3 a^2 b \sqrt{-c^2 x^2 + 1}}{c} + 3 a b^2 x \arcsin^2(cx) - 6 a b^2 x + \frac{6 a b^2 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{c} + b^3 x \arcsin^3(cx) - 6 b^3 \\ a^3 x \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*3,x)

[Out]  $Piecewise((a**3*x + 3*a**2*b*x*asin(c*x) + 3*a**2*b*sqrt(-c**2*x**2 + 1)/c + 3*a*b**2*x*asin(c*x)**2 - 6*a*b**2*x + 6*a*b**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/c + b**3*x*asin(c*x)**3 - 6*b**3*x*asin(c*x) + 3*b**3*sqrt(-c**2*x**2 + 1)*asin(c*x)**2/c - 6*b**3*sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (a**3*x, True))$

$$3.156 \quad \int \frac{(a+b \sin^{-1}(cx))^3}{x} dx$$

**Optimal.** Leaf size=123

$$\frac{3}{2}b^2\text{Li}_3\left(e^{2i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)-\frac{3}{2}ib\text{Li}_2\left(e^{2i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)^2-\frac{i\left(a+b\sin^{-1}(cx)\right)^4}{4b}+\log\left(1-e^{2i\sin^{-1}(cx)}\right)$$

[Out]  $-1/4*I*(a+b*\arcsin(c*x))^4/b+(a+b*\arcsin(c*x))^3*\ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-3/2*I*b*(a+b*\arcsin(c*x))^2*\text{polylog}(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)+3/2*b^2*(a+b*\arcsin(c*x))*\text{polylog}(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)+3/4*I*b^3*\text{polylog}(4,(I*c*x+(-c^2*x^2+1)^(1/2))^2)$

**Rubi [A]** time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4625, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}b^2\text{PolyLog}\left(3,e^{2i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)-\frac{3}{2}ib\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)^2+\frac{3}{4}ib^3\text{PolyLog}\left(4,e^{2i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)^3$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^3/x, x]

[Out]  $((-I/4)*(a + b*\text{ArcSin}[c*x])^4)/b + (a + b*\text{ArcSin}[c*x])^3*\text{Log}[1 - E^{((2*I)*\text{ArcSin}[c*x])}] - ((3*I)/2)*b*(a + b*\text{ArcSin}[c*x])^2*\text{PolyLog}[2, E^{((2*I)*\text{ArcSin}[c*x])}] + (3*b^2*(a + b*\text{ArcSin}[c*x])*\text{PolyLog}[3, E^{((2*I)*\text{ArcSin}[c*x])}])/2 + ((3*I)/4)*b^3*\text{PolyLog}[4, E^{((2*I)*\text{ArcSin}[c*x])}]$

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^(c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/ (b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 3717

Int[(((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[(((c + d\*x)^m \* E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sin^{-1}(cx))^3}{x} dx &= \text{Subst} \left( \int (a + bx)^3 \cot(x) dx, x, \sin^{-1}(cx) \right) \\
 &= -\frac{i(a + b \sin^{-1}(cx))^4}{4b} - 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)^3}{1 - e^{2ix}} dx, x, \sin^{-1}(cx) \right) \\
 &= -\frac{i(a + b \sin^{-1}(cx))^4}{4b} + (a + b \sin^{-1}(cx))^3 \log(1 - e^{2i \sin^{-1}(cx)}) - (3b) \text{Subst} \left( \int (a + bx)^2 \cot(x) dx, x, \sin^{-1}(cx) \right) \\
 &= -\frac{i(a + b \sin^{-1}(cx))^4}{4b} + (a + b \sin^{-1}(cx))^3 \log(1 - e^{2i \sin^{-1}(cx)}) - \frac{3}{2}ib(a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)}) \\
 &= -\frac{i(a + b \sin^{-1}(cx))^4}{4b} + (a + b \sin^{-1}(cx))^3 \log(1 - e^{2i \sin^{-1}(cx)}) - \frac{3}{2}ib(a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)}) \\
 &= -\frac{i(a + b \sin^{-1}(cx))^4}{4b} + (a + b \sin^{-1}(cx))^3 \log(1 - e^{2i \sin^{-1}(cx)}) - \frac{3}{2}ib(a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)}) \\
 &= -\frac{i(a + b \sin^{-1}(cx))^4}{4b} + (a + b \sin^{-1}(cx))^3 \log(1 - e^{2i \sin^{-1}(cx)}) - \frac{3}{2}ib(a + b \sin^{-1}(cx))^2 \log(1 - e^{2i \sin^{-1}(cx)})
 \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 244, normalized size = 1.98

$$a^3 \log(cx) + 3a^2b \left( \sin^{-1}(cx) \log(1 - e^{2i \sin^{-1}(cx)}) - \frac{1}{2}i(\sin^{-1}(cx)^2 + \text{Li}_2(e^{2i \sin^{-1}(cx)})) \right) + \frac{1}{8}ab^2(24i \sin^{-1}(cx) \text{Li}_2(e^{2i \sin^{-1}(cx)}))$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSin[c\*x])^3/x, x]

[Out] a^3\*Log[c\*x] + 3\*a^2\*b\*(ArcSin[c\*x]\*Log[1 - E^((2\*I)\*ArcSin[c\*x])] - (I/2)\*(ArcSin[c\*x]^2 + PolyLog[2, E^((2\*I)\*ArcSin[c\*x])])) + (a\*b^2\*((-I)\*Pi^3 + (8\*I)\*ArcSin[c\*x]^3 + 24\*ArcSin[c\*x]^2\*Log[1 - E^((-2\*I)\*ArcSin[c\*x])]) + (24\*I)\*ArcSin[c\*x]\*PolyLog[2, E^((-2\*I)\*ArcSin[c\*x])] + 12\*PolyLog[3, E^((-2\*I)\*ArcSin[c\*x])])/8 - (I/64)\*b^3\*(Pi^4 - 16\*ArcSin[c\*x]^4 + (64\*I)\*ArcSin[c\*x]^3\*Log[1 - E^((-2\*I)\*ArcSin[c\*x])] - 96\*ArcSin[c\*x]^2\*PolyLog[2, E^((-2\*I)\*ArcSin[c\*x])] + (96\*I)\*ArcSin[c\*x]\*PolyLog[3, E^((-2\*I)\*ArcSin[c\*x])] + 48\*PolyLog[4, E^((-2\*I)\*ArcSin[c\*x])])

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/x,x, algorithm="fricas")

[Out] integral((b^3\*arcsin(c\*x)^3 + 3\*a\*b^2\*arcsin(c\*x)^2 + 3\*a^2\*b\*arcsin(c\*x) + a^3)/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/x,x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^3/x, x)

**maple** [B] time = 0.03, size = 592, normalized size = 4.81

$$a^3 \ln(cx) - 6iab^2 \arcsin(cx) \text{polylog}\left(2, icx + \sqrt{-c^2x^2 + 1}\right) + b^3 \arcsin(cx)^3 \ln\left(1 + icx + \sqrt{-c^2x^2 + 1}\right) - 3ia^2b \text{polylog}\left(2, icx + \sqrt{-c^2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^3/x,x)

[Out] a^3\*ln(c\*x)+6\*I\*b^3\*polylog(4,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+b^3\*arcsin(c\*x)^3\*ln(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))-6\*I\*a\*b^2\*arcsin(c\*x)\*polylog(2,I\*c\*x+(-c^2\*x^2+1)^(1/2))+6\*b^3\*arcsin(c\*x)\*polylog(3,-I\*c\*x-(-c^2\*x^2+1)^(1/2))-6\*I\*a\*b^2\*arcsin(c\*x)\*polylog(2,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+b^3\*arcsin(c\*x)^3\*ln(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))-1/4\*I\*b^3\*arcsin(c\*x)^4+6\*b^3\*arcsin(c\*x)\*polylog(3,I\*c\*x+(-c^2\*x^2+1)^(1/2))-3\*I\*a^2\*b\*polylog(2,-I\*c\*x-(-c^2\*x^2+1)^(1/2))-I\*a\*b^2\*arcsin(c\*x)^3-3\*I\*b^3\*arcsin(c\*x)^2\*polylog(2,I\*c\*x+(-c^2\*x^2+1)^(1/2))+3\*a\*b^2\*arcsin(c\*x)^2\*ln(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))-3\*I\*b^3\*arcsin(c\*x)^2\*polylog(2,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+3\*a\*b^2\*arcsin(c\*x)^2\*ln(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))+6\*a\*b^2\*polylog(3,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+6\*a\*b^2\*polylog(3,I\*c\*x+(-c^2\*x^2+1)^(1/2))+6\*I\*b^3\*polylog(4,I\*c\*x+(-c^2\*x^2+1)^(1/2))+3\*a^2\*b\*arcsin(c\*x)\*ln(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))+3\*a^2\*b\*arcsin(c\*x)\*ln(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))-3/2\*I\*a^2\*b\*arcsin(c\*x)^2-3\*I\*a^2\*b\*polylog(2,I\*c\*x+(-c^2\*x^2+1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \log(x) + \int \frac{b^3 \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^3 + 3ab^2 \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^2 + 3a^2b \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/x,x, algorithm="maxima")

[Out] a^3\*log(x) + integrate((b^3\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^3 + 3\*a\*b^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 3\*a^2\*b\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/x, x)

**mapad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \arcsin(cx))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))^3/x, x)`

[Out] `int((a + b*asin(c*x))^3/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asin(c*x))**3/x, x)`

[Out] `Integral((a + b*asin(c*x))**3/x, x)`

$$3.157 \quad \int \frac{(a+b \sin^{-1}(cx))^3}{x^2} dx$$

Optimal. Leaf size=137

$$6ib^2c\text{Li}_2\left(-e^{i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)-6ib^2c\text{Li}_2\left(e^{i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)-\frac{\left(a+b\sin^{-1}(cx)\right)^3}{x}-6bc\tanh^{-1}\left(e^{i\sin^{-1}(cx)}\right)$$

[Out]  $-(a+b*\arcsin(c*x))^3/x-6*b*c*(a+b*\arcsin(c*x))^2*\arctanh(I*c*x+(-c^2*x^2+1)^{(1/2)})+6*I*b^2*c*(a+b*\arcsin(c*x))*\text{polylog}(2,-I*c*x+(-c^2*x^2+1)^{(1/2)})-6*I*b^2*c*(a+b*\arcsin(c*x))*\text{polylog}(2,I*c*x+(-c^2*x^2+1)^{(1/2)})-6*b^3*c*\text{polylog}(3,-I*c*x+(-c^2*x^2+1)^{(1/2)})+6*b^3*c*\text{polylog}(3,I*c*x+(-c^2*x^2+1)^{(1/2)})$

**Rubi [A]** time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {4627, 4709, 4183, 2531, 2282, 6589}

$$6ib^2c\text{PolyLog}\left(2,-e^{i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)-6ib^2c\text{PolyLog}\left(2,e^{i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)-6b^3c\text{PolyLog}\left(3,-e^{i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)+6b^3c\text{PolyLog}\left(3,e^{i\sin^{-1}(cx)}\right)\left(a+b\sin^{-1}(cx)\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^3/x^2,x]

[Out]  $-\left((a+b*\text{ArcSin}[c*x])^3/x\right)-6*b*c*(a+b*\text{ArcSin}[c*x])^2*\text{ArcTanh}\left[E^{(I*\text{ArcSin}[c*x])}\right]+(6*I)*b^2*c*(a+b*\text{ArcSin}[c*x])*PolyLog[2,-E^{(I*\text{ArcSin}[c*x])}]-6*(I)*b^2*c*(a+b*\text{ArcSin}[c*x])*PolyLog[2,E^{(I*\text{ArcSin}[c*x])}]-6*b^3*c*PolyLog[3,-E^{(I*\text{ArcSin}[c*x])}]+6*b^3*c*PolyLog[3,E^{(I*\text{ArcSin}[c*x])}]$

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m-1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 4183

Int[csc[(e\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Log[1 - E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Log[1 + E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4709

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^n_.)*(x_.)^m_.)/Sqrt[(d_.) + (e_.)*
(x_.)^2], x_Symbol] := Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin
[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d +
e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^p_.]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + b \sin^{-1}(cx))^3}{x^2} dx &= -\frac{(a + b \sin^{-1}(cx))^3}{x} + (3bc) \int \frac{(a + b \sin^{-1}(cx))^2}{x\sqrt{1 - c^2x^2}} dx \\ &= -\frac{(a + b \sin^{-1}(cx))^3}{x} + (3bc) \text{Subst} \left( \int (a + bx)^2 \csc(x) dx, x, \sin^{-1}(cx) \right) \\ &= -\frac{(a + b \sin^{-1}(cx))^3}{x} - 6bc (a + b \sin^{-1}(cx))^2 \tanh^{-1}(e^{i \sin^{-1}(cx)}) - (6b^2c) \text{Subst} \left( \int \frac{(a + bx)^2}{x} dx, x, \sin^{-1}(cx) \right) \\ &= -\frac{(a + b \sin^{-1}(cx))^3}{x} - 6bc (a + b \sin^{-1}(cx))^2 \tanh^{-1}(e^{i \sin^{-1}(cx)}) + 6ib^2c (a + b \sin^{-1}(cx))^2 \\ &= -\frac{(a + b \sin^{-1}(cx))^3}{x} - 6bc (a + b \sin^{-1}(cx))^2 \tanh^{-1}(e^{i \sin^{-1}(cx)}) + 6ib^2c (a + b \sin^{-1}(cx))^2 \\ &= -\frac{(a + b \sin^{-1}(cx))^3}{x} - 6bc (a + b \sin^{-1}(cx))^2 \tanh^{-1}(e^{i \sin^{-1}(cx)}) + 6ib^2c (a + b \sin^{-1}(cx))^2 \end{aligned}$$

**Mathematica [B]** time = 0.32, size = 283, normalized size = 2.07

$$-\frac{a^3}{x} - 3a^2bc \log(\sqrt{1 - c^2x^2} + 1) + 3a^2bc \log(x) - \frac{3a^2b \sin^{-1}(cx)}{x} + 3ab^2c \left( 2i \text{Li}_2(-e^{i \sin^{-1}(cx)}) - 2i \text{Li}_2(e^{i \sin^{-1}(cx)}) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSin[c*x])^3/x^2, x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcSin[c*x])/x + 3*a^2*b*c*Log[x] - 3*a^2*b*c*Log[1 + S
qrt[1 - c^2*x^2]] + 3*a*b^2*c*(-(ArcSin[c*x]*(ArcSin[c*x]/(c*x) - 2*Log[1 -
E^(I*ArcSin[c*x])]) + 2*Log[1 + E^(I*ArcSin[c*x])])) + (2*I)*PolyLog[2, -E^
(I*ArcSin[c*x])] - (2*I)*PolyLog[2, E^(I*ArcSin[c*x])] + b^3*c*(-(ArcSin[c
*x]^3/(c*x)) + 3*ArcSin[c*x]^2*Log[1 - E^(I*ArcSin[c*x])] - 3*ArcSin[c*x]^2
*Log[1 + E^(I*ArcSin[c*x])] + (6*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x
])] - (6*I)*ArcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])] - 6*PolyLog[3, -E^(I*
ArcSin[c*x])] + 6*PolyLog[3, E^(I*ArcSin[c*x])])
```

**fricas [F]** time = 0.75, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsin(c*x))^3/x^2, x, algorithm="fricas")
```

[Out] integral((b^3\*arcsin(c\*x)^3 + 3\*a\*b^2\*arcsin(c\*x)^2 + 3\*a^2\*b\*arcsin(c\*x) + a^3)/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/x^2,x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^3/x^2, x)

**maple** [B] time = 0.10, size = 378, normalized size = 2.76

$$-\frac{a^3}{x} - \frac{b^3 \arcsin(cx)^3}{x} - 3c b^3 \arcsin(cx)^2 \ln\left(1 + icx + \sqrt{-c^2x^2 + 1}\right) + 6ic b^3 \arcsin(cx) \operatorname{polylog}\left(2, -icx - \sqrt{-c^2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^3/x^2,x)

[Out] -a^3/x - b^3/x\*arcsin(c\*x)^3 - 3\*c\*b^3\*arcsin(c\*x)^2\*ln(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))+6\*I\*c\*b^3\*arcsin(c\*x)\*polylog(2,-I\*c\*x-(-c^2\*x^2+1)^(1/2))-6\*b^3\*c\*polylog(3,-I\*c\*x-(-c^2\*x^2+1)^(1/2))+3\*c\*b^3\*arcsin(c\*x)^2\*ln(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))-6\*I\*c\*b^3\*arcsin(c\*x)\*polylog(2,I\*c\*x+(-c^2\*x^2+1)^(1/2))+6\*b^3\*c\*polylog(3,I\*c\*x+(-c^2\*x^2+1)^(1/2))-3\*a\*b^2/x\*arcsin(c\*x)^2+6\*c\*a\*b^2\*arcsin(c\*x)\*ln(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))-6\*c\*a\*b^2\*arcsin(c\*x)\*ln(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))+6\*I\*c\*a\*b^2\*dilog(1+I\*c\*x+(-c^2\*x^2+1)^(1/2))-6\*I\*c\*a\*b^2\*dilog(1-I\*c\*x-(-c^2\*x^2+1)^(1/2))-3\*a^2\*b/x\*arcsin(c\*x)-3\*c\*a^2\*b\*arctanh(1/(-c^2\*x^2+1)^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-3 \left( c \log \left( \frac{2 \sqrt{-c^2x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) a^2 b - \frac{a^3}{x} - \frac{b^3 \arctan \left( cx, \sqrt{cx + 1} \sqrt{-cx + 1} \right)^3}{x} + \frac{3}{2} \left( ab^2 c^2 \left( \frac{\log(cx+1)}{c} \right) - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/x^2,x, algorithm="maxima")

[Out] -3\*(c\*log(2\*sqrt(-c^2\*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c\*x)/x)\*a^2\*b - a^3/x - (b^3\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^3 + x\*integrate(3\*(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b^3\*c\*x\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 - (a\*b^2\*c^2\*x^2 - a\*b^2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2)/(c^2\*x^4 - x^2), x))/x

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asin}(cx))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^3/x^2,x)

[Out] int((a + b\*asin(c\*x))^3/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^3}{x^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asin(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*asin(c*x))**3/x**2, x)
```

$$3.158 \quad \int \frac{x^2}{a+b \sin^{-1}(cx)} dx$$

**Optimal.** Leaf size=121

$$\frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{3a}{b}\right) \text{Ci}\left(\frac{3(a+b \sin^{-1}(cx))}{b}\right)}{4bc^3} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{4bc^3} - \frac{\sin\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b \sin^{-1}(cx))}{b}\right)}{4bc^3}$$

[Out] 1/4\*Ci((a+b\*arcsin(c\*x))/b)\*cos(a/b)/b/c^3-1/4\*Ci(3\*(a+b\*arcsin(c\*x))/b)\*cos(3\*a/b)/b/c^3+1/4\*Si((a+b\*arcsin(c\*x))/b)\*sin(a/b)/b/c^3-1/4\*Si(3\*(a+b\*arcsin(c\*x))/b)\*sin(3\*a/b)/b/c^3

**Rubi [A]** time = 0.25, antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {4635, 4406, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{4bc^3} - \frac{\cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b} + 3 \sin^{-1}(cx)\right)}{4bc^3} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{4bc^3} - \frac{\sin\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3a}{b} + 3 \sin^{-1}(cx)\right)}{4bc^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*ArcSin[c\*x]),x]

[Out] (Cos[a/b]\*CosIntegral[a/b + ArcSin[c\*x]])/(4\*b\*c^3) - (Cos[(3\*a)/b]\*CosIntegral[(3\*a)/b + 3\*ArcSin[c\*x]])/(4\*b\*c^3) + (Sin[a/b]\*SinIntegral[a/b + ArcSin[c\*x]])/(4\*b\*c^3) - (Sin[(3\*a)/b]\*SinIntegral[(3\*a)/b + 3\*ArcSin[c\*x]])/(4\*b\*c^3)

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^(n)\*Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + b \sin^{-1}(cx)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x) \sin^2(x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{4(a+bx)} - \frac{\cos(3x)}{4(a+bx)}\right) dx, x, \sin^{-1}(cx)\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \frac{\cos(x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4c^3} - \frac{\text{Subst}\left(\int \frac{\cos(3x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4c^3} \\
&= \frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4c^3} - \frac{\cos\left(\frac{3a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{3a}{b}+3x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4c^3} \\
&= \frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{4bc^3} - \frac{\cos\left(\frac{3a}{b}\right) \text{Ci}\left(\frac{3a}{b} + 3 \sin^{-1}(cx)\right)}{4bc^3} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{4bc^3}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 91, normalized size = 0.75

$$\frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right) - \cos\left(\frac{3a}{b}\right) \text{Ci}\left(3\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right) + \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right) - \sin\left(\frac{3a}{b}\right) \text{Si}\left(3\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right)}{4bc^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*ArcSin[c\*x]), x]

[Out] (Cos[a/b]\*CosIntegral[a/b + ArcSin[c\*x]] - Cos[(3\*a)/b]\*CosIntegral[3\*(a/b + ArcSin[c\*x])]) + Sin[a/b]\*SinIntegral[a/b + ArcSin[c\*x]] - Sin[(3\*a)/b]\*SinIntegral[3\*(a/b + ArcSin[c\*x])]/(4\*b\*c^3)

**fricas [F]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{b \arcsin(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(x^2/(b\*arcsin(c\*x) + a), x)

**giac [A]** time = 0.45, size = 173, normalized size = 1.43

$$\frac{\cos\left(\frac{a}{b}\right)^3 \text{Ci}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) - \cos\left(\frac{a}{b}\right)^2 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) - 3 \cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{3a}{b} + 3 \arcsin(cx)\right)}{4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] -cos(a/b)^3\*cos\_integral(3\*a/b + 3\*arcsin(c\*x))/(b\*c^3) - cos(a/b)^2\*sin(a/b)\*sin\_integral(3\*a/b + 3\*arcsin(c\*x))/(b\*c^3) + 3/4\*cos(a/b)\*cos\_integral(3\*a/b + 3\*arcsin(c\*x))/(b\*c^3) + 1/4\*cos(a/b)\*cos\_integral(a/b + arcsin(c\*x))/(b\*c^3) + 1/4\*sin(a/b)\*sin\_integral(3\*a/b + 3\*arcsin(c\*x))/(b\*c^3) + 1/4\*sin(a/b)\*sin\_integral(a/b + arcsin(c\*x))/(b\*c^3)

**maple** [A] time = 0.03, size = 102, normalized size = 0.84

$$\frac{\frac{\text{Si}\left(\arcsin(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{4b} + \frac{\text{Ci}\left(\arcsin(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{4b} - \frac{\text{Si}\left(3\arcsin(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)}{4b} - \frac{\text{Ci}\left(3\arcsin(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)}{4b}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*arcsin(c\*x)),x)

[Out] 1/c^3\*(1/4\*Si(arcsin(c\*x)+a/b)\*sin(a/b)/b+1/4\*Ci(arcsin(c\*x)+a/b)\*cos(a/b)/b-1/4\*Si(3\*arcsin(c\*x)+3\*a/b)\*sin(3\*a/b)/b-1/4\*Ci(3\*arcsin(c\*x)+3\*a/b)\*cos(3\*a/b)/b)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(x^2/(b\*arcsin(c\*x) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*asin(c\*x)),x)

[Out] int(x^2/(a + b\*asin(c\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*asin(c\*x)),x)

[Out] Integral(x\*\*2/(a + b\*asin(c\*x)), x)

$$3.159 \quad \int \frac{x}{a+b \sin^{-1}(cx)} dx$$

**Optimal.** Leaf size=63

$$\frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \sin^{-1}(cx))}{b}\right)}{2bc^2} - \frac{\sin\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2(a+b \sin^{-1}(cx))}{b}\right)}{2bc^2}$$

[Out] 1/2\*cos(2\*a/b)\*Si(2\*(a+b\*arcsin(c\*x))/b)/b/c^2-1/2\*Ci(2\*(a+b\*arcsin(c\*x))/b)\*sin(2\*a/b)/b/c^2

**Rubi [A]** time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4635, 4406, 12, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{2bc^2} - \frac{\sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{2bc^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*ArcSin[c\*x]),x]

[Out] -(CosIntegral[(2\*a)/b + 2\*ArcSin[c\*x]]\*Sin[(2\*a)/b])/(2\*b\*c^2) + (Cos[(2\*a)/b]\*SinIntegral[(2\*a)/b + 2\*ArcSin[c\*x]])/(2\*b\*c^2)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^n \* Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.)^(n\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n \* Sin[x]^m \* Cos[x], x], x, ArcSin[c\*x]], x]

/; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{a + b \sin^{-1}(cx)} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x) \sin(x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{c^2} \\
 &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)} dx, x, \sin^{-1}(cx)\right)}{c^2} \\
 &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{2c^2} \\
 &= \frac{\cos\left(\frac{2a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{2c^2} - \frac{\sin\left(\frac{2a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{2c^2} \\
 &= -\frac{\text{Ci}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right) \sin\left(\frac{2a}{b}\right)}{2bc^2} + \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{2bc^2}
 \end{aligned}$$

**Mathematica** [A] time = 0.08, size = 56, normalized size = 0.89

$$\frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right) - \sin\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{2bc^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*ArcSin[c\*x]),x]

[Out] (-(CosIntegral[(2\*a)/b + 2\*ArcSin[c\*x]]\*Sin[(2\*a)/b]) + Cos[(2\*a)/b]\*SinIntegral[(2\*a)/b + 2\*ArcSin[c\*x]])/(2\*b\*c^2)

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b \arcsin(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x)),x, algorithm="fricas")

[Out] integral(x/(b\*arcsin(c\*x) + a), x)

**giac** [A] time = 0.32, size = 86, normalized size = 1.37

$$-\frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{bc^2} + \frac{\cos\left(\frac{a}{b}\right)^2 \text{Si}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{bc^2} - \frac{\text{Si}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x)),x, algorithm="giac")

[Out] -cos(a/b)\*cos\_integral(2\*a/b + 2\*arcsin(c\*x))\*sin(a/b)/(b\*c^2) + cos(a/b)^2\*sin\_integral(2\*a/b + 2\*arcsin(c\*x))/(b\*c^2) - 1/2\*sin\_integral(2\*a/b + 2\*arcsin(c\*x))/(b\*c^2)

**maple** [A] time = 0.03, size = 58, normalized size = 0.92

$$\frac{\frac{\operatorname{Si}\left(2\arcsin(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{2b} - \frac{\operatorname{Ci}\left(2\arcsin(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)}{2b}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arcsin(c\*x)),x)

[Out] 1/c^2\*(1/2\*Si(2\*arcsin(c\*x)+2\*a/b)\*cos(2\*a/b)/b-1/2\*Ci(2\*arcsin(c\*x)+2\*a/b)\*sin(2\*a/b)/b)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(x/(b\*arcsin(c\*x) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*asin(c\*x)),x)

[Out] int(x/(a + b\*asin(c\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*asin(c\*x)),x)

[Out] Integral(x/(a + b\*asin(c\*x)), x)

$$3.160 \quad \int \frac{1}{a+b \sin^{-1}(cx)} dx$$

Optimal. Leaf size=53

$$\frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{bc}$$

[Out] Ci((a+b\*arcsin(c\*x))/b)\*cos(a/b)/b/c+Si((a+b\*arcsin(c\*x))/b)\*sin(a/b)/b/c

**Rubi [A]** time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {4623, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^(-1), x]

[Out] (Cos[a/b]\*CosIntegral[(a + b\*ArcSin[c\*x])/b])/(b\*c) + (Sin[a/b]\*SinIntegral[(a + b\*ArcSin[c\*x])/b])/(b\*c)

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rubi steps



$$\begin{aligned} \int \frac{1}{a + b \sin^{-1}(cx)} dx &= \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a-x}{b}\right)}{x} dx, x, a + b \sin^{-1}(cx)\right)}{bc} \\ &= \frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x} dx, x, a + b \sin^{-1}(cx)\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x} dx, x, a + b \sin^{-1}(cx)\right)}{bc} \\ &= \frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{bc} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 44, normalized size = 0.83

$$\frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right) + \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^(-1), x]

[Out] (Cos[a/b]\*CosIntegral[a/b + ArcSin[c\*x]] + Sin[a/b]\*SinIntegral[a/b + ArcSin[c\*x]])/(b\*c)

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b \arcsin(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(1/(b\*arcsin(c\*x) + a), x)

**giac [A]** time = 0.64, size = 49, normalized size = 0.92

$$\frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \arcsin(cx)\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] cos(a/b)\*cos\_integral(a/b + arcsin(c\*x))/(b\*c) + sin(a/b)\*sin\_integral(a/b + arcsin(c\*x))/(b\*c)

**maple [A]** time = 0.04, size = 48, normalized size = 0.91

$$\frac{\frac{\text{Si}\left(\arcsin(cx) + \frac{a}{b}\right) \sin\left(\frac{a}{b}\right)}{b} + \frac{\text{Ci}\left(\arcsin(cx) + \frac{a}{b}\right) \cos\left(\frac{a}{b}\right)}{b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arcsin(c\*x)), x)

[Out] 1/c\*(Si(arcsin(c\*x)+a/b)\*sin(a/b)/b+Ci(arcsin(c\*x)+a/b)\*cos(a/b)/b)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(1/(b\*arcsin(c\*x) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*asin(c\*x)),x)

[Out] int(1/(a + b\*asin(c\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*asin(c\*x)),x)

[Out] Integral(1/(a + b\*asin(c\*x)), x)

$$3.161 \quad \int \frac{1}{x(a+b \sin^{-1}(cx))} dx$$

**Optimal.** Leaf size=17

$$\text{Int}\left(\frac{1}{x(a+b \sin^{-1}(cx))}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*arcsin(c\*x)), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSin[c\*x])), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSin[c\*x])), x]

Rubi steps

$$\int \frac{1}{x(a+b \sin^{-1}(cx))} dx = \int \frac{1}{x(a+b \sin^{-1}(cx))} dx$$

**Mathematica [A]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSin[c\*x])), x]

[Out] Integrate[1/(x\*(a + b\*ArcSin[c\*x])), x]

**fricas [A]** time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx \arcsin(cx) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(1/(b\*x\*arcsin(c\*x) + a\*x), x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x)::OUTPUT:Not invertible Error: Bad Argument Value

**maple** [A] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \arcsin(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsin(c\*x)),x)

[Out] int(1/x/(a+b\*arcsin(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsin(c\*x) + a)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x(a + b \arcsin(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*asin(c\*x))),x)

[Out] int(1/(x\*(a + b\*asin(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \arcsin(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*asin(c\*x)),x)

[Out] Integral(1/(x\*(a + b\*asin(c\*x))), x)

$$3.162 \quad \int \frac{1}{x^2(a+b \sin^{-1}(cx))} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x^2(a+b \sin^{-1}(cx))}, x\right)$$

[Out] Unintegrable(1/x^2/(a+b\*arcsin(c\*x)), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*ArcSin[c\*x])), x]

[Out] Defer[Int][1/(x^2\*(a + b\*ArcSin[c\*x])), x]

Rubi steps

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))} dx = \int \frac{1}{x^2(a+b \sin^{-1}(cx))} dx$$

Mathematica [A] time = 2.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])), x]

[Out] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])), x]

fricas [A] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^2 \arcsin(cx) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(1/(b\*x^2\*arcsin(c\*x) + a\*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] integrate(1/((b\*arcsin(c\*x) + a)\*x^2), x)

**maple** [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \arcsin(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsin(c\*x)),x)

[Out] int(1/x^2/(a+b\*arcsin(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsin(c\*x) + a)\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*asin(c\*x))),x)

[Out] int(1/(x^2\*(a + b\*asin(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asin(c\*x)),x)

[Out] Integral(1/(x\*\*2\*(a + b\*asin(c\*x))), x)

$$3.163 \quad \int \frac{x^2}{(a+b \sin^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=156

$$\frac{\sin\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{4b^2c^3} - \frac{3 \sin\left(\frac{3a}{b}\right) \operatorname{Ci}\left(\frac{3(a+b \sin^{-1}(cx))}{b}\right)}{4b^2c^3} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{4b^2c^3} + \frac{3 \cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \sin^{-1}(cx))}{b}\right)}{4b^2c^3}$$

[Out]  $-1/4*\cos(a/b)*\operatorname{Si}((a+b*\arcsin(c*x))/b)/b^2/c^3+3/4*\cos(3*a/b)*\operatorname{Si}(3*(a+b*\arcsin(c*x))/b)/b^2/c^3+1/4*\operatorname{Ci}((a+b*\arcsin(c*x))/b)*\sin(a/b)/b^2/c^3-3/4*\operatorname{Ci}(3*(a+b*\arcsin(c*x))/b)*\sin(3*a/b)/b^2/c^3-x^2*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))$

**Rubi [A]** time = 0.18, antiderivative size = 152, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4631, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{4b^2c^3} - \frac{3 \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3a}{b} + 3 \sin^{-1}(cx)\right)}{4b^2c^3} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{4b^2c^3} + \frac{3 \cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3a}{b} + 3 \sin^{-1}(cx)\right)}{4b^2c^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/(a + b*\operatorname{ArcSin}[c*x])^2, x]$

[Out]  $-((x^2*\sqrt{1 - c^2*x^2})/(b*c*(a + b*\operatorname{ArcSin}[c*x]))) + (\operatorname{CosIntegral}[a/b + \operatorname{ArcSin}[c*x]]*\sin[a/b])/(4*b^2*c^3) - (3*\operatorname{CosIntegral}[(3*a)/b + 3*\operatorname{ArcSin}[c*x]]*\sin[(3*a)/b])/(4*b^2*c^3) - (\operatorname{Cos}[a/b]*\operatorname{SinIntegral}[a/b + \operatorname{ArcSin}[c*x]])/(4*b^2*c^3) + (3*\operatorname{Cos}[(3*a)/b]*\operatorname{SinIntegral}[(3*a)/b + 3*\operatorname{ArcSin}[c*x]])/(4*b^2*c^3)$

**Rule 3299**

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{SinIntegral}[e + f*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{EqQ}[d*e - c*f, 0]$

**Rule 3302**

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CosIntegral}[e - \pi/2 + f*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{EqQ}[d*(e - \pi/2) - c*f, 0]$

**Rule 3303**

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{NeQ}[d*e - c*f, 0]$

**Rule 4631**

$\operatorname{Int}[(c_. + \operatorname{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(x^m*\sqrt{1 - c^2*x^2}*(a + b*\operatorname{ArcSin}[c*x])^{(n+1)})/(b*c*(n+1)), x] - \operatorname{Dist}[1/(b*c^{(m+1)}*(n+1)), \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandTrigReduce}[(a + b*x)^{(n+1)}, \operatorname{Sin}[x]^{(m-1)}*(m - (m+1)*\operatorname{Sin}[x]^2), x], x], x, \operatorname{ArcSin}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{GeQ}[n, -2] \&\& \operatorname{LtQ}[n, -1]$

**Rubi steps**

$$\begin{aligned}
\int \frac{x^2}{(a + b \sin^{-1}(cx))^2} dx &= -\frac{x^2 \sqrt{1 - c^2 x^2}}{bc(a + b \sin^{-1}(cx))} + \frac{\text{Subst}\left(\int \left(-\frac{\sin(x)}{4(a+bx)} + \frac{3 \sin(3x)}{4(a+bx)}\right) dx, x, \sin^{-1}(cx)\right)}{bc^3} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{bc(a + b \sin^{-1}(cx))} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4bc^3} + \frac{3 \text{Subst}\left(\int \frac{\sin(3x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4bc^3} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{bc(a + b \sin^{-1}(cx))} - \frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{a}{b} + x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4bc^3} + \frac{\left(3 \cos\left(\frac{3a}{b}\right)\right) \text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b} + x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{4bc^3} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{bc(a + b \sin^{-1}(cx))} + \frac{\text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right) \sin\left(\frac{a}{b}\right)}{4b^2 c^3} - \frac{3 \text{Ci}\left(\frac{3a}{b} + 3 \sin^{-1}(cx)\right) \sin\left(\frac{3a}{b}\right)}{4b^2 c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.56, size = 125, normalized size = 0.80

$$\frac{-\frac{4bc^2x^2\sqrt{1-c^2x^2}}{a+b\sin^{-1}(cx)} + \sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right) - 3 \sin\left(\frac{3a}{b}\right) \text{Ci}\left(3\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right) - \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right) + 3 \cos\left(\frac{3a}{b}\right) \text{Si}\left(3\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right)}{4b^2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*ArcSin[c\*x])^2,x]

[Out] ((-4\*b\*c^2\*x^2\*Sqrt[1 - c^2\*x^2])/(a + b\*ArcSin[c\*x]) + CosIntegral[a/b + ArcSin[c\*x]]\*Sin[a/b] - 3\*CosIntegral[3\*(a/b + ArcSin[c\*x])]\*Sin[(3\*a)/b] - Cos[a/b]\*SinIntegral[a/b + ArcSin[c\*x]] + 3\*Cos[(3\*a)/b]\*SinIntegral[3\*(a/b + ArcSin[c\*x])])/(4\*b^2\*c^3)

**fricas [F]** time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral(x^2/(b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2), x)

**giac [B]** time = 0.61, size = 646, normalized size = 4.14

$$\frac{3b \arcsin(cx) \cos\left(\frac{a}{b}\right)^2 \text{Ci}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - 3b \arcsin(cx) \cos\left(\frac{a}{b}\right)^3 \text{Si}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) - 3a \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arcsin(cx)\right) + 3a \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{3a}{b} + \arcsin(cx)\right)}{b^3c^3 \arcsin(cx) + ab^2c^3} + \frac{3b \arcsin(cx) \cos\left(\frac{a}{b}\right)^2 \text{Ci}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - 3b \arcsin(cx) \cos\left(\frac{a}{b}\right)^3 \text{Si}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) - 3a \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arcsin(cx)\right) + 3a \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{3a}{b} + \arcsin(cx)\right)}{b^3c^3 \arcsin(cx) + ab^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] -3\*b\*arcsin(c\*x)\*cos(a/b)^2\*cos\_integral(3\*a/b + 3\*arcsin(c\*x))\*sin(a/b)/(b^3\*c^3\*arcsin(c\*x) + a\*b^2\*c^3) + 3\*b\*arcsin(c\*x)\*cos(a/b)^3\*sin\_integral(3\*a/b + 3\*arcsin(c\*x))/(b^3\*c^3\*arcsin(c\*x) + a\*b^2\*c^3) - 3\*a\*cos(a/b)^2\*cos\_integral(3\*a/b + 3\*arcsin(c\*x))\*sin(a/b)/(b^3\*c^3\*arcsin(c\*x) + a\*b^2\*c^3) + 3\*a\*cos(a/b)^3\*sin\_integral(3\*a/b + 3\*arcsin(c\*x))/(b^3\*c^3\*arcsin(c\*x) + a\*b^2\*c^3) + 3/4\*b\*arcsin(c\*x)\*cos\_integral(3\*a/b + 3\*arcsin(c\*x))\*sin(a/b)/(b^3\*c^3\*arcsin(c\*x) + a\*b^2\*c^3) + 1/4\*b\*arcsin(c\*x)\*cos\_integral(a/b + arcsin(c\*x))\*sin(a/b)/(b^3\*c^3\*arcsin(c\*x) + a\*b^2\*c^3)



$$+ \arcsin(cx) \sin(a/b) / (b^3 c^3 \arcsin(cx) + a b^2 c^3) - 9/4 b \arcsin(cx) \cos(a/b) \sin_{\text{integral}}(3a/b + 3 \arcsin(cx)) / (b^3 c^3 \arcsin(cx) + a b^2 c^3) - 1/4 b \arcsin(cx) \cos(a/b) \sin_{\text{integral}}(a/b + \arcsin(cx)) / (b^3 c^3 \arcsin(cx) + a b^2 c^3) + 3/4 a \cos_{\text{integral}}(3a/b + 3 \arcsin(cx)) \sin(a/b) / (b^3 c^3 \arcsin(cx) + a b^2 c^3) + 1/4 a \cos_{\text{integral}}(a/b + \arcsin(cx)) \sin(a/b) / (b^3 c^3 \arcsin(cx) + a b^2 c^3) - 9/4 a \cos(a/b) \sin_{\text{integral}}(3a/b + 3 \arcsin(cx)) / (b^3 c^3 \arcsin(cx) + a b^2 c^3) - 1/4 a \cos(a/b) \sin_{\text{integral}}(a/b + \arcsin(cx)) / (b^3 c^3 \arcsin(cx) + a b^2 c^3) + (-c^2 x^2 + 1)^{3/2} b / (b^3 c^3 \arcsin(cx) + a b^2 c^3) - \sqrt{-c^2 x^2 + 1} b / (b^3 c^3 \arcsin(cx) + a b^2 c^3)$$

**maple [A]** time = 0.04, size = 149, normalized size = 0.96

$$\frac{-\frac{\sqrt{-c^2 x^2 + 1}}{4(a+b \arcsin(cx))b} - \frac{\text{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) - \text{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})}{4b^2} + \frac{\cos(3 \arcsin(cx))}{4(a+b \arcsin(cx))b} + \frac{\frac{3 \text{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b})}{4} - \frac{3 \text{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b})}{4}}{b^2}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*arcsin(c\*x))^2,x)

[Out]  $1/c^3 * (-1/4 * (-c^2 * x^2 + 1)^{1/2} / (a + b * \arcsin(c * x)) / b - 1/4 * (\text{Si}(\arcsin(c * x) + a/b) * \cos(a/b) - \text{Ci}(\arcsin(c * x) + a/b) * \sin(a/b)) / b^2 + 1/4 * \cos(3 * \arcsin(c * x)) / (a + b * \arcsin(c * x)) / b + 3/4 * (\text{Si}(3 * \arcsin(c * x) + 3 * a/b) * \cos(3 * a/b) - \text{Ci}(3 * \arcsin(c * x) + 3 * a/b) * \sin(3 * a/b)) / b^2)$

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] Timed out

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*asin(c\*x))^2,x)

[Out] int(x^2/(a + b\*asin(c\*x))^2, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*asin(c\*x))\*\*2,x)

[Out] Integral(x\*\*2/(a + b\*asin(c\*x))\*\*2, x)

$$3.164 \quad \int \frac{x}{(a+b \sin^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=90

$$\frac{\cos\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2(a+b \sin^{-1}(cx))}{b}\right)}{b^2 c^2} + \frac{\sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \sin^{-1}(cx))}{b}\right)}{b^2 c^2} - \frac{x\sqrt{1-c^2 x^2}}{bc(a+b \sin^{-1}(cx))}$$

[Out] Ci(2\*(a+b\*arcsin(c\*x))/b)\*cos(2\*a/b)/b^2/c^2+Si(2\*(a+b\*arcsin(c\*x))/b)\*sin(2\*a/b)/b^2/c^2-x\*(-c^2\*x^2+1)^(1/2)/b/c/(a+b\*arcsin(c\*x))

**Rubi [A]** time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4631, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{b^2 c^2} + \frac{\sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{b^2 c^2} - \frac{x\sqrt{1-c^2 x^2}}{bc(a+b \sin^{-1}(cx))}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*ArcSin[c\*x])^2,x]

[Out] -((x\*sqrt[1 - c^2\*x^2])/(b\*c\*(a + b\*ArcSin[c\*x]))) + (Cos[(2\*a)/b]\*CosIntegral[(2\*a)/b + 2\*ArcSin[c\*x]])/(b^2\*c^2) + (Sin[(2\*a)/b]\*SinIntegral[(2\*a)/b + 2\*ArcSin[c\*x]])/(b^2\*c^2)

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^m, x\_Symbol] := Simp[(x^m\*sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{x}{(a + b \sin^{-1}(cx))^2} dx &= -\frac{x\sqrt{1-c^2x^2}}{bc(a + b \sin^{-1}(cx))} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{bc^2} \\ &= -\frac{x\sqrt{1-c^2x^2}}{bc(a + b \sin^{-1}(cx))} + \frac{\cos\left(\frac{2a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{bc^2} + \frac{\sin\left(\frac{2a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{bc^2} \\ &= -\frac{x\sqrt{1-c^2x^2}}{bc(a + b \sin^{-1}(cx))} + \frac{\cos\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{b^2c^2} + \frac{\sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{b^2c^2} \end{aligned}$$

**Mathematica [A]** time = 0.29, size = 79, normalized size = 0.88

$$\frac{-\frac{bcx\sqrt{1-c^2x^2}}{a+b\sin^{-1}(cx)} + \cos\left(\frac{2a}{b}\right) \text{Ci}\left(2\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right) + \sin\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right)}{b^2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*ArcSin[c\*x])^2,x]

[Out]  $-\left(\frac{b*c*x*\text{Sqrt}[1 - c^2*x^2]}{a + b*\text{ArcSin}[c*x]}\right) + \text{Cos}[(2*a)/b]*\text{CosIntegral}[2*(a/b + \text{ArcSin}[c*x])] + \text{Sin}[(2*a)/b]*\text{SinIntegral}[2*(a/b + \text{ArcSin}[c*x])]$   
/(b^2\*c^2)

**fricas [F]** time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral(x/(b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2), x)

**giac [B]** time = 0.66, size = 326, normalized size = 3.62

$$\frac{2b \arcsin(cx) \cos\left(\frac{a}{b}\right)^2 \text{Ci}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{b^3c^2 \arcsin(cx) + ab^2c^2} + \frac{2b \arcsin(cx) \cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{b^3c^2 \arcsin(cx) + ab^2c^2} + \frac{2a \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{b^3c^2 \arcsin(cx) + ab^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out]  $2*b*\arcsin(c*x)*\cos(a/b)^2*\text{cos\_integral}(2*a/b + 2*\arcsin(c*x))/(b^3*c^2*\arcsin(c*x) + a*b^2*c^2) + 2*b*\arcsin(c*x)*\cos(a/b)*\sin(a/b)*\text{sin\_integral}(2*a/b + 2*\arcsin(c*x))/(b^3*c^2*\arcsin(c*x) + a*b^2*c^2) + 2*a*\cos(a/b)^2*\text{cos\_integral}(2*a/b + 2*\arcsin(c*x))/(b^3*c^2*\arcsin(c*x) + a*b^2*c^2) + 2*a*\cos(a/b)*\sin(a/b)*\text{sin\_integral}(2*a/b + 2*\arcsin(c*x))/(b^3*c^2*\arcsin(c*x) + a*b^2*c^2) - \text{sqrt}(-c^2*x^2 + 1)*b*c*x/(b^3*c^2*\arcsin(c*x) + a*b^2*c^2) - b*a*\arcsin(c*x)*\text{cos\_integral}(2*a/b + 2*\arcsin(c*x))/(b^3*c^2*\arcsin(c*x) + a*b^2*c^2) - a*\text{cos\_integral}(2*a/b + 2*\arcsin(c*x))/(b^3*c^2*\arcsin(c*x) + a*b^2*c^2)$

**maple [A]** time = 0.03, size = 77, normalized size = 0.86

$$\frac{-\frac{\sin(2\arcsin(cx))}{2(a+b\arcsin(cx))b} + \frac{\text{Si}\left(2\arcsin(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)+\text{Ci}\left(2\arcsin(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{b^2}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*arcsin(c*x))^2,x)`

[Out]  $1/c^2*(-1/2*\sin(2*\arcsin(c*x))/(a+b*\arcsin(c*x))/b+(Si(2*\arcsin(c*x)+2*a/b)*\sin(2*a/b)+Ci(2*\arcsin(c*x)+2*a/b)*\cos(2*a/b))/b^2$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*asin(c*x))^2,x)`

[Out] `int(x/(a + b*asin(c*x))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*asin(c*x))**2,x)`

[Out] `Integral(x/(a + b*asin(c*x))**2, x)`

$$3.165 \quad \int \frac{1}{(a+b \sin^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=86

$$\frac{\sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{b^2c} - \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{b^2c} - \frac{\sqrt{1-c^2x^2}}{bc(a+b \sin^{-1}(cx))}$$

[Out]  $-\cos(a/b) \cdot \text{Si}((a+b \cdot \arcsin(cx))/b) / b^2/c + \text{Ci}((a+b \cdot \arcsin(cx))/b) \cdot \sin(a/b) / b^2/c - (\sqrt{1-c^2x^2})^{1/2} / b/c / (a+b \cdot \arcsin(cx))$

**Rubi [A]** time = 0.17, antiderivative size = 82, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4621, 4723, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{b^2c} - \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{b^2c} - \frac{\sqrt{1-c^2x^2}}{bc(a+b \sin^{-1}(cx))}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^(-2), x]

[Out]  $-(\text{Sqrt}[1 - c^2x^2] / (b*c*(a + b*ArcSin[c*x]))) + (\text{CosIntegral}[a/b + ArcSin[c*x]] * \text{Sin}[a/b]) / (b^2*c) - (\text{Cos}[a/b] * \text{SinIntegral}[a/b + ArcSin[c*x]]) / (b^2*c)$

**Rule 3299**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

**Rule 3302**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

**Rule 3303**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

**Rule 4621**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

**Rule 4723**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^(m\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sin^{-1}(cx))^2} dx &= -\frac{\sqrt{1-c^2x^2}}{bc(a + b \sin^{-1}(cx))} - \frac{c \int \frac{x}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))} dx}{b} \\
&= -\frac{\sqrt{1-c^2x^2}}{bc(a + b \sin^{-1}(cx))} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{bc} \\
&= -\frac{\sqrt{1-c^2x^2}}{bc(a + b \sin^{-1}(cx))} - \frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx} dx, x, \sin^{-1}(cx)\right)}{bc} \\
&= -\frac{\sqrt{1-c^2x^2}}{bc(a + b \sin^{-1}(cx))} + \frac{\text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right) \sin\left(\frac{a}{b}\right)}{b^2c} - \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{b^2c}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 72, normalized size = 0.84

$$\frac{-\frac{b\sqrt{1-c^2x^2}}{a+b \sin^{-1}(cx)} + \sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right) - \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^(-2), x]

[Out] (-((b\*Sqrt[1 - c^2\*x^2])/(a + b\*ArcSin[c\*x])) + CosIntegral[a/b + ArcSin[c\*x]]\*Sin[a/b] - Cos[a/b]\*SinIntegral[a/b + ArcSin[c\*x]])/(b^2\*c)

**fricas [F]** time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2), x)

**giac [B]** time = 0.49, size = 192, normalized size = 2.23

$$\frac{b \arcsin(cx) \text{Ci}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{b^3c \arcsin(cx) + ab^2c} - \frac{b \arcsin(cx) \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{b^3c \arcsin(cx) + ab^2c} + \frac{a \text{Ci}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{b^3c \arcsin(cx) + ab^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] b\*arcsin(c\*x)\*cos\_integral(a/b + arcsin(c\*x))\*sin(a/b)/(b^3\*c\*arcsin(c\*x) + a\*b^2\*c) - b\*arcsin(c\*x)\*cos(a/b)\*sin\_integral(a/b + arcsin(c\*x))/(b^3\*c\*arcsin(c\*x) + a\*b^2\*c) + a\*cos\_integral(a/b + arcsin(c\*x))\*sin(a/b)/(b^3\*c\*arcsin(c\*x) + a\*b^2\*c) - a\*cos(a/b)\*sin\_integral(a/b + arcsin(c\*x))/(b^3\*c\*arcsin(c\*x) + a\*b^2\*c) - sqrt(-c^2\*x^2 + 1)\*b/(b^3\*c\*arcsin(c\*x) + a\*b^2\*c)

**maple [A]** time = 0.04, size = 76, normalized size = 0.88

$$\frac{\sqrt{-c^2x^2+1}}{(a+b \arcsin(cx))b} - \frac{\text{Si}\left(\arcsin(cx)+\frac{a}{b}\right) \cos\left(\frac{a}{b}\right) - \text{Ci}\left(\arcsin(cx)+\frac{a}{b}\right) \sin\left(\frac{a}{b}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*arcsin(c*x))^2,x)`

[Out] `1/c*(-(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))/b-(Si(arcsin(c*x)+a/b)*cos(a/b)-Ci(arcsin(c*x)+a/b)*sin(a/b))/b^2)`

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*asin(c*x))^2,x)`

[Out] `int(1/(a + b*asin(c*x))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*asin(c*x))**2,x)`

[Out] `Integral((a + b*asin(c*x))**(-2), x)`

$$3.166 \quad \int \frac{1}{x(a+b \sin^{-1}(cx))^2} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x(a+b \sin^{-1}(cx))^2}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*arcsin(c\*x))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSin[c\*x])^2), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSin[c\*x])^2), x]

Rubi steps

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^2} dx = \int \frac{1}{x(a+b \sin^{-1}(cx))^2} dx$$

Mathematica [A] time = 4.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^2), x]

[Out] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^2), x]

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x \arcsin(cx)^2 + 2abx \arcsin(cx) + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x\*arcsin(c\*x)^2 + 2\*a\*b\*x\*arcsin(c\*x) + a^2\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")



[Out] integrate(1/((b\*arcsin(c\*x) + a)^2\*x), x)

**maple** [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsin(c\*x))^2,x)

[Out] int(1/x/(a+b\*arcsin(c\*x))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x(a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*asin(c\*x))^2),x)

[Out] int(1/(x\*(a + b\*asin(c\*x))^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*asin(c\*x))\*\*2,x)

[Out] Integral(1/(x\*(a + b\*asin(c\*x))\*\*2), x)

$$3.167 \quad \int \frac{1}{x^2(a+b \sin^{-1}(cx))^2} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x^2(a+b \sin^{-1}(cx))^2}, x\right)$$

[Out] Unintegrable(1/x^2/(a+b\*arcsin(c\*x))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*ArcSin[c\*x])^2), x]

[Out] Defer[Int][1/(x^2\*(a + b\*ArcSin[c\*x])^2), x]

Rubi steps

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))^2} dx = \int \frac{1}{x^2(a+b \sin^{-1}(cx))^2} dx$$

Mathematica [A] time = 34.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^2), x]

[Out] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^2), x]

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x^2 \arcsin(cx)^2 + 2abx^2 \arcsin(cx) + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^2\*arcsin(c\*x)^2 + 2\*a\*b\*x^2\*arcsin(c\*x) + a^2\*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^2\*x^2), x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsin(c\*x))^2,x)

[Out] int(1/x^2/(a+b\*arcsin(c\*x))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^2 (a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*asin(c\*x))^2),x)

[Out] int(1/(x^2\*(a + b\*asin(c\*x))^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asin(c\*x))\*\*2,x)

[Out] Integral(1/(x\*\*2\*(a + b\*asin(c\*x))\*\*2), x)

$$3.168 \quad \int \frac{x^2}{(a+b \sin^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=197

$$-\frac{\cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{8b^3c^3} + \frac{9 \cos\left(\frac{3a}{b}\right) \operatorname{Ci}\left(\frac{3(a+b \sin^{-1}(cx))}{b}\right)}{8b^3c^3} - \frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{8b^3c^3} + \frac{9 \sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \sin^{-1}(cx))}{b}\right)}{8b^3c^3}$$

[Out]  $-x/b^2/c^2/(a+b*\arcsin(c*x))+3/2*x^3/b^2/(a+b*\arcsin(c*x))-1/8*\operatorname{Ci}((a+b*\arcsin(c*x))/b)*\cos(a/b)/b^3/c^3+9/8*\operatorname{Ci}(3*(a+b*\arcsin(c*x))/b)*\cos(3*a/b)/b^3/c^3-1/8*\operatorname{Si}((a+b*\arcsin(c*x))/b)*\sin(a/b)/b^3/c^3+9/8*\operatorname{Si}(3*(a+b*\arcsin(c*x))/b)*\sin(3*a/b)/b^3/c^3-1/2*x^2*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))^2$

**Rubi [A]** time = 0.54, antiderivative size = 245, normalized size of antiderivative = 1.24, number of steps used = 16, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {4633, 4719, 4635, 4406, 3303, 3299, 3302, 4623}

$$-\frac{9 \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{8b^3c^3} + \frac{9 \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3a}{b} + 3 \sin^{-1}(cx)\right)}{8b^3c^3} + \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{b^3c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*ArcSin[c\*x])^3,x]

[Out]  $-(x^2*\sqrt{1-c^2*x^2})/(2*b*c*(a+b*\operatorname{ArcSin}[c*x])^2) - x/(b^2*c^2*(a+b*\operatorname{ArcSin}[c*x])) + (3*x^3)/(2*b^2*(a+b*\operatorname{ArcSin}[c*x])) - (9*\operatorname{Cos}[a/b]*\operatorname{CosIntegral}[a/b + \operatorname{ArcSin}[c*x]])/(8*b^3*c^3) + (9*\operatorname{Cos}[(3*a)/b]*\operatorname{CosIntegral}[(3*a)/b + 3*\operatorname{ArcSin}[c*x]])/(8*b^3*c^3) + (\operatorname{Cos}[a/b]*\operatorname{CosIntegral}[(a+b*\operatorname{ArcSin}[c*x])/b])/(b^3*c^3) - (9*\operatorname{Sin}[a/b]*\operatorname{SinIntegral}[a/b + \operatorname{ArcSin}[c*x]])/(8*b^3*c^3) + (9*\operatorname{Sin}[(3*a)/b]*\operatorname{SinIntegral}[(3*a)/b + 3*\operatorname{ArcSin}[c*x]])/(8*b^3*c^3) + (\operatorname{Sin}[a/b]*\operatorname{SinIntegral}[(a+b*\operatorname{ArcSin}[c*x])/b])/(b^3*c^3)$

#### Rule 3299

Int[sin[(e.) + (f.)\*(x\_)]/((c.) + (d.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e.) + (f.)\*(x\_)]/((c.) + (d.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 3303

Int[sin[(e.) + (f.)\*(x\_)]/((c.) + (d.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 4406

Int[Cos[(a.) + (b.)\*(x\_)]^(p.)\*((c.) + (d.)\*(x\_))^(m.)\*Sin[(a.) + (b.)\*(x\_)]^(n.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]]^n\*Cos[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*((f\_.)\*(x\_)^(m\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + b \sin^{-1}(cx))^3} dx &= -\frac{x^2 \sqrt{1 - c^2 x^2}}{2bc (a + b \sin^{-1}(cx))^2} + \frac{\int \frac{x}{\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^2} dx}{bc} - \frac{(3c) \int \frac{x^3}{\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^2} dx}{2b} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{2bc (a + b \sin^{-1}(cx))^2} - \frac{x}{b^2 c^2 (a + b \sin^{-1}(cx))} + \frac{3x^3}{2b^2 (a + b \sin^{-1}(cx))} - \frac{9 \int \frac{x^2}{a + b \sin^{-1}(cx)} dx}{2b^2} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{2bc (a + b \sin^{-1}(cx))^2} - \frac{x}{b^2 c^2 (a + b \sin^{-1}(cx))} + \frac{3x^3}{2b^2 (a + b \sin^{-1}(cx))} + \frac{\text{Subst} \left( \int \frac{x^2}{a + b \sin^{-1}(cx)} dx \right)}{2b^2} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{2bc (a + b \sin^{-1}(cx))^2} - \frac{x}{b^2 c^2 (a + b \sin^{-1}(cx))} + \frac{3x^3}{2b^2 (a + b \sin^{-1}(cx))} - \frac{9 \text{Subst} \left( \int \frac{x^2}{a + b \sin^{-1}(cx)} dx \right)}{2b^2} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{2bc (a + b \sin^{-1}(cx))^2} - \frac{x}{b^2 c^2 (a + b \sin^{-1}(cx))} + \frac{3x^3}{2b^2 (a + b \sin^{-1}(cx))} + \frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{b^2} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{2bc (a + b \sin^{-1}(cx))^2} - \frac{x}{b^2 c^2 (a + b \sin^{-1}(cx))} + \frac{3x^3}{2b^2 (a + b \sin^{-1}(cx))} + \frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{b^2} \\
&= -\frac{x^2 \sqrt{1 - c^2 x^2}}{2bc (a + b \sin^{-1}(cx))^2} - \frac{x}{b^2 c^2 (a + b \sin^{-1}(cx))} + \frac{3x^3}{2b^2 (a + b \sin^{-1}(cx))} - \frac{9 \cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{2b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.87, size = 168, normalized size = 0.85

$$\frac{4b^2 x^2 \sqrt{1 - c^2 x^2}}{c(a + b \sin^{-1}(cx))^2} + \frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{c^3} - \frac{9 \cos\left(\frac{3a}{b}\right) \text{Ci}\left(3\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right)}{c^3} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{c^3} - \frac{9 \sin\left(\frac{3a}{b}\right) \text{Si}\left(3\left(\frac{a}{b} + \sin^{-1}(cx)\right)\right)}{c^3} + \frac{c^2}{8b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*ArcSin[c\*x])^3,x]

[Out] -1/8\*((4\*b^2\*x^2\*Sqrt[1 - c^2\*x^2])/(c\*(a + b\*ArcSin[c\*x])^2) + (8\*b\*x)/(c^2\*(a + b\*ArcSin[c\*x])) - (12\*b\*x^3)/(a + b\*ArcSin[c\*x]) + (Cos[a/b]\*CosIntegral[a/b + ArcSin[c\*x]])/c^3 - (9\*Cos[(3\*a)/b]\*CosIntegral[3\*(a/b + ArcSin[c\*x])])/c^3 + (Sin[a/b]\*SinIntegral[a/b + ArcSin[c\*x]])/c^3 - (9\*Sin[(3\*a)/b]\*SinIntegral[3\*(a/b + ArcSin[c\*x])])/c^3)/b^3

**fricas [F]** time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out] integral(x^2/(b^3\*arcsin(c\*x)^3 + 3\*a\*b^2\*arcsin(c\*x)^2 + 3\*a^2\*b\*arcsin(c\*x) + a^3), x)

**giac [B]** time = 0.82, size = 1539, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out] 
$$\frac{9/2*b^2*arcsin(c*x)^2*cos(a/b)^3*cos\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 9/2*b^2*arcsin(c*x)^2*cos(a/b)^2*sin(a/b)*sin\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 9*a*b*arcsin(c*x)*cos(a/b)^3*cos\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 9*a*b*arcsin(c*x)*cos(a/b)^2*sin(a/b)*sin\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 3/2*(c^2*x^2 - 1)*b^2*c*x*arcsin(c*x)/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 27/8*b^2*arcsin(c*x)^2*cos(a/b)*cos\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 9/2*a^2*cos(a/b)^3*cos\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 1/8*b^2*arcsin(c*x)^2*cos(a/b)*cos\_integral(a/b + arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 9/8*b^2*arcsin(c*x)^2*sin(a/b)*sin\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 9/2*a^2*cos(a/b)^2*sin(a/b)*sin\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 1/8*b^2*arcsin(c*x)^2*sin(a/b)*sin\_integral(a/b + arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 3/2*(c^2*x^2 - 1)*a*b*c*x/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 1/2*b^2*c*x*arcsin(c*x)/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 27/4*a*b*arcsin(c*x)*cos(a/b)*cos\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + a^2*b^3*c^3) - 1/4*a*b*arcsin(c*x)*cos(a/b)*cos\_integral(a/b + arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 9/4*a*b*arcsin(c*x)*sin(a/b)*sin\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 1/4*a*b*arcsin(c*x)*sin(a/b)*sin\_integral(a/b + arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 1/2*a*b*c*x/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 27/8*a^2*cos(a/b)*cos\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 1/8*a^2*cos(a/b)*cos\_integral(a/b + arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 9/8*a^2*sin(a/b)*sin\_integral(3*a/b + 3*arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 1/8*a^2*sin(a/b)*sin\_integral(a/b + arcsin(c*x))/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) + 1/2*(-c^2*x^2 + 1)^(3/2)*b^2/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3) - 1/2*sqrt(-c^2*x^2 + 1)*b^2/(b^5*c^3*arcsin(c*x)^2 + 2*a*b^4*c^3*arcsin(c*x) + a^2*b^3*c^3)$$

**maple [A]** time = 0.04, size = 290, normalized size = 1.47

$$\frac{\sqrt{-c^2x^2+1}}{8(a+b\arcsin(cx))^2b} - \frac{\arcsin(cx)\operatorname{Si}\left(\arcsin(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\arcsin(cx)\operatorname{Ci}\left(\arcsin(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b+\operatorname{Si}\left(\arcsin(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)a+\operatorname{Ci}\left(\arcsin(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)a}{8(a+b\arcsin(cx))b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*arcsin(c\*x))^3,x)

[Out] 
$$\frac{1}{c^3}*(-1/8*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2/b-1/8*(arcsin(c*x)*\operatorname{Si}\left(\arcsin(c*x)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)*b+\arcsin(c*x)*\operatorname{Ci}\left(\arcsin(c*x)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)*b+\operatorname{Si}\left(\arcsin(c*x)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)*a+\operatorname{Ci}\left(\arcsin(c*x)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)*a-x*b*c)/(a+b*arcsin(c*x))/b^3+1/8*\cos(3*arcsin(c*x))/(a+b*arcsin(c*x))^2/b+3/8*(3*arcsin(c*x)*\operatorname{Si}\left(3*arcsin(c*x)+3*a/b\right)\sin\left(3*a/b\right)*b+3*arcsin(c*x)*\operatorname{Ci}\left(3*arcsin(c*x)+3*a/b\right)\cos\left(3*a/b\right)*b+3*\operatorname{Si}\left(3*arcsin(c*x)+3*a/b\right)\sin\left(3*a/b\right)*a+3*\operatorname{Ci}\left(3*arcsin(c*x)+3*a/b\right)\cos\left(3*a/b\right)*a-\sin(3*arcsin(c*x))*b)/(a+b*arcsin(c*x))/b^3)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3ac^2x^3 - \sqrt{cx+1}\sqrt{-cx+1}bcx^2 - 2ax + (3bc^2x^3 - 2bx)\arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}) - (b^4c^2\arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}))}{2(b^4c^2\arctan(cx, \sqrt{cx+1}\sqrt{-cx+1})^2 + 2ab^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out] 1/2\*(3\*a\*c^2\*x^3 - sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b\*c\*x^2 - 2\*a\*x + (3\*b\*c^2\*x^3 - 2\*b\*x)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) - 2\*(b^4\*c^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*a\*b^3\*c^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a^2\*b^2\*c^2)\*integrate(1/2\*(9\*c^2\*x^2 - 2)/(b^3\*c^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b^2\*c^2), x))/(b^4\*c^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*a\*b^3\*c^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a^2\*b^2\*c^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*asin(c\*x))^3,x)

[Out] int(x^2/(a + b\*asin(c\*x))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*asin(c\*x))\*\*3,x)

[Out] Integral(x\*\*2/(a + b\*asin(c\*x))\*\*3, x)



$$3.169 \quad \int \frac{x}{(a+b \sin^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=130

$$\frac{\sin\left(\frac{2a}{b}\right) \operatorname{Ci}\left(\frac{2(a+b \sin^{-1}(cx))}{b}\right)}{b^3 c^2} - \frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \sin^{-1}(cx))}{b}\right)}{b^3 c^2} - \frac{1}{2b^2 c^2 (a+b \sin^{-1}(cx))} + \frac{x^2}{b^2 (a+b \sin^{-1}(cx))} - \frac{x}{2bc(a+b \sin^{-1}(cx))}$$

[Out]  $-1/2/b^2/c^2/(a+b*\arcsin(c*x))+x^2/b^2/(a+b*\arcsin(c*x))-\cos(2*a/b)*\operatorname{Si}(2*(a+b*\arcsin(c*x))/b)/b^3/c^2+\operatorname{Ci}(2*(a+b*\arcsin(c*x))/b)*\sin(2*a/b)/b^3/c^2-1/2*x*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))^2$

**Rubi [A]** time = 0.32, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {4633, 4719, 4635, 4406, 12, 3303, 3299, 3302, 4641}

$$\frac{\sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{b^3 c^2} - \frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2a}{b} + 2 \sin^{-1}(cx)\right)}{b^3 c^2} - \frac{1}{2b^2 c^2 (a+b \sin^{-1}(cx))} + \frac{x^2}{b^2 (a+b \sin^{-1}(cx))} - \frac{x}{2bc(a+b \sin^{-1}(cx))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x/(a + b*\operatorname{ArcSin}[c*x])^3, x]$

[Out]  $-(x*\operatorname{Sqrt}[1 - c^2*x^2])/(2*b*c*(a + b*\operatorname{ArcSin}[c*x])^2) - 1/(2*b^2*c^2*(a + b*\operatorname{ArcSin}[c*x])) + x^2/(b^2*(a + b*\operatorname{ArcSin}[c*x])) + (\operatorname{CosIntegral}[(2*a)/b + 2*\operatorname{ArcSin}[c*x])*\operatorname{Sin}[(2*a)/b])/(b^3*c^2) - (\operatorname{Cos}[(2*a)/b]*\operatorname{SinIntegral}[(2*a)/b + 2*\operatorname{ArcSin}[c*x]])/(b^3*c^2)$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)*(v_)] /;$   $\operatorname{FreeQ}[b, x]$

#### Rule 3299

$\operatorname{Int}[\sin[(e_*) + (f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{SinIntegral}[e + f*x]/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \ \operatorname{EqQ}[d*e - c*f, 0]$

#### Rule 3302

$\operatorname{Int}[\sin[(e_*) + (f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CosIntegral}[e - \operatorname{Pi}/2 + f*x]/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \ \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f, 0]$

#### Rule 3303

$\operatorname{Int}[\sin[(e_*) + (f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \ \operatorname{NeQ}[d*e - c*f, 0]$

#### Rule 4406

$\operatorname{Int}[\operatorname{Cos}[(a_*) + (b_*)*(x_)]^{(p_*)}*((c_*) + (d_*)*(x_))^{(m_*)}*\operatorname{Sin}[(a_*) + (b_*)*(x_)]^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + d*x)^m, \operatorname{Sin}[a + b*x]^n*\operatorname{Cos}[a + b*x]^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{IGtQ}[p, 0]$

Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> Simp[(a + b\*ArcSin[c\*x])^(n + 1)/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_.))^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + b \sin^{-1}(cx))^3} dx &= -\frac{x\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} + \frac{\int \frac{1}{\sqrt{1 - c^2x^2}(a + b \sin^{-1}(cx))^2} dx}{2bc} - \frac{c \int \frac{x^2}{\sqrt{1 - c^2x^2}(a + b \sin^{-1}(cx))^2} dx}{b} \\
&= -\frac{x\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} - \frac{1}{2b^2c^2(a + b \sin^{-1}(cx))} + \frac{x^2}{b^2(a + b \sin^{-1}(cx))} - \frac{2 \int \frac{x}{a + b \sin^{-1}(cx)} dx}{b^2} \\
&= -\frac{x\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} - \frac{1}{2b^2c^2(a + b \sin^{-1}(cx))} + \frac{x^2}{b^2(a + b \sin^{-1}(cx))} - \frac{2 \operatorname{Subst}\left(\int \frac{x}{a + b \sin^{-1}(cx)} dx\right)}{b^2} \\
&= -\frac{x\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} - \frac{1}{2b^2c^2(a + b \sin^{-1}(cx))} + \frac{x^2}{b^2(a + b \sin^{-1}(cx))} - \frac{2 \operatorname{Subst}\left(\int \frac{x}{a + b \sin^{-1}(cx)} dx\right)}{b^2} \\
&= -\frac{x\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} - \frac{1}{2b^2c^2(a + b \sin^{-1}(cx))} + \frac{x^2}{b^2(a + b \sin^{-1}(cx))} - \frac{\operatorname{Subst}\left(\int \frac{x}{a + b \sin^{-1}(cx)} dx\right)}{b^2} \\
&= -\frac{x\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} - \frac{1}{2b^2c^2(a + b \sin^{-1}(cx))} + \frac{x^2}{b^2(a + b \sin^{-1}(cx))} - \frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2a}{b} + 2 \operatorname{arcsin}(cx)\right)}{b^2} \\
&= -\frac{x\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} - \frac{1}{2b^2c^2(a + b \sin^{-1}(cx))} + \frac{x^2}{b^2(a + b \sin^{-1}(cx))} + \frac{\operatorname{Ci}\left(\frac{2a}{b} + 2 \operatorname{arcsin}(cx)\right)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.39, size = 108, normalized size = 0.83

$$\frac{-\frac{b^2 c x \sqrt{1-c^2 x^2}}{(a+b \sin^{-1}(c x))^2} + \frac{b(2c^2 x^2 - 1)}{a+b \sin^{-1}(c x)} + 2 \sin\left(\frac{2a}{b}\right) \text{Ci}\left(2\left(\frac{a}{b} + \sin^{-1}(c x)\right)\right) - 2 \cos\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \sin^{-1}(c x)\right)\right)}{2b^3 c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*ArcSin[c\*x])^3,x]

[Out]  $-(b^2 c x \sqrt{1-c^2 x^2}) / (a + b \text{ArcSin}[c x])^2 + (b(-1 + 2c^2 x^2)) / (a + b \text{ArcSin}[c x]) + 2 \text{CosIntegral}[2(a/b + \text{ArcSin}[c x])] * \text{Sin}[(2a)/b] - 2 \text{Cos}[(2a)/b] * \text{SinIntegral}[2(a/b + \text{ArcSin}[c x])] / (2b^3 c^2)$

**fricas [F]** time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out] integral(x/(b^3\*arcsin(c\*x)^3 + 3\*a\*b^2\*arcsin(c\*x)^2 + 3\*a^2\*b\*arcsin(c\*x) + a^3), x)

**giac [B]** time = 1.52, size = 864, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out]  $2b^2 \arcsin(cx)^2 \cos(a/b) \cos\_integral(2a/b + 2 \arcsin(cx)) \sin(a/b) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) - 2b^2 \arcsin(cx)^2 \cos(a/b)^2 \sin\_integral(2a/b + 2 \arcsin(cx)) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + 4a b \arcsin(cx) \cos(a/b) \cos\_integral(2a/b + 2 \arcsin(cx)) \sin(a/b) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) - 4a b \arcsin(cx) \cos(a/b)^2 \sin\_integral(2a/b + 2 \arcsin(cx)) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + 2a^2 \cos(a/b) \cos\_integral(2a/b + 2 \arcsin(cx)) \sin(a/b) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + b^2 \arcsin(cx)^2 \sin\_integral(2a/b + 2 \arcsin(cx)) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) - 2a^2 \cos(a/b)^2 \sin\_integral(2a/b + 2 \arcsin(cx)) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) - 1/2 \sqrt{-c^2 x^2 + 1} b^2 c x / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + (c^2 x^2 - 1) b^2 \arcsin(cx) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + 2a b \arcsin(cx) \sin\_integral(2a/b + 2 \arcsin(cx)) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + (c^2 x^2 - 1) a b / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + 1/2 b^2 \arcsin(cx) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + a^2 \sin\_integral(2a/b + 2 \arcsin(cx)) / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2) + 1/2 a b / (b^5 c^2 \arcsin(cx)^2 + 2a b^4 c^2 \arcsin(cx) + a^2 b^3 c^2)$

**maple [A]** time = 0.03, size = 157, normalized size = 1.21

$$\frac{\frac{\sin(2 \arcsin(cx))}{4(a+b \arcsin(cx))^2 b} - \frac{2 \text{Si}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) \arcsin(cx) b - 2 \text{Ci}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right) \arcsin(cx) b + 2 \text{Si}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) a}{2(a+b \arcsin(cx)) b^3}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*arcsin(c*x))^3,x)`

[Out]  $\frac{1}{c^2} \left( -\frac{1}{4} \sin(2 \arcsin(cx)) / (a+b \arcsin(cx))^2 / b - \frac{1}{2} (2 \operatorname{Si}(2 \arcsin(cx)) + 2a/b) \cos(2a/b) \arcsin(cx) * b - 2 \operatorname{Ci}(2 \arcsin(cx) + 2a/b) \sin(2a/b) \arcsin(cx) * b + 2 \operatorname{Si}(2 \arcsin(cx) + 2a/b) \cos(2a/b) a - 2 \operatorname{Ci}(2 \arcsin(cx) + 2a/b) \sin(2a/b) a + \cos(2 \arcsin(cx)) * b \right) / (a+b \arcsin(cx)) / b^3$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2ac^2x^2 - \sqrt{cx+1}\sqrt{-cx+1}bcx + (2bc^2x^2 - b) \arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}) - a - \frac{4(b^4c^2 \arctan(cx, \sqrt{cx+1}\sqrt{-cx+1})^2}{2(b^4c^2 \arctan(cx, \sqrt{cx+1}\sqrt{-cx+1})^2 + 2ab^3c^2 \arctan(cx, \sqrt{cx+1}\sqrt{-cx+1})^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arcsin(c*x))^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} (2ac^2x^2 - \sqrt{cx+1}\sqrt{-cx+1}bcx + (2bc^2x^2 - b) \arctan2(cx, \sqrt{cx+1}\sqrt{-cx+1}) - 4(b^4c^2 \arctan2(cx, \sqrt{cx+1}\sqrt{-cx+1})^2 + 2ab^3c^2 \arctan2(cx, \sqrt{cx+1}\sqrt{-cx+1})^2 + a^2b^2c^2) \int \frac{x}{(a+b \arcsin(cx))^3} dx - a) / (b^4c^2 \arctan2(cx, \sqrt{cx+1}\sqrt{-cx+1})^2 + 2ab^3c^2 \arctan2(cx, \sqrt{cx+1}\sqrt{-cx+1})^2 + a^2b^2c^2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*asin(c*x))^3,x)`

[Out] `int(x/(a + b*asin(c*x))^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*asin(c*x))**3,x)`

[Out] `Integral(x/(a + b*asin(c*x))**3, x)`

$$3.170 \quad \int \frac{1}{(a+b \sin^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=111

$$\frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{2b^3c} - \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{2b^3c} + \frac{x}{2b^2(a+b \sin^{-1}(cx))} - \frac{\sqrt{1-c^2x^2}}{2bc(a+b \sin^{-1}(cx))^2}$$

[Out] 1/2\*x/b^2/(a+b\*arcsin(c\*x))-1/2\*Ci((a+b\*arcsin(c\*x))/b)\*cos(a/b)/b^3/c-1/2\*Si((a+b\*arcsin(c\*x))/b)\*sin(a/b)/b^3/c-1/2\*(-c^2\*x^2+1)^(1/2)/b/c/(a+b\*arcsin(c\*x))^2

**Rubi [A]** time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {4621, 4719, 4623, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{2b^3c} - \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \sin^{-1}(cx)}{b}\right)}{2b^3c} + \frac{x}{2b^2(a+b \sin^{-1}(cx))} - \frac{\sqrt{1-c^2x^2}}{2bc(a+b \sin^{-1}(cx))^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^(-3), x]

[Out] -Sqrt[1 - c^2\*x^2]/(2\*b\*c\*(a + b\*ArcSin[c\*x])^2) + x/(2\*b^2\*(a + b\*ArcSin[c\*x])) - (Cos[a/b]\*CosIntegral[(a + b\*ArcSin[c\*x])/b])/(2\*b^3\*c) - (Sin[a/b]\*SinIntegral[(a + b\*ArcSin[c\*x])/b])/(2\*b^3\*c)

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 4621

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Simp[(Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] + Dist[c/(b\*(n + 1)), Int[(x\*(a + b\*ArcSin[c\*x])^(n + 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n, x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n \* Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

## Rule 4719

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*((f\_.)\*(x\_.))^ (m\_.)]/Sqrt[(d\_. + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[((f\*x)^m\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] & & EqQ[c^2\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]

## Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \sin^{-1}(cx))^3} dx &= -\frac{\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} - \frac{c \int \frac{x}{\sqrt{1 - c^2x^2}(a + b \sin^{-1}(cx))^2} dx}{2b} \\ &= -\frac{\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} + \frac{x}{2b^2(a + b \sin^{-1}(cx))} - \frac{\int \frac{1}{a + b \sin^{-1}(cx)} dx}{2b^2} \\ &= -\frac{\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} + \frac{x}{2b^2(a + b \sin^{-1}(cx))} - \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a-x}{b}\right)}{x} dx, x, a + b \sin^{-1}(cx)\right)}{2b^3c} \\ &= -\frac{\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} + \frac{x}{2b^2(a + b \sin^{-1}(cx))} - \frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x} dx, x, a + b \sin^{-1}(cx)\right)}{2b^3c} \\ &= -\frac{\sqrt{1 - c^2x^2}}{2bc(a + b \sin^{-1}(cx))^2} + \frac{x}{2b^2(a + b \sin^{-1}(cx))} - \frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a + b \sin^{-1}(cx)}{b}\right)}{2b^3c} - \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a + b \sin^{-1}(cx)}{b}\right)}{2b^3c} \end{aligned}$$

**Mathematica [A]** time = 0.46, size = 93, normalized size = 0.84

$$\frac{b\left(\frac{b\sqrt{1-c^2x^2}}{c} - x(a + b \sin^{-1}(cx))\right)}{(a + b \sin^{-1}(cx))^2} + \frac{\cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{c} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sin^{-1}(cx)\right)}{c}$$


---


$$2b^3$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^(-3), x]

[Out] -1/2\*((b\*((b\*Sqrt[1 - c^2\*x^2])/c - x\*(a + b\*ArcSin[c\*x])))/(a + b\*ArcSin[c\*x])^2 + (Cos[a/b]\*CosIntegral[a/b + ArcSin[c\*x]])/c + (Sin[a/b]\*SinIntegral[a/b + ArcSin[c\*x]])/c)/b^3

**fricas [F]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*arcsin(c\*x)^3 + 3\*a\*b^2\*arcsin(c\*x)^2 + 3\*a^2\*b\*arcsin(c\*x) + a^3), x)

**giac [B]** time = 1.07, size = 482, normalized size = 4.34

$$\frac{b^2 \arcsin(cx)^2 \cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \arcsin(cx)\right)}{2(b^5c \arcsin(cx)^2 + 2ab^4c \arcsin(cx) + a^2b^3c)} - \frac{b^2 \arcsin(cx)^2 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{2(b^5c \arcsin(cx)^2 + 2ab^4c \arcsin(cx) + a^2b^3c)} + \frac{\dots}{2(b^5c \arcsin(cx)^2 + 2ab^4c \arcsin(cx) + a^2b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out] 
$$-1/2*b^2*arcsin(c*x)^2*cos(a/b)*cos\_integral(a/b + arcsin(c*x))/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) - 1/2*b^2*arcsin(c*x)^2*sin(a/b)*sin\_integral(a/b + arcsin(c*x))/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) + 1/2*b^2*c*x*arcsin(c*x)/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) - a*b*arcsin(c*x)*cos(a/b)*cos\_integral(a/b + arcsin(c*x))/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) - a*b*arcsin(c*x)*sin(a/b)*sin\_integral(a/b + arcsin(c*x))/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) + 1/2*a*b*c*x/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) - 1/2*a^2*cos(a/b)*cos\_integral(a/b + arcsin(c*x))/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) - 1/2*a^2*sin(a/b)*sin\_integral(a/b + arcsin(c*x))/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c) - 1/2*sqrt(-c^2*x^2 + 1)*b^2/(b^5*c*arcsin(c*x)^2 + 2*a*b^4*c*arcsin(c*x) + a^2*b^3*c)$$

**maple** [A] time = 0.04, size = 138, normalized size = 1.24

$$\frac{\sqrt{-c^2x^2+1}}{2(a+b \arcsin(cx))^2b} - \frac{\arcsin(cx) \operatorname{Si}\left(\arcsin(cx)+\frac{a}{b}\right) \sin\left(\frac{a}{b}\right)b + \arcsin(cx) \operatorname{Ci}\left(\arcsin(cx)+\frac{a}{b}\right) \cos\left(\frac{a}{b}\right)b + \operatorname{Si}\left(\arcsin(cx)+\frac{a}{b}\right) \sin\left(\frac{a}{b}\right)a + \operatorname{Ci}\left(\arcsin(cx)+\frac{a}{b}\right) \cos\left(\frac{a}{b}\right)a}{2(a+b \arcsin(cx))b^3}$$

c

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arcsin(c\*x))^3,x)

[Out] 
$$1/c*(-1/2*(-c^2*x^2+1)^{(1/2)}/(a+b*arcsin(c*x))^2/b-1/2*(arcsin(c*x)*\operatorname{Si}(arcsin(c*x)+a/b)*\sin(a/b)*b+arcsin(c*x)*\operatorname{Ci}(arcsin(c*x)+a/b)*\cos(a/b)*b+\operatorname{Si}(arcsin(c*x)+a/b)*\sin(a/b)*a+\operatorname{Ci}(arcsin(c*x)+a/b)*\cos(a/b)*a-x*b*c)/(a+b*arcsin(c*x))/b^3)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcx \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right) + acx - \sqrt{cx+1} \sqrt{-cx+1} b - \left(b^4c \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^2 + 2ab^3c \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)\right)}{2\left(b^4c \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^2 + 2ab^3c \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out] 
$$1/2*(b*c*x*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1}) + a*c*x - \sqrt{c*x + 1}*\sqrt{-c*x + 1}*b - 2*(b^4*c*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1})^2 + 2*a*b^3*c*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1}) + a^2*b^2*c)*integrate(1/2/(b^3*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1}) + a*b^2), x))/(b^4*c*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1})^2 + 2*a*b^3*c*\arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1}) + a^2*b^2*c)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*asin(c\*x))^3,x)

[Out] int(1/(a + b\*asin(c\*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \sin(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asin(c*x))**3,x)
```

```
[Out] Integral((a + b*asin(c*x))**(-3), x)
```



$$3.171 \quad \int \frac{1}{x(a+b \sin^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=17

$$\text{Int} \left( \frac{1}{x(a+b \sin^{-1}(cx))^3}, x \right)$$

[Out] Unintegrable(1/x/(a+b\*arcsin(c\*x))^3,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSin[c\*x])^3), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSin[c\*x])^3), x]

Rubi steps

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^3} dx = \int \frac{1}{x(a+b \sin^{-1}(cx))^3} dx$$

**Mathematica [A]** time = 2.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^3), x]

[Out] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^3), x]

**fricas [A]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^3 x \arcsin(cx)^3 + 3 a b^2 x \arcsin(cx)^2 + 3 a^2 b x \arcsin(cx) + a^3 x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*x\*arcsin(c\*x)^3 + 3\*a\*b^2\*x\*arcsin(c\*x)^2 + 3\*a^2\*b\*x\*arcsin(c\*x) + a^3\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^3\*x), x)

**maple** [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \arcsin(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsin(c\*x))^3,x)

[Out] int(1/x/(a+b\*arcsin(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{cx+1}\sqrt{-cx+1}bcx - b \arctan\left(cx, \sqrt{cx+1}\sqrt{-cx+1}\right) - a - \frac{2\left(b^4c^2x^2 \arctan\left(cx, \sqrt{cx+1}\sqrt{-cx+1}\right)^2 + 2ab^3c^2x^2 \arctan\left(cx, \sqrt{cx+1}\sqrt{-cx+1}\right)\right)}{b^2}}{2\left(b^4c^2x^2 \arctan\left(cx, \sqrt{cx+1}\sqrt{-cx+1}\right)^2 + 2ab^3c^2x^2 \arctan\left(cx, \sqrt{cx+1}\sqrt{-cx+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out] -1/2\*(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b\*c\*x - b\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) - 2\*(b^4\*c^2\*x^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*a\*b^3\*c^2\*x^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a^2\*b^2\*c^2\*x^2)\*integrate(1/(b^3\*c^2\*x^3\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b^2\*c^2\*x^3), x) - a)/(b^4\*c^2\*x^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*a\*b^3\*c^2\*x^2\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a^2\*b^2\*c^2\*x^2)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x(a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*asin(c\*x))^3),x)

[Out] int(1/(x\*(a + b\*asin(c\*x))^3), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*asin(c\*x))\*\*3,x)

[Out] Integral(1/(x\*(a + b\*asin(c\*x))\*\*3), x)

$$3.172 \quad \int \frac{1}{x^2(a+b \sin^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=17

$$\text{Int}\left(\frac{1}{x^2(a+b \sin^{-1}(cx))^3}, x\right)$$

[Out] Unintegrable(1/x^2/(a+b\*arcsin(c\*x))^3,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*ArcSin[c\*x])^3), x]

[Out] Defer[Int][1/(x^2\*(a + b\*ArcSin[c\*x])^3), x]

Rubi steps

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))^3} dx = \int \frac{1}{x^2(a+b \sin^{-1}(cx))^3} dx$$

**Mathematica [A]** time = 19.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+b \sin^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^3), x]

[Out] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^3), x]

**fricas [A]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x^2 \arcsin(cx)^3 + 3ab^2x^2 \arcsin(cx)^2 + 3a^2bx^2 \arcsin(cx) + a^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*x^2\*arcsin(c\*x)^3 + 3\*a\*b^2\*x^2\*arcsin(c\*x)^2 + 3\*a^2\*b\*x^2\*arcsin(c\*x) + a^3\*x^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^3,x, algorithm="giac")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^3\*x^2), x)

**maple** [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \arcsin(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsin(c\*x))^3,x)

[Out] int(1/x^2/(a+b\*arcsin(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac^2x^2 + \sqrt{cx+1}\sqrt{-cx+1}bcx + (bc^2x^2 - 2b)\arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}) + (b^4c^2x^3\arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}) - 2ab^3)}{2(b^4c^2x^3\arctan(cx, \sqrt{cx+1}\sqrt{-cx+1})^2 + 2ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out] -1/2\*(a\*c^2\*x^2 + sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b\*c\*x + (b\*c^2\*x^2 - 2\*b)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + 2\*(b^4\*c^2\*x^3\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*a\*b^3\*c^2\*x^3\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a^2\*b^2\*c^2\*x^3)\*integrate(1/2\*(c^2\*x^2 - 6)/(b^3\*c^2\*x^4\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b^2\*c^2\*x^4), x) - 2\*a)/(b^4\*c^2\*x^3\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 2\*a\*b^3\*c^2\*x^3\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a^2\*b^2\*c^2\*x^3)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*asin(c\*x))^3),x)

[Out] int(1/(x^2\*(a + b\*asin(c\*x))^3), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asin(c\*x))\*\*3,x)

[Out] Integral(1/(x\*\*2\*(a + b\*asin(c\*x))\*\*3), x)

### 3.173 $\int x^2 \sqrt{a + b \sin^{-1}(cx)} dx$

**Optimal.** Leaf size=242

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3} - \frac{\sqrt{\frac{\pi}{6}} \sqrt{b} \sin\left(\frac{3a}{b}\right) C\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{12c^3} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3}$$

[Out]  $1/72*\cos(3*a/b)*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*6^{(1/2)}*\text{Pi}^{(1/2)}/c^3-1/72*\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(3*a/b)*b^{(1/2)}*6^{(1/2)}*\text{Pi}^{(1/2)}/c^3-1/8*\cos(a/b)*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\text{Pi}^{(1/2)}/c^3+1/8*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(a/b)*b^{(1/2)}*2^{(1/2)}*\text{Pi}^{(1/2)}/c^3+1/3*x^3*(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 0.76, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4629, 4723, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3} - \frac{\sqrt{\frac{\pi}{6}} \sqrt{b} \sin\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{12c^3} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sqrt[a + b*ArcSin[c*x]],x]`

[Out]  $(x^3*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/3 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a/b]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(4*c^3) + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[(3*a)/b]*\text{FresnelS}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(12*c^3) + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(4*c^3) - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(3*a)/b])/(12*c^3)$

#### Rule 3304

`Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

#### Rule 3305

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

#### Rule 3306

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]`

#### Rule 3312

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)</sup>(x\_)<sup>(m\_.)</sup>, x\_Symbol] := Simp[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>n</sup>)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n - 1)</sup>)/Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)</sup>(x\_)<sup>(m\_.)</sup>((d\_.) + (e\_.)\*(x\_)<sup>2</sup>)<sup>(p\_.)</sup>, x\_Symbol] := Dist[d<sup>p</sup>/c<sup>(m + 1)</sup>, Subst[Int[(a + b\*x)<sup>n</sup>\*Sin[x]<sup>m</sup>\*Cos[x]<sup>(2\*p + 1)</sup>, x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
 \int x^2 \sqrt{a + b \sin^{-1}(cx)} dx &= \frac{1}{3} x^3 \sqrt{a + b \sin^{-1}(cx)} - \frac{1}{6} (bc) \int \frac{x^3}{\sqrt{1 - c^2 x^2} \sqrt{a + b \sin^{-1}(cx)}} dx \\
 &= \frac{1}{3} x^3 \sqrt{a + b \sin^{-1}(cx)} - \frac{b \operatorname{Subst}\left(\int \frac{\sin^3(x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{6c^3} \\
 &= \frac{1}{3} x^3 \sqrt{a + b \sin^{-1}(cx)} - \frac{b \operatorname{Subst}\left(\int \left(\frac{3 \sin(x)}{4\sqrt{a+bx}} - \frac{\sin(3x)}{4\sqrt{a+bx}}\right) dx, x, \sin^{-1}(cx)\right)}{6c^3} \\
 &= \frac{1}{3} x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{b \operatorname{Subst}\left(\int \frac{\sin(3x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{24c^3} - \frac{b \operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{8c^3} \\
 &= \frac{1}{3} x^3 \sqrt{a + b \sin^{-1}(cx)} - \frac{(b \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{a}{b} + x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{8c^3} + \frac{(b \cos\left(\frac{3a}{b}\right)) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{3a}{b} + x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{8c^3} \\
 &= \frac{1}{3} x^3 \sqrt{a + b \sin^{-1}(cx)} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{4c^3} + \frac{\cos\left(\frac{3a}{b}\right) \operatorname{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{4c^3} \\
 &= \frac{1}{3} x^3 \sqrt{a + b \sin^{-1}(cx)} - \frac{\sqrt{b} \sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c^3} + \frac{\sqrt{b} \sqrt{\frac{\pi}{6}} \cos\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{12c^3}
 \end{aligned}$$

**Mathematica** [C] time = 0.32, size = 246, normalized size = 1.02

$$\frac{ie^{-\frac{3ia}{b}} \sqrt{a + b \sin^{-1}(cx)} \left( 9e^{\frac{2ia}{b}} \sqrt{\frac{i(a + b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{3}{2}, -\frac{i(a + b \sin^{-1}(cx))}{b}\right) - 9e^{\frac{4ia}{b}} \sqrt{-\frac{i(a + b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{3}{2}, \frac{i(a + b \sin^{-1}(cx))}{b}\right) \right) + 72c^3 \sqrt{\frac{(a + b \sin^{-1}(cx))^2}{b^2}}}{72c^3 \sqrt{\frac{(a + b \sin^{-1}(cx))^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[a + b*ArcSin[c*x]],x]
```

```
[Out] ((-1/72*I)*Sqrt[a + b*ArcSin[c*x]]*(9*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*ArcSin[c*x]))/b] - 9*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcSin[c*x]))/b] + Sqrt[3]*(-(Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcSin[c*x]))/b]) + E^(((6*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*ArcSin[c*x]))/b]))/(c^3*E^(((3*I)*a)/b)*Sqrt[(a + b*ArcSin[c*x])^2/b^2])
```

```
fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsin(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

```
giac [C] time = 2.13, size = 1057, normalized size = 4.37
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsin(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))c^3) + 1/16*I*sqrt(2)*sqrt(pi)*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))c^3) + 1/8*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))c^3) - 1/16*I*sqrt(2)*sqrt(pi)*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))c^3) - 1/4*sqrt(pi)*a*sqrt(b)*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/abs(b))c^3) - 1/24*I*sqrt(pi)*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/abs(b))c^3) - 1/4*sqrt(pi)*a*sqrt(b)*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b - I*sqrt(6)*b^2/abs(b))c^3) + 1/24*I*sqrt(pi)*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b - I*sqrt(6)*b^2/abs(b))c^3) + 1/4*sqrt(pi)*a*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b))c^3) - 1/4*sqrt(pi)*a*erf(-1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c^3*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - 1/4*sqrt(pi)*a*erf(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c^3*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) + 1/4*sqrt(pi)*a*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*sqrt(b) - I*sqrt(6)*b^(3/2)/abs(b))c^3) + 1/24*I*sqrt(b*arcsin(c*x) + a)*e^(3*I*arcsin(c*x))/c^3 - 1/
```

$8I\sqrt{b\arcsin(cx) + a}e^{I\arcsin(cx)}/c^3 + 1/8I\sqrt{b\arcsin(cx) + a}e^{-I\arcsin(cx)}/c^3 - 1/24I\sqrt{b\arcsin(cx) + a}e^{-3I\arcsin(cx)}/c^3$

**maple** [A] time = 0.15, size = 356, normalized size = 1.47

$$\frac{\sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{\frac{1}{b}}\sqrt{a+b\arcsin(cx)}\cos\left(\frac{3a}{b}\right)S\left(\frac{\sqrt{2}\sqrt{3}\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{\frac{1}{b}}b}\right)b - \sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{\frac{1}{b}}\sqrt{a+b\arcsin(cx)}\sin\left(\frac{3a}{b}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsin(c\*x))^(1/2),x)

[Out]  $\frac{1}{72}c^3/(a+b\arcsin(cx))^{1/2}*(3^{1/2}*2^{1/2}*Pi^{1/2}*(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}*\cos(3a/b)*FresnelS(2^{1/2}/Pi^{1/2}*3^{1/2}/(1/b)^{1/2})*(a+b\arcsin(cx))^{1/2}/b)*b-3^{1/2}*2^{1/2}*Pi^{1/2}*(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}*\sin(3a/b)*FresnelC(2^{1/2}/Pi^{1/2}*3^{1/2}/(1/b)^{1/2})*(a+b\arcsin(cx))^{1/2}/b)*b-9*2^{1/2}*Pi^{1/2}*(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}*\cos(a/b)*FresnelS(2^{1/2}/Pi^{1/2}/(1/b)^{1/2})*(a+b\arcsin(cx))^{1/2}/b)*b+9*2^{1/2}*Pi^{1/2}*(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}*\sin(a/b)*FresnelC(2^{1/2}/Pi^{1/2}/(1/b)^{1/2})*(a+b\arcsin(cx))^{1/2}/b)*b+18*\arcsin(cx)*\sin((a+b\arcsin(cx))/b-a/b)*b+18*\sin((a+b\arcsin(cx))/b-a/b)*a-6*\arcsin(cx)*\sin(3*(a+b\arcsin(cx))/b-3a/b)*b-6*\sin(3*(a+b\arcsin(cx))/b-3a/b)*a)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\arcsin(cx) + a} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*arcsin(c\*x) + a)\*x^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a + b \arcsin(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*asin(c\*x))^(1/2),x)

[Out] int(x^2\*(a + b\*asin(c\*x))^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \arcsin(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*asin(c\*x))\*\*(1/2),x)

[Out] Integral(x\*\*2\*sqrt(a + b\*asin(c\*x)), x)



### 3.174 $\int x\sqrt{a + b \sin^{-1}(cx)} dx$

**Optimal.** Leaf size=137

$$\frac{\sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{8c^2} + \frac{\sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{8c^2} - \frac{\sqrt{a + b \sin^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a + b \sin^{-1}(cx)}$$

[Out]  $1/8*\cos(2*a/b)*\text{FresnelC}(2*(a+b*\arcsin(c*x))^{1/2}/b^{1/2}/\text{Pi}^{1/2})*b^{1/2}*\text{Pi}^{1/2}/c^2+1/8*\text{FresnelS}(2*(a+b*\arcsin(c*x))^{1/2}/b^{1/2}/\text{Pi}^{1/2})*\sin(2*a/b)*b^{1/2}*\text{Pi}^{1/2}/c^2-1/4*(a+b*\arcsin(c*x))^{1/2}/c^2+1/2*x^2*(a+b*\arcsin(c*x))^{1/2}$

**Rubi [A]** time = 0.45, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {4629, 4723, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{\pi} \sqrt{b}}\right)}{8c^2} + \frac{\sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{8c^2} - \frac{\sqrt{a + b \sin^{-1}(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a + b \sin^{-1}(cx)}$$

Antiderivative was successfully verified.

[In] `Int[x*Sqrt[a + b*ArcSin[c*x]], x]`

[Out]  $-\text{Sqrt}[a + b*\text{ArcSin}[c*x]]/(4*c^2) + (x^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/2 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{Cos}[(2*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])]/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))/(8*c^2) + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])]/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[(2*a)/b]/(8*c^2)$

#### Rule 3304

`Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

#### Rule 3305

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

#### Rule 3306

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]`

#### Rule 3312

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

#### Rule 3351

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]`

Rule 3352

Int[Cos[(d\_.)\*(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m, x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
 \int x \sqrt{a + b \sin^{-1}(cx)} dx &= \frac{1}{2} x^2 \sqrt{a + b \sin^{-1}(cx)} - \frac{1}{4} (bc) \int \frac{x^2}{\sqrt{1 - c^2 x^2} \sqrt{a + b \sin^{-1}(cx)}} dx \\
 &= \frac{1}{2} x^2 \sqrt{a + b \sin^{-1}(cx)} - \frac{b \operatorname{Subst}\left(\int \frac{\sin^2(x)}{\sqrt{a + bx}} dx, x, \sin^{-1}(cx)\right)}{4c^2} \\
 &= \frac{1}{2} x^2 \sqrt{a + b \sin^{-1}(cx)} - \frac{b \operatorname{Subst}\left(\int \left(\frac{1}{2\sqrt{a + bx}} - \frac{\cos(2x)}{2\sqrt{a + bx}}\right) dx, x, \sin^{-1}(cx)\right)}{4c^2} \\
 &= -\frac{\sqrt{a + b \sin^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{b \operatorname{Subst}\left(\int \frac{\cos(2x)}{\sqrt{a + bx}} dx, x, \sin^{-1}(cx)\right)}{8c^2} \\
 &= -\frac{\sqrt{a + b \sin^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{\left(b \cos\left(\frac{2a}{b}\right)\right) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{2a}{b} + 2x\right)}{\sqrt{a + bx}} dx, x, \sin^{-1}(cx)\right)}{8c^2} \\
 &= -\frac{\sqrt{a + b \sin^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{\cos\left(\frac{2a}{b}\right) \operatorname{Subst}\left(\int \cos\left(\frac{2x}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{4c^2} \\
 &= -\frac{\sqrt{a + b \sin^{-1}(cx)}}{4c^2} + \frac{1}{2} x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{\sqrt{b} \sqrt{\pi} \cos\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{8c^2} + \dots
 \end{aligned}$$

**Mathematica** [C] time = 0.08, size = 141, normalized size = 1.03

$$\frac{e^{-\frac{2ia}{b}} \sqrt{a + b \sin^{-1}(cx)} \left( \sqrt{\frac{i(a + b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{3}{2}, -\frac{2i(a + b \sin^{-1}(cx))}{b}\right) + e^{\frac{4ia}{b}} \sqrt{-\frac{i(a + b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{3}{2}, \frac{2i(a + b \sin^{-1}(cx))}{b}\right) \right)}{8\sqrt{2} c^2 \sqrt{\frac{(a + b \sin^{-1}(cx))^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sqrt[a + b\*ArcSin[c\*x]],x]

[Out] -1/8\*(Sqrt[a + b\*ArcSin[c\*x]]\*(Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[3/2, ((-2\*I)\*(a + b\*ArcSin[c\*x]))/b] + E^(((4\*I)\*a)/b)\*Sqrt[(-I)\*(a + b\*ArcSin[c



[In] integrate(x\*(a+b\*arcsin(c\*x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*arcsin(c\*x) + a)\*x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asin(c\*x))^(1/2),x)

[Out] int(x\*(a + b\*asin(c\*x))^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asin(c\*x))\*\*(1/2),x)

[Out] Integral(x\*sqrt(a + b\*asin(c\*x)), x)

### 3.175 $\int \sqrt{a + b \sin^{-1}(cx)} dx$

**Optimal.** Leaf size=120

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{c} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{c} + x \sqrt{a + b \sin^{-1}(cx)}$$

[Out]  $-1/2*\cos(a/b)*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\text{Pi}^{(1/2)}/c+1/2*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(a/b)*b^{(1/2)}*2^{(1/2)}*\text{Pi}^{(1/2)}/c+x*(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 0.33, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {4619, 4723, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{c} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{c} + x \sqrt{a + b \sin^{-1}(cx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*ArcSin[c\*x]], x]

[Out]  $x*\text{Sqrt}[a + b*\text{ArcSin}[c*x]] - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a/b]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/c + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/c$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/Sqrt[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

### Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x]^(2*p + 1), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])
```

### Rubi steps

$$\begin{aligned} \int \sqrt{a + b \sin^{-1}(cx)} dx &= x\sqrt{a + b \sin^{-1}(cx)} - \frac{1}{2}(bc) \int \frac{x}{\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}} dx \\ &= x\sqrt{a + b \sin^{-1}(cx)} - \frac{b \operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2c} \\ &= x\sqrt{a + b \sin^{-1}(cx)} - \frac{(b \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2c} + \frac{(b \sin\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2c} \\ &= x\sqrt{a + b \sin^{-1}(cx)} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{c} + \frac{\sin\left(\frac{a}{b}\right) \operatorname{Subst}\left(\int \cos\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{c} \\ &= x\sqrt{a + b \sin^{-1}(cx)} - \frac{\sqrt{b} \sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{c} + \frac{\sqrt{b} \sqrt{\frac{\pi}{2}} C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{c} \end{aligned}$$

**Mathematica [C]** time = 0.10, size = 119, normalized size = 0.99

$$\frac{be^{-\frac{ia}{b}} \left( \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{3}{2}, -\frac{i(a+b \sin^{-1}(cx))}{b}\right) + e^{\frac{2ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{3}{2}, \frac{i(a+b \sin^{-1}(cx))}{b}\right) \right)}{2c\sqrt{a + b \sin^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSin[c*x]], x]
```

```
[Out] (b*(Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*ArcSin[c*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcSin[c*x]))/b])/(2*c*E^((I*a)/b)*Sqrt[a + b*ArcSin[c*x]])
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsin(c*x))^(1/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

**giac** [C] time = 2.05, size = 531, normalized size = 4.42

$$\frac{\sqrt{2} \sqrt{\pi} a b \operatorname{erf}\left(-\frac{i \sqrt{2} \sqrt{b \arcsin(cx)+a}}{2 \sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arcsin(cx)+a} \sqrt{|b|}}{2 b}\right) e^{\left(\frac{ia}{b}\right)} + i \sqrt{2} \sqrt{\pi} b^2 \operatorname{erf}\left(-\frac{i \sqrt{2} \sqrt{b \arcsin(cx)+a}}{2 \sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arcsin(cx)+a} \sqrt{|b|}}{2 b}\right)}{2 \left(\frac{ib^2}{\sqrt{|b|}} + b \sqrt{|b|}\right) c} + \frac{i \sqrt{2} \sqrt{\pi} b^2 \operatorname{erf}\left(-\frac{i \sqrt{2} \sqrt{b \arcsin(cx)+a}}{2 \sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arcsin(cx)+a} \sqrt{|b|}}{2 b}\right)}{4 \left(\frac{ib^2}{\sqrt{|b|}} + b \sqrt{|b|}\right) c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2), x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*sqrt(pi)\*a\*b\*erf(-1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)/sqrt(abs(b)) - 1/2\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*sqrt(abs(b))/b)\*e^(I\*a/b)/((I\*b^2/sqrt(abs(b)) + b\*sqrt(abs(b))) \* c) + 1/4\*I\*sqrt(2)\*sqrt(pi)\*b^2\*erf(-1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)/sqrt(abs(b)) - 1/2\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*sqrt(abs(b))/b)\*e^(I\*a/b)/((I\*b^2/sqrt(abs(b)) + b\*sqrt(abs(b))) \* c) + 1/2\*sqrt(2)\*sqrt(pi)\*a\*b\*erf(1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)/sqrt(abs(b)) - 1/2\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*sqrt(abs(b))/b)\*e^(-I\*a/b)/((-I\*b^2/sqrt(abs(b)) + b\*sqrt(abs(b))) \* c) - 1/4\*I\*sqrt(2)\*sqrt(pi)\*b^2\*erf(1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)/sqrt(abs(b)) - 1/2\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*sqrt(abs(b))/b)\*e^(-I\*a/b)/((-I\*b^2/sqrt(abs(b)) + b\*sqrt(abs(b))) \* c) - sqrt(pi)\*a\*erf(-1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)/sqrt(abs(b)) - 1/2\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*sqrt(abs(b))/b)\*e^(I\*a/b)/(c\*(I\*sqrt(2)\*b/sqrt(abs(b)) + sqrt(2)\*sqrt(abs(b)))) - sqrt(pi)\*a\*erf(1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)/sqrt(abs(b)) - 1/2\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*sqrt(abs(b))/b)\*e^(-I\*a/b)/(c\*(-I\*sqrt(2)\*b/sqrt(abs(b)) + sqrt(2)\*sqrt(abs(b)))) - 1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*e^(I\*arcsin(c\*x))/c + 1/2\*I\*sqrt(2)\*sqrt(b\*arcsin(c\*x) + a)\*e^(-I\*arcsin(c\*x))/c

**maple** [A] time = 0.07, size = 178, normalized size = 1.48

$$\frac{-\sqrt{2} \sqrt{\pi} \sqrt{\frac{1}{b}} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) b + \sqrt{2} \sqrt{\pi} \sqrt{\frac{1}{b}} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) b}{2c \sqrt{a+b \arcsin(cx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^(1/2), x)

[Out] 1/2/c/(a+b\*arcsin(c\*x))^(1/2)\*(-2^(1/2)\*Pi^(1/2)\*(1/b)^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)\*cos(a/b)\*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)/b)\*b+2^(1/2)\*Pi^(1/2)\*(1/b)^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)\*sin(a/b)\*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)/b)\*b+2\*arcsin(c\*x)\*sin((a+b\*arcsin(c\*x))/b-a/b)\*b+2\*sin((a+b\*arcsin(c\*x))/b-a/b)\*a

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b\*arcsin(c\*x) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \arcsin(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(c*x))^(1/2),x)
```

```
[Out] int((a + b*asin(c*x))^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asin(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asin(c*x)), x)
```



$$3.176 \quad \int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x} dx$$

**Optimal.** Leaf size=19

$$\text{Int}\left(\frac{\sqrt{a+b \sin^{-1}(cx)}}{x}, x\right)$$

[Out] Unintegrable((a+b\*arcsin(c\*x))^(1/2)/x,x)

**Rubi [A]** time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[a + b\*ArcSin[c\*x]]/x,x]

[Out] Defer[Int][Sqrt[a + b\*ArcSin[c\*x]]/x, x]

Rubi steps

$$\int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x} dx = \int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x} dx$$

**Mathematica [A]** time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[a + b\*ArcSin[c\*x]]/x,x]

[Out] Integrate[Sqrt[a + b\*ArcSin[c\*x]]/x, x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \arcsin(cx) + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(b\*arcsin(c\*x) + a)/x, x)

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \arcsin(cx)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^(1/2)/x,x)

[Out] int((a+b\*arcsin(c\*x))^(1/2)/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \arcsin(cx) + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(b\*arcsin(c\*x) + a)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{a + b \arcsin(cx)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^(1/2)/x,x)

[Out] int((a + b\*asin(c\*x))^(1/2)/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \arcsin(cx)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*(1/2)/x,x)

[Out] Integral(sqrt(a + b\*asin(c\*x))/x, x)

$$3.177 \quad \int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x^2} dx$$

**Optimal.** Leaf size=19

$$\text{Int}\left(\frac{\sqrt{a+b \sin^{-1}(cx)}}{x^2}, x\right)$$

[Out] Unintegrable((a+b\*arcsin(c\*x))^(1/2)/x^2, x)

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[a + b\*ArcSin[c\*x]]/x^2, x]

[Out] Defer[Int][Sqrt[a + b\*ArcSin[c\*x]]/x^2, x]

Rubi steps

$$\int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x^2} dx = \int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x^2} dx$$

**Mathematica [A]** time = 11.90, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b \sin^{-1}(cx)}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[a + b\*ArcSin[c\*x]]/x^2, x]

[Out] Integrate[Sqrt[a + b\*ArcSin[c\*x]]/x^2, x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2)/x^2, x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \arcsin(cx) + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2)/x^2, x, algorithm="giac")

[Out] integrate(sqrt(b\*arcsin(c\*x) + a)/x^2, x)

**maple** [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \arcsin(cx)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^(1/2)/x^2,x)

[Out] int((a+b\*arcsin(c\*x))^(1/2)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \arcsin(cx) + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(b\*arcsin(c\*x) + a)/x^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{a + b \operatorname{asin}(cx)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^(1/2)/x^2,x)

[Out] int((a + b\*asin(c\*x))^(1/2)/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \operatorname{asin}(cx)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*(1/2)/x\*\*2,x)

[Out] Integral(sqrt(a + b\*asin(c\*x))/x\*\*2, x)

### 3.178 $\int x^2 \left( a + b \sin^{-1}(cx) \right)^{3/2} dx$

**Optimal.** Leaf size=313

$$\frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{\sqrt{\frac{\pi}{6}} b^{3/2} \cos\left(\frac{3a}{b}\right) C\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}$$

[Out]  $1/3*x^3*(a+b*\arcsin(c*x))^{(3/2)}+1/144*b^{(3/2)}*\cos(3*a/b)*\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/c^3+1/144*b^{(3/2)}*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(3*a/b)*6^{(1/2)}*\text{Pi}^{(1/2)}/c^3-3/16*b^{(3/2)}*\cos(a/b)*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/c^3-3/16*b^{(3/2)}*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(a/b)*2^{(1/2)}*\text{Pi}^{(1/2)}/c^3+1/3*b*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/c^3+1/6*b*x^2*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/c$

**Rubi [A]** time = 1.05, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$ , Rules used = {4629, 4707, 4677, 4623, 3306, 3305, 3351, 3304, 3352, 4635, 4406}

$$\frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{\sqrt{\frac{\pi}{6}} b^{3/2} \cos\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \sin\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{ArcSin}[c*x])^{(3/2)}, x]$

[Out]  $(b*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(3*c^3) + (b*x^2*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(6*c) + (x^3*(a + b*\text{ArcSin}[c*x])^{(3/2)})/3 - (3*b^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a/b]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(8*c^3) + (b^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[(3*a)/b]*\text{FresnelC}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(24*c^3) - (3*b^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(8*c^3) + (b^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(3*a)/b])/(24*c^3)$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]<sup>(p\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]<sup>(n\_.)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[a + b\*x]<sup>n</sup>\*Cos[a + b\*x]<sup>p</sup>, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]</sup></sup>

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)</sup>, x\_Symbol] := Dist[1/(b\*c), Subst[Int[x<sup>n</sup>\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)\*(x\_)<sup>(m\_)</sup>, x\_Symbol] := Simp[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>n</sup>)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n - 1)</sup>)/Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]</sup>

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)\*(x\_)<sup>(m\_)</sup>, x\_Symbol] := Dist[1/c<sup>(m + 1)</sup>, Subst[Int[(a + b\*x)<sup>n</sup>\*Sin[x]<sup>m</sup>\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]</sup>

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)\*(x\_)\*((d\_) + (e\_.)\*(x\_)<sup>2</sup>)<sup>(p\_.)</sup>, x\_Symbol] := Simp[((d + e\*x<sup>2</sup>)<sup>(p + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>n</sup>)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d<sup>IntPart[p]</sup>\*(d + e\*x<sup>2</sup>)<sup>FracPart[p]</sup>)/(2\*c\*(p + 1)\*(1 - c<sup>2</sup>\*x<sup>2</sup>)<sup>FracPart[p]</sup>), Int[(1 - c<sup>2</sup>\*x<sup>2</sup>)<sup>(p + 1/2)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n - 1)</sup>, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]</sup>

#### Rule 4707

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)\*((f\_.)\*(x\_))<sup>(m\_)</sup>)/Sqrt[(d\_ + (e\_.)\*(x\_)<sup>2</sup>], x\_Symbol] := Simp[(f\*(f\*x)<sup>(m - 1)</sup>\*Sqrt[d + e\*x<sup>2</sup>]\*(a + b\*ArcSin[c\*x])<sup>n</sup>)/(e\*m), x] + (Dist[(f<sup>2</sup>\*(m - 1))/(c<sup>2</sup>\*m), Int[(f\*x)<sup>(m - 2)</sup>\*(a + b\*ArcSin[c\*x])<sup>n</sup>]/Sqrt[d + e\*x<sup>2</sup>], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>])/(c\*m\*Sqrt[d + e\*x<sup>2</sup>]), Int[(f\*x)<sup>(m - 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n - 1)</sup>, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]</sup>

#### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \sin^{-1}(cx))^{3/2} dx &= \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2} - \frac{1}{2}(bc) \int \frac{x^3 \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{1 - c^2x^2}} dx \\
&= \frac{bx^2 \sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{6c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2} - \frac{1}{12}b^2 \int \frac{x^2}{\sqrt{a + b \sin^{-1}(cx)}} dx \\
&= \frac{b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{3c^3} + \frac{bx^2 \sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{6c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{3c^3} + \frac{bx^2 \sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{6c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{3c^3} + \frac{bx^2 \sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{6c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{3c^3} + \frac{bx^2 \sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{6c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{3c^3} + \frac{bx^2 \sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{6c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{3c^3} + \frac{bx^2 \sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{6c} + \frac{1}{3}x^3 (a + b \sin^{-1}(cx))^{3/2}
\end{aligned}$$

**Mathematica [C]** time = 0.31, size = 245, normalized size = 0.78

$$\frac{be^{-\frac{3ia}{b}} \sqrt{a + b \sin^{-1}(cx)} \left( 27e^{\frac{2ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{5}{2}, -\frac{i(a+b \sin^{-1}(cx))}{b}\right) + 27e^{\frac{4ia}{b}} \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{5}{2}, \frac{i(a+b \sin^{-1}(cx))}{b}\right) \right)}{216c^3 \sqrt{\frac{(a+b \sin^{-1}(cx))}{b^2}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcSin[c\*x])^(3/2),x]

[Out] (b\*Sqrt[a + b\*ArcSin[c\*x]]\*(27\*E^(((2\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[5/2, ((-I)\*(a + b\*ArcSin[c\*x]))/b] + 27\*E^(((4\*I)\*a)/b)\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[5/2, (I\*(a + b\*ArcSin[c\*x]))/b] - Sqrt[3]\*(Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[5/2, ((-3\*I)\*(a + b\*ArcSin[c\*x]))/b] + E^(((6\*I)\*a)/b)\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[5/2, ((3\*I)\*(a + b\*ArcSin[c\*x]))/b]))/(216\*c^3\*E^(((3\*I)\*a)/b)\*Sqrt[(a + b\*ArcSin[c\*x])^2/b^2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [C] time = 4.17, size = 1967, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^(3/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} & \frac{1}{8}\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(-\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{Ia/b}/\left(\frac{Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c^3 + \frac{1}{8}I\sqrt{2}\sqrt{\pi}a^2b^3 \\ & \operatorname{erf}\left(-\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{Ia/b}/\left(\frac{Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & + \frac{1}{8}\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{-Ia/b}/\left(-\frac{Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & - \frac{1}{8}I\sqrt{2}\sqrt{\pi}a^2b^3\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{-Ia/b}/\left(-\frac{Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & - \frac{1}{4}\sqrt{\pi}a^2b^{3/2}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} - \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{3Ia/b}/\left(\sqrt{6}b^2 + I\sqrt{6}b^3/\operatorname{abs}(b)\right)c^3 \\ & - \frac{1}{12}I\sqrt{\pi}a^2b^{5/2}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} - \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{3Ia/b}/\left(\sqrt{6}b^2 + I\sqrt{6}b^3/\operatorname{abs}(b)\right)c^3 \\ & - \frac{1}{8}I\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(-\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{Ia/b}/\left(\frac{Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & + \frac{3}{32}\sqrt{2}\sqrt{\pi}b^3\operatorname{erf}\left(-\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{Ia/b}/\left(\frac{Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & + \frac{1}{8}I\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{-Ia/b}/\left(-\frac{Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & + \frac{3}{32}\sqrt{2}\sqrt{\pi}b^3\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{-Ia/b}/\left(-\frac{Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & - \frac{1}{4}\sqrt{\pi}a^2b^{3/2}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} + \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{-3Ia/b}/\left(\sqrt{6}b^2 - I\sqrt{6}b^3/\operatorname{abs}(b)\right)c^3 \\ & + \frac{1}{12}I\sqrt{\pi}a^2b^{5/2}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} + \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{-3Ia/b}/\left(\sqrt{6}b^2 - I\sqrt{6}b^3/\operatorname{abs}(b)\right)c^3 \\ & + \frac{1}{4}\sqrt{\pi}a^2b^2\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} - \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{3Ia/b}/\left(\sqrt{6}b^{3/2} + I\sqrt{6}b^{5/2}/\operatorname{abs}(b)\right)c^3 \\ & + \frac{1}{12}I\sqrt{\pi}a^2b^2\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} - \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{3Ia/b}/\left(\sqrt{6}b^{3/2} + I\sqrt{6}b^{5/2}/\operatorname{abs}(b)\right)c^3 \\ & - \frac{1}{4}\sqrt{\pi}a^2b^2\operatorname{erf}\left(-\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{Ia/b}/\left(\frac{I\sqrt{2}}{\sqrt{\operatorname{abs}(b)}} + \sqrt{2}b\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & - \frac{1}{4}\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b e^{-Ia/b}/\left(-\frac{I\sqrt{2}}{\sqrt{\operatorname{abs}(b)}} + \sqrt{2}b\sqrt{\operatorname{abs}(b)}\right)c^3 \\ & + \frac{1}{4}\sqrt{\pi}a^2b^2\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} + \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{-3Ia/b}/\left(\sqrt{6}b^{3/2} - I\sqrt{6}b^{5/2}/\operatorname{abs}(b)\right)c^3 \\ & - \frac{1}{12}I\sqrt{\pi}a^2b^2\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} + \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{-3Ia/b}/\left(\sqrt{6}b^{3/2} - I\sqrt{6}b^{5/2}/\operatorname{abs}(b)\right)c^3 \\ & - \frac{1}{48}\sqrt{\pi}b^{5/2}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} - \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{3Ia/b}/\left(\sqrt{6}b + I\sqrt{6}b^2/\operatorname{abs}(b)\right)c^3 \\ & - \frac{1}{48}\sqrt{\pi}b^{5/2}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{b} + \frac{1}{2}I\sqrt{6}\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b) e^{-3Ia/b}/\left(\sqrt{6}b - I\sqrt{6}b^2/\operatorname{abs}(b)\right)c^3 \\ & + \frac{1}{24}I\sqrt{b\arcsin(cx)+a}b\arcsin(cx)e^{3Ia/b} \end{aligned}$$



$$\begin{aligned} & I \arcsin(cx) / c^3 - 1/8 I \sqrt{b \arcsin(cx) + a} b \arcsin(cx) e^{(I \arcsin(cx))} / c^3 + 1/8 I \sqrt{b \arcsin(cx) + a} b \arcsin(cx) e^{(-I \arcsin(cx))} / c^3 \\ & - 1/24 I \sqrt{b \arcsin(cx) + a} b \arcsin(cx) e^{(-3 I \arcsin(cx))} / c^3 + 1/24 I \sqrt{b \arcsin(cx) + a} a e^{(3 I \arcsin(cx))} / c^3 - 1/48 \sqrt{b \arcsin(cx) + a} b e^{(3 I \arcsin(cx))} / c^3 \\ & - 1/8 I \sqrt{b \arcsin(cx) + a} a e^{(I \arcsin(cx))} / c^3 + 3/16 \sqrt{b \arcsin(cx) + a} b e^{(I \arcsin(cx))} / c^3 + 1/8 I \sqrt{b \arcsin(cx) + a} a e^{(-I \arcsin(cx))} / c^3 \\ & + 3/16 \sqrt{b \arcsin(cx) + a} b e^{(-I \arcsin(cx))} / c^3 - 1/24 I \sqrt{b \arcsin(cx) + a} a e^{(-3 I \arcsin(cx))} / c^3 - 1/48 \sqrt{b \arcsin(cx) + a} b e^{(-3 I \arcsin(cx))} / c^3 \end{aligned}$$

**maple [B]** time = 0.17, size = 542, normalized size = 1.73

$$-\sqrt{\pi} \sqrt{2} \sqrt{3} \sqrt{a + b \arcsin(cx)} \cos\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{a + b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b} b}}\right) \sqrt{\frac{1}{b}} b^2 - \sqrt{\pi} \sqrt{2} \sqrt{3} \sqrt{a + b \arcsin(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsin(c\*x))^(3/2),x)

[Out] 
$$\begin{aligned} & -1/144/c^3 * (-\pi^{(1/2)} * 2^{(1/2)} * 3^{(1/2)} * (a+b \arcsin(cx))^{(1/2)} * \cos(3a/b) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)}/(1/b)^{(1/2)} * (a+b \arcsin(cx))^{(1/2)}/b) * (1/b)^{(1/2)} * b^2 \\ & - \pi^{(1/2)} * 2^{(1/2)} * 3^{(1/2)} * (a+b \arcsin(cx))^{(1/2)} * \sin(3a/b) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)}/(1/b)^{(1/2)} * (a+b \arcsin(cx))^{(1/2)}/b) * (1/b)^{(1/2)} * b^2 \\ & + 27 * \pi^{(1/2)} * 2^{(1/2)} * (a+b \arcsin(cx))^{(1/2)} * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}/(1/b)^{(1/2)} * (a+b \arcsin(cx))^{(1/2)}/b) * \cos(a/b) * (1/b)^{(1/2)} * b^2 \\ & + 27 * \pi^{(1/2)} * 2^{(1/2)} * (a+b \arcsin(cx))^{(1/2)} * \sin(a/b) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}/(1/b)^{(1/2)} * (a+b \arcsin(cx))^{(1/2)}/b) * (1/b)^{(1/2)} * b^2 \\ & + 12 * \arcsin(cx)^2 * \sin(3(a+b \arcsin(cx))/b - 3a/b) * b^2 - 36 * \arcsin(cx)^2 * \sin((a+b \arcsin(cx))/b - a/b) * b^2 \\ & - 54 * \arcsin(cx) * \cos((a+b \arcsin(cx))/b - a/b) * b^2 + 24 * \arcsin(cx) * \sin(3(a+b \arcsin(cx))/b - 3a/b) * a * b + 6 * \arcsin(cx) * \cos(3(a+b \arcsin(cx))/b - 3a/b) * b^2 \\ & - 72 * \arcsin(cx) * \sin((a+b \arcsin(cx))/b - a/b) * a * b - 54 * \cos((a+b \arcsin(cx))/b - a/b) * a * b + 12 * \sin(3(a+b \arcsin(cx))/b - 3a/b) * a^2 + 6 * \cos(3(a+b \arcsin(cx))/b - 3a/b) * a * b \\ & - 36 * \sin((a+b \arcsin(cx))/b - a/b) * a^2 / (a+b \arcsin(cx))^{(1/2)} \end{aligned}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \arcsin(cx) + a)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^(3/2),x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(3/2)\*x^2, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b \arcsin(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*asin(c\*x))^(3/2),x)

[Out] int(x^2\*(a + b\*asin(c\*x))^(3/2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \arcsin(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asin(c*x))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asin(c*x))**(3/2), x)
```

$$3.179 \quad \int x \left( a + b \sin^{-1}(cx) \right)^{3/2} dx$$

**Optimal.** Leaf size=172

$$\frac{3\sqrt{\pi} b^{3/2} \sin\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2} - \frac{3\sqrt{\pi} b^{3/2} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2} + \frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\sin^{-1}(cx)}}{8c}$$

[Out]  $-1/4*(a+b*\arcsin(c*x))^{(3/2)}/c^2+1/2*x^2*(a+b*\arcsin(c*x))^{(3/2)}-3/32*b^{(3/2)}*\cos(2*a/b)*\text{FresnelS}(2*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/c^2+3/32*b^{(3/2)}*\text{FresnelC}(2*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)}/\text{Pi}^{(1/2)})*\sin(2*a/b)*\text{Pi}^{(1/2)}/c^2+3/8*b*x*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/c$

**Rubi [A]** time = 0.52, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$ , Rules used = {4629, 4707, 4641, 4635, 4406, 12, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\pi} b^{3/2} \sin\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{32c^2} - \frac{3\sqrt{\pi} b^{3/2} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2} + \frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\sin^{-1}(cx)}}{8c}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*ArcSin[c*x])^(3/2), x]`

[Out]  $(3*b*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(8*c) - (a + b*\text{ArcSin}[c*x])^{(3/2)}/(4*c^2) + (x^2*(a + b*\text{ArcSin}[c*x])^{(3/2)})/2 - (3*b^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[(2*a)/b]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))/(32*c^2) + (3*b^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[(2*a)/b])/(32*c^2)$

**Rule 12**

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

**Rule 3304**

`Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

**Rule 3305**

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

**Rule 3306**

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]`

**Rule 3351**

$\text{Int}[\text{Sin}[(d_.)*(e_.) + (f_.)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] \text{ /; FreeQ}\{d, e, f\}, x]$

#### Rule 3352

$\text{Int}[\text{Cos}[(d_.)*(e_.) + (f_.)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] \text{ /; FreeQ}\{d, e, f\}, x]$

#### Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]^{(p_.)*((c_.) + (d_.)*(x_))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^{n*Cos}[a + b*x]^p, x], x] \text{ /; FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

#### Rule 4629

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)]^{(n_.)*(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*(a + b*\text{ArcSin}[c*x])^n)/(m+1), x] - \text{Dist}[(b*c*n)/(m+1), \text{Int}[(x^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2], x], x] \text{ /; FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{GtQ}[n, 0]$

#### Rule 4635

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)]^{(n_.)*(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sin}[x]^m*\text{Cos}[x], x], x, \text{ArcSin}[c*x]], x] \text{ /; FreeQ}\{a, b, c, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 4641

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)]^{(n_.)/\text{Sqrt}[(d_.) + (e_.)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{ArcSin}[c*x])^{(n+1)}/(b*c*\text{Sqrt}[d]*(n+1)), x] \text{ /; FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[d, 0] \&\& \text{NeQ}[n, -1]$

#### Rule 4707

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)]^{(n_.)*((f_.)*(x_))^{(m_.)}/\text{Sqrt}[(d_.) + (e_.)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(f*(f*x)^{(m-1)}*\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcSin}[c*x])^n)/(e*m), x] + (\text{Dist}[(f^2*(m-1))/(c^2*m), \text{Int}[(f*x)^{(m-2)}*(a + b*\text{ArcSin}[c*x])^n]/\text{Sqrt}[d + e*x^2], x], x] + \text{Dist}[(b*f*n*\text{Sqrt}[1 - c^2*x^2])/(c*m*\text{Sqrt}[d + e*x^2]), \text{Int}[(f*x)^{(m-1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ /; FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{GtQ}[m, 1] \&\& \text{IntegerQ}[m]$

#### Rubi steps

$$\begin{aligned}
\int x(a + b \sin^{-1}(cx))^{3/2} dx &= \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2} - \frac{1}{4}(3bc) \int \frac{x^2 \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{1 - c^2x^2}} dx \\
&= \frac{3bx\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{8c} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2} - \frac{1}{16}(3b^2) \int \frac{1}{\sqrt{a + b \sin^{-1}(cx)}} dx \\
&= \frac{3bx\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{3bx\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{3bx\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{3bx\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{3bx\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2} \\
&= \frac{3bx\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a + b \sin^{-1}(cx))^{3/2}
\end{aligned}$$

**Mathematica [C]** time = 0.07, size = 126, normalized size = 0.73

$$\frac{b^2 e^{-\frac{2ia}{b}} \left( \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{5}{2}, -\frac{2i(a+b \sin^{-1}(cx))}{b}\right) + e^{\frac{4ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{5}{2}, \frac{2i(a+b \sin^{-1}(cx))}{b}\right) \right)}{16\sqrt{2}c^2\sqrt{a + b \sin^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcSin[c\*x])^(3/2), x]

[Out] (b^2\*(Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[5/2, ((-2\*I)\*(a + b\*ArcSin[c\*x]))/b] + E^(((4\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[5/2, ((2\*I)\*(a + b\*ArcSin[c\*x]))/b]))/(16\*Sqrt[2]\*c^2\*E^(((2\*I)\*a)/b)\*Sqrt[a + b\*ArcSin[c\*x]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [C]** time = 2.09, size = 845, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^(3/2),x, algorithm="giac")

[Out]  $\frac{1}{4}I\sqrt{\pi}a^2b^{3/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)-I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b^2+Ib^3/\operatorname{abs}(b))c^2)-1/8\sqrt{\pi}ab^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)-I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b^2+Ib^3/\operatorname{abs}(b))c^2)-1/4I\sqrt{\pi}a^2b^{3/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)+I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^2-Ib^3/\operatorname{abs}(b))c^2)-1/8\sqrt{\pi}ab^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)+I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^2-Ib^3/\operatorname{abs}(b))c^2)+1/8\sqrt{\pi}ab^2\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)-I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b^{3/2}+Ib^{5/2}/\operatorname{abs}(b))c^2)+1/4I\sqrt{\pi}a^2b\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)+I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^{3/2}-Ib^{5/2}/\operatorname{abs}(b))c^2)+1/8\sqrt{\pi}ab^2\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)+I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^{3/2}-Ib^{5/2}/\operatorname{abs}(b))c^2)-1/4I\sqrt{\pi}a^2\sqrt{b}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)-I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b+Ib^2/\operatorname{abs}(b))c^2)+3/64I\sqrt{\pi}b^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)-I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b+Ib^2/\operatorname{abs}(b))c^2)-3/64I\sqrt{\pi}b^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right)+I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b-Ib^2/\operatorname{abs}(b))c^2)-1/8\sqrt{b\arcsin(cx)+a}b\arcsin(cx)e^{2I\arcsin(cx)}/c^2-1/8\sqrt{b\arcsin(cx)+a}b\arcsin(cx)e^{-2I\arcsin(cx)}/c^2-1/8\sqrt{b\arcsin(cx)+a}a^2e^{2I\arcsin(cx)}/c^2-3/32I\sqrt{b\arcsin(cx)+a}b^2e^{2I\arcsin(cx)}/c^2-1/8\sqrt{b\arcsin(cx)+a}a^2e^{-2I\arcsin(cx)}/c^2+3/32I\sqrt{b\arcsin(cx)+a}b^2e^{-2I\arcsin(cx)}/c^2$

**maple** [A] time = 0.10, size = 267, normalized size = 1.55

$$-3\sqrt{\frac{1}{b}}\sqrt{a+b\arcsin(cx)}\sin\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{\frac{1}{b}}}\right)\sqrt{\pi}b^2+3\sqrt{\frac{1}{b}}\sqrt{a+b\arcsin(cx)}\cos\left(\frac{2a}{b}\right)\operatorname{S}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{\frac{1}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsin(c\*x))^(3/2),x)

[Out]  $-1/32/c^2*(-3*(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}*\sin(2*a/b)*\operatorname{FresnelC}(2/\operatorname{Pi}^{1/2}/(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}/b)*\operatorname{Pi}^{1/2}*b^2+3*(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}*\cos(2*a/b)*\operatorname{FresnelS}(2/\operatorname{Pi}^{1/2}/(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}/b)*\operatorname{Pi}^{1/2}*b^2+8*\arcsin(cx)^2*\cos(2*(a+b\arcsin(cx)))/b-2*a/b)*b^2-6*\arcsin(cx)*\sin(2*(a+b\arcsin(cx)))/b-2*a/b)*b^2+16*\arcsin(cx)*\cos(2*(a+b\arcsin(cx)))/b-2*a/b)*a*b-6*\sin(2*(a+b\arcsin(cx)))/b-2*a/b)*a*b+8*\cos(2*(a+b\arcsin(cx)))/b-2*a/b)*a^2)/(a+b\arcsin(cx))^{1/2}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \arcsin(cx) + a)^{3/2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^(3/2),x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(3/2)\*x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x(a+b\arcsin(cx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(c*x))^(3/2), x)
```

```
[Out] int(x*(a + b*asin(c*x))^(3/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x (a + b \operatorname{asin}(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asin(c*x))**(3/2), x)
```

```
[Out] Integral(x*(a + b*asin(c*x))**(3/2), x)
```

### 3.180 $\int (a + b \sin^{-1}(cx))^{3/2} dx$

**Optimal.** Leaf size=159

$$\frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2c} - \frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2c} + \frac{3b\sqrt{1-c^2x^2} \sqrt{a+b \sin^{-1}(cx)}}{2c} + \dots$$

[Out]  $x*(a+b*\arcsin(c*x))^{3/2}-3/4*b^{3/2}*\cos(a/b)*\text{FresnelC}(2^{1/2}/\text{Pi}^{1/2}*(a+b*\arcsin(c*x))^{1/2}/b^{1/2})*2^{1/2}*\text{Pi}^{1/2}/c-3/4*b^{3/2}*\text{FresnelS}(2^{1/2}/\text{Pi}^{1/2}*(a+b*\arcsin(c*x))^{1/2}/b^{1/2})*\sin(a/b)*2^{1/2}*\text{Pi}^{1/2}/c+3/2*b*(-c^2*x^2+1)^{1/2}*(a+b*\arcsin(c*x))^{1/2}/c$

**Rubi [A]** time = 0.28, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4619, 4677, 4623, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2c} - \frac{3\sqrt{\frac{\pi}{2}} b^{3/2} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2c} + \frac{3b\sqrt{1-c^2x^2} \sqrt{a+b \sin^{-1}(cx)}}{2c} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSin}[c*x])^{3/2}, x]$

[Out]  $(3*b*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(2*c) + x*(a + b*\text{ArcSin}[c*x])^{3/2} - (3*b^{3/2}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a/b]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(2*c) - (3*b^{3/2}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(2*c)$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

#### Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

#### Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$



Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.), x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int (a + b \sin^{-1}(cx))^{3/2} dx &= x(a + b \sin^{-1}(cx))^{3/2} - \frac{1}{2}(3bc) \int \frac{x\sqrt{a + b \sin^{-1}(cx)}}{\sqrt{1 - c^2x^2}} dx \\
 &= \frac{3b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{2c} + x(a + b \sin^{-1}(cx))^{3/2} - \frac{1}{4}(3b^2) \int \frac{1}{\sqrt{a + b \sin^{-1}(cx)}} dx \\
 &= \frac{3b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{2c} + x(a + b \sin^{-1}(cx))^{3/2} - \frac{(3b) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{a}{b} - \frac{x}{b}\right)}{\sqrt{x}} dx\right)}{4c} \\
 &= \frac{3b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{2c} + x(a + b \sin^{-1}(cx))^{3/2} - \frac{(3b \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}} dx\right)}{4c} \\
 &= \frac{3b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{2c} + x(a + b \sin^{-1}(cx))^{3/2} - \frac{(3b \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}} dx\right)}{4c} \\
 &= \frac{3b\sqrt{1 - c^2x^2} \sqrt{a + b \sin^{-1}(cx)}}{2c} + x(a + b \sin^{-1}(cx))^{3/2} - \frac{3b^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) C\left(\sqrt{\frac{2}{\pi}}\right)}{2c}
 \end{aligned}$$

**Mathematica [C]** time = 2.98, size = 291, normalized size = 1.83

$$b \left[ 2 \left( 3\sqrt{1 - c^2x^2} + 2cx \sin^{-1}(cx) \right) \sqrt{a + b \sin^{-1}(cx)} - \sqrt{2\pi} \sqrt{\frac{1}{b}} \left( 2a \sin\left(\frac{a}{b}\right) + 3b \cos\left(\frac{a}{b}\right) \right) C\left(\sqrt{\frac{1}{b}} \sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}\right) \right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSin[c\*x])^(3/2), x]

[Out] (b\*(2\*Sqrt[a + b\*ArcSin[c\*x]]\*(3\*Sqrt[1 - c^2\*x^2] + 2\*c\*x\*ArcSin[c\*x]) + (2\*a\*(Sqrt[(-I)\*(a + b\*ArcSin[c\*x]])/b]\*Gamma[3/2, (-I)\*(a + b\*ArcSin[c\*x]]

))/b] + E^(((2\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[3/2, (I\*(a + b\*ArcSin[c\*x]))/b)]/(E^((I\*a)/b)\*Sqrt[a + b\*ArcSin[c\*x]]) - Sqrt[b^(-1)]\*Sqrt[2\*Pi]\*FresnelC[Sqrt[b^(-1)]\*Sqrt[2/Pi]\*Sqrt[a + b\*ArcSin[c\*x]]]\*(3\*b\*Cos[a/b] + 2\*a\*Sin[a/b]) + Sqrt[b^(-1)]\*Sqrt[2\*Pi]\*FresnelS[Sqrt[b^(-1)]\*Sqrt[2/Pi]\*Sqrt[a + b\*ArcSin[c\*x]]]\*(2\*a\*Cos[a/b] - 3\*b\*Sin[a/b])))/(4\*c)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 2.79, size = 993, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2),x, algorithm="giac")

[Out] 
$$\frac{1}{2}\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{-1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{Ia/b}/\left(\frac{Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c + \frac{1}{2}I\sqrt{2}\sqrt{\pi}a^2b^3\operatorname{erf}\left(\frac{-1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{Ia/b}/\left(\frac{Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c + \frac{1}{2}\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{-Ia/b}/\left(\frac{-Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c - \frac{1}{2}I\sqrt{2}\sqrt{\pi}a^2b^3\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{-Ia/b}/\left(\frac{-Ib^3}{\sqrt{\operatorname{abs}(b)}} + b^2\sqrt{\operatorname{abs}(b)}\right)c - \frac{1}{2}I\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{-1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{Ia/b}/\left(\frac{Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c + \frac{3}{8}\sqrt{2}\sqrt{\pi}b^3\operatorname{erf}\left(\frac{-1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{Ia/b}/\left(\frac{Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c + \frac{1}{2}I\sqrt{2}\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{-Ia/b}/\left(\frac{-Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c + \frac{3}{8}\sqrt{2}\sqrt{\pi}b^3\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{-Ia/b}/\left(\frac{-Ib^2}{\sqrt{\operatorname{abs}(b)}} + b\sqrt{\operatorname{abs}(b)}\right)c - \sqrt{\pi}a^2b\operatorname{erf}\left(\frac{-1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{Ia/b}/\left(\frac{I\sqrt{2}b^2}{\sqrt{\operatorname{abs}(b)}} + \sqrt{2}b\sqrt{\operatorname{abs}(b)}\right)c - \sqrt{\pi}a^2b\operatorname{erf}\left(\frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\right)/\sqrt{\operatorname{abs}(b)} - \frac{1}{2}\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{-Ia/b}/\left(\frac{-I\sqrt{2}b^2}{\sqrt{\operatorname{abs}(b)}} + \sqrt{2}b\sqrt{\operatorname{abs}(b)}\right)c - \frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{Ia/b}/\left(\frac{I\sqrt{2}b^2}{\sqrt{\operatorname{abs}(b)}} + \sqrt{2}b\sqrt{\operatorname{abs}(b)}\right)c - \frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/b\cdot e^{-Ia/b}/\left(\frac{-I\sqrt{2}b^2}{\sqrt{\operatorname{abs}(b)}} + \sqrt{2}b\sqrt{\operatorname{abs}(b)}\right)c + \frac{3}{4}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/c + \frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/c + \frac{3}{4}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/c + \frac{1}{2}I\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/c + \frac{3}{4}\sqrt{b\arcsin(cx)+a}\sqrt{\operatorname{abs}(b)}/c$$

**maple** [B] time = 0.09, size = 270, normalized size = 1.70

$$-3\sqrt{\pi}\sqrt{2}\sqrt{a+b\arcsin(cx)}\operatorname{FresnelC}\left(\frac{\sqrt{2}\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{\frac{1}{b}}}\right)\cos\left(\frac{a}{b}\right)\sqrt{\frac{1}{b}}b^2 - 3\sqrt{\pi}\sqrt{2}\sqrt{a+b\arcsin(cx)}\sin\left(\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsin(c*x))^(3/2),x)`

[Out]  $\frac{1}{4}c/(a+b\arcsin(cx))^{1/2}(-3\pi^{1/2}2^{1/2}(a+b\arcsin(cx))^{1/2} \text{FresnelC}(2^{1/2}/\pi^{1/2}/(1/b)^{1/2}(a+b\arcsin(cx))^{1/2}/b)\cos(a/b)*(1/b)^{1/2}b^2-3\pi^{1/2}2^{1/2}(a+b\arcsin(cx))^{1/2}\sin(a/b)\text{FresnelS}(2^{1/2}/\pi^{1/2}/(1/b)^{1/2}(a+b\arcsin(cx))^{1/2}/b)*(1/b)^{1/2}b^2+4\arcsin(cx)^2\sin((a+b\arcsin(cx))/b-a/b)*b^2+8\arcsin(cx)\sin((a+b\arcsin(cx))/b-a/b)*a+b+6\arcsin(cx)\cos((a+b\arcsin(cx))/b-a/b)*b^2+4\sin((a+b\arcsin(cx))/b-a/b)*a^2+6\cos((a+b\arcsin(cx))/b-a/b)*ab)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \arcsin(cx) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*arcsin(c*x) + a)^(3/2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \arcsin(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))^(3/2),x)`

[Out] `int((a + b*asin(c*x))^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \arcsin(cx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asin(c*x))**(3/2),x)`

[Out] `Integral((a + b*asin(c*x))**(3/2), x)`

$$3.181 \quad \int \frac{(a+b \sin^{-1}(cx))^{3/2}}{x} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{(a + b \sin^{-1}(cx))^{3/2}}{x}, x \right)$$

[Out] Unintegrable((a+b\*arcsin(c\*x))^(3/2)/x,x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a + b \sin^{-1}(cx))^{3/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSin[c\*x])^(3/2)/x,x]

[Out] Defer[Int] [(a + b\*ArcSin[c\*x])^(3/2)/x, x]

Rubi steps

$$\int \frac{(a + b \sin^{-1}(cx))^{3/2}}{x} dx = \int \frac{(a + b \sin^{-1}(cx))^{3/2}}{x} dx$$

Mathematica [A] time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sin^{-1}(cx))^{3/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSin[c\*x])^(3/2)/x,x]

[Out] Integrate[(a + b\*ArcSin[c\*x])^(3/2)/x, x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2)/x,x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^(3/2)/x, x)

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^(3/2)/x,x)

[Out] int((a+b\*arcsin(c\*x))^(3/2)/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2)/x,x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(3/2)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^(3/2)/x,x)

[Out] int((a + b\*asin(c\*x))^(3/2)/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*(3/2)/x,x)

[Out] Integral((a + b\*asin(c\*x))\*\*(3/2)/x, x)

$$3.182 \quad \int \frac{(a+b \sin^{-1}(cx))^{3/2}}{x^2} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{(a+b \sin^{-1}(cx))^{3/2}}{x^2}, x \right)$$

[Out] Unintegrable((a+b\*arcsin(c\*x))^(3/2)/x^2,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sin^{-1}(cx))^{3/2}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSin[c\*x])^(3/2)/x^2,x]

[Out] Defer[Int] [(a + b\*ArcSin[c\*x])^(3/2)/x^2, x]

Rubi steps

$$\int \frac{(a+b \sin^{-1}(cx))^{3/2}}{x^2} dx = \int \frac{(a+b \sin^{-1}(cx))^{3/2}}{x^2} dx$$

Mathematica [A] time = 10.57, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sin^{-1}(cx))^{3/2}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSin[c\*x])^(3/2)/x^2,x]

[Out] Integrate[(a + b\*ArcSin[c\*x])^(3/2)/x^2, x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2)/x^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2)/x^2,x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^(3/2)/x^2, x)

**maple** [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^(3/2)/x^2,x)

[Out] int((a+b\*arcsin(c\*x))^(3/2)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(3/2)/x^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^(3/2)/x^2,x)

[Out] int((a + b\*asin(c\*x))^(3/2)/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*(3/2)/x\*\*2,x)

[Out] Integral((a + b\*asin(c\*x))\*\*(3/2)/x\*\*2, x)

### 3.183 $\int x^2 (a + b \sin^{-1}(cx))^{5/2} dx$

**Optimal.** Leaf size=358

$$\frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \sin\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3} + \frac{5\sqrt{\frac{\pi}{6}} b^{5/2} \sin\left(\frac{3a}{b}\right) C\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{144c^3} + \frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}$$

[Out]  $\frac{1}{3}x^3(a+b\arcsin(cx))^{5/2} - \frac{5}{864}b^{5/2}\cos(3a/b)\text{FresnelS}(6^{1/2}/\text{Pi}^{1/2})(a+b\arcsin(cx))^{1/2}/b^{1/2})^{6^{1/2}}\text{Pi}^{1/2}/c^3 + \frac{5}{864}b^{5/2}\text{FresnelC}(6^{1/2}/\text{Pi}^{1/2})(a+b\arcsin(cx))^{1/2}/b^{1/2})\sin(3a/b)6^{1/2}\text{Pi}^{1/2}/c^3 + \frac{15}{32}b^{5/2}\cos(a/b)\text{FresnelS}(2^{1/2}/\text{Pi}^{1/2})(a+b\arcsin(cx))^{1/2}/b^{1/2})2^{1/2}\text{Pi}^{1/2}/c^3 - \frac{15}{32}b^{5/2}\text{FresnelC}(2^{1/2}/\text{Pi}^{1/2})(a+b\arcsin(cx))^{1/2}/b^{1/2})\sin(a/b)2^{1/2}\text{Pi}^{1/2}/c^3 + \frac{9}{9}b^{5/2}(a+b\arcsin(cx))^{3/2}(-c^2x^2+1)^{1/2}/c^3 + \frac{5}{18}b^{5/2}x^2(a+b\arcsin(cx))^{3/2}(-c^2x^2+1)^{1/2}/c^3 - \frac{5}{36}b^{5/2}x^3(a+b\arcsin(cx))^{1/2}$

**Rubi [A]** time = 1.41, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$ , Rules used = {4629, 4707, 4677, 4619, 4723, 3306, 3305, 3351, 3304, 3352, 3312}

$$\frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3} + \frac{5\sqrt{\frac{\pi}{6}} b^{5/2} \sin\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{144c^3} + \frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \cos\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcSin[c\*x])^(5/2), x]

[Out]  $\frac{-5b^2x\sqrt{a+b\text{ArcSin}[cx]}}{6c^2} - \frac{5b^2x^3\sqrt{a+b\text{ArcSin}[cx]}}{36} + \frac{5b\sqrt{1-c^2x^2}(a+b\text{ArcSin}[cx])^{3/2}}{9c^3} + \frac{5bx^2\sqrt{1-c^2x^2}(a+b\text{ArcSin}[cx])^{3/2}}{18c} + \frac{x^3(a+b\text{ArcSin}[cx])^{5/2}}{3} + \frac{15b^{5/2}\sqrt{\text{Pi}/2}\cos[a/b]\text{FresnelS}[\sqrt{2/\text{Pi}}\sqrt{a+b\text{ArcSin}[cx]}/\sqrt{b}]}{16c^3} - \frac{5b^{5/2}\sqrt{\text{Pi}/6}\cos[(3a)/b]\text{FresnelS}[\sqrt{6/\text{Pi}}\sqrt{a+b\text{ArcSin}[cx]}/\sqrt{b}]}{144c^3} - \frac{15b^{5/2}\sqrt{\text{Pi}/2}\text{FresnelC}[\sqrt{2/\text{Pi}}\sqrt{a+b\text{ArcSin}[cx]}/\sqrt{b}]\sin[a/b]}{16c^3} + \frac{5b^{5/2}\sqrt{\text{Pi}/6}\text{FresnelC}[\sqrt{6/\text{Pi}}\sqrt{a+b\text{ArcSin}[cx]}/\sqrt{b}]\sin[(3a)/b]}{144c^3}$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]



Rule 3312

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 4707

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*((f\_.)\*(x\_)^(m\_))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(f\*(f\*x)^(m - 1)\*Sqrt[d + e\*x^2]\*(a + b\*ArcSin[c\*x])^n)/(e\*m), x] + (Dist[(f^2\*(m - 1))/(c^2\*m), Int[(f\*x)^(m - 2)\*(a + b\*ArcSin[c\*x])^n/Sqrt[d + e\*x^2], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c^2\*x^2])/(c\*m\*Sqrt[d + e\*x^2]), Int[(f\*x)^(m - 1)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^n\*(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \sin^{-1}(cx))^{5/2} dx &= \frac{1}{3} x^3 (a + b \sin^{-1}(cx))^{5/2} - \frac{1}{6} (5bc) \int \frac{x^3 (a + b \sin^{-1}(cx))^{3/2}}{\sqrt{1 - c^2 x^2}} dx \\
&= \frac{5bx^2 \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{18c} + \frac{1}{3} x^3 (a + b \sin^{-1}(cx))^{5/2} - \frac{1}{12} (5b^2) \int x^2 \sqrt{a + b \sin^{-1}(cx)} dx \\
&= -\frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3} + \frac{5bx^2 \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{1/2}}{18c} \\
&= -\frac{5b^2 x \sqrt{a + b \sin^{-1}(cx)}}{6c^2} - \frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3} \\
&= -\frac{5b^2 x \sqrt{a + b \sin^{-1}(cx)}}{6c^2} - \frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3} \\
&= -\frac{5b^2 x \sqrt{a + b \sin^{-1}(cx)}}{6c^2} - \frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3} \\
&= -\frac{5b^2 x \sqrt{a + b \sin^{-1}(cx)}}{6c^2} - \frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3} \\
&= -\frac{5b^2 x \sqrt{a + b \sin^{-1}(cx)}}{6c^2} - \frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3} \\
&= -\frac{5b^2 x \sqrt{a + b \sin^{-1}(cx)}}{6c^2} - \frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3} \\
&= -\frac{5b^2 x \sqrt{a + b \sin^{-1}(cx)}}{6c^2} - \frac{5}{36} b^2 x^3 \sqrt{a + b \sin^{-1}(cx)} + \frac{5b \sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{9c^3}
\end{aligned}$$

**Mathematica [C]** time = 0.28, size = 228, normalized size = 0.64

$$\frac{b^3 e^{-\frac{3ia}{b}} \left( -81 e^{\frac{2ia}{b}} \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{7}{2}, -\frac{i(a+b \sin^{-1}(cx))}{b}\right) - 81 e^{\frac{4ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{7}{2}, \frac{i(a+b \sin^{-1}(cx))}{b}\right) + \sqrt{3} \left( \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{7}{2}, -\frac{i(a+b \sin^{-1}(cx))}{b}\right) + \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{7}{2}, \frac{i(a+b \sin^{-1}(cx))}{b}\right) \right) \right)}{648c^3 \sqrt{a + b \sin^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcSin[c\*x])^(5/2),x]

[Out] (b^3\*(-81\*E^(((2\*I)\*a)/b)\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[7/2, ((-I)\*(a + b\*ArcSin[c\*x]))/b] - 81\*E^(((4\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[7/2, (I\*(a + b\*ArcSin[c\*x]))/b] + Sqrt[3]\*(Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[7/2, ((-3\*I)\*(a + b\*ArcSin[c\*x]))/b] + E^(((6\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[7/2, ((3\*I)\*(a + b\*ArcSin[c\*x]))/b]))/(648\*c^3\*E^(((3\*I)\*a)/b)\*Sqrt[a + b\*ArcSin[c\*x]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)



```

) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b^(3/2) - I*sqrt(6)*b^(5/2)/a
bs(b))*c^3) + 1/16*sqrt(pi)*a*b^3*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/
sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b
)/((sqrt(6)*b^(3/2) - I*sqrt(6)*b^(5/2)/abs(b))*c^3) - 1/16*sqrt(pi)*a*b^(5
/2)*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b
*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/a
bs(b))*c^3) + 5/288*I*sqrt(pi)*b^(7/2)*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x)
+ a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(3*I
*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/abs(b))*c^3) - 1/16*sqrt(pi)*a*b^(5/2)*er
f(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcsi
n(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b - I*sqrt(6)*b^2/abs(b)
)*c^3) - 5/288*I*sqrt(pi)*b^(7/2)*erf(-1/2*sqrt(6)*sqrt(b*arcsin(c*x) + a)/
sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcsin(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b
)/((sqrt(6)*b - I*sqrt(6)*b^2/abs(b))*c^3) + 1/12*I*sqrt(b*arcsin(c*x) + a)
*a*b*arcsin(c*x)*e^(3*I*arcsin(c*x))/c^3 - 5/144*sqrt(b*arcsin(c*x) + a)*b^
2*arcsin(c*x)*e^(3*I*arcsin(c*x))/c^3 - 1/4*I*sqrt(b*arcsin(c*x) + a)*a*b*a
rcsin(c*x)*e^(I*arcsin(c*x))/c^3 + 5/16*sqrt(b*arcsin(c*x) + a)*b^2*arcsin(
c*x)*e^(I*arcsin(c*x))/c^3 + 1/4*I*sqrt(b*arcsin(c*x) + a)*a*b*arcsin(c*x)*
e^(-I*arcsin(c*x))/c^3 + 5/16*sqrt(b*arcsin(c*x) + a)*b^2*arcsin(c*x)*e^(-I
*arcsin(c*x))/c^3 - 1/12*I*sqrt(b*arcsin(c*x) + a)*a*b*arcsin(c*x)*e^(-3*I*
arcsin(c*x))/c^3 - 5/144*sqrt(b*arcsin(c*x) + a)*b^2*arcsin(c*x)*e^(-3*I*ar
csin(c*x))/c^3 + 1/24*I*sqrt(b*arcsin(c*x) + a)*a^2*e^(3*I*arcsin(c*x))/c^3
- 5/144*sqrt(b*arcsin(c*x) + a)*a*b*e^(3*I*arcsin(c*x))/c^3 - 5/288*I*sqrt
(b*arcsin(c*x) + a)*b^2*e^(3*I*arcsin(c*x))/c^3 - 1/8*I*sqrt(b*arcsin(c*x)
+ a)*a^2*e^(I*arcsin(c*x))/c^3 + 5/16*sqrt(b*arcsin(c*x) + a)*a*b*e^(I*arcs
in(c*x))/c^3 + 15/32*I*sqrt(b*arcsin(c*x) + a)*b^2*e^(I*arcsin(c*x))/c^3 +
1/8*I*sqrt(b*arcsin(c*x) + a)*a^2*e^(-I*arcsin(c*x))/c^3 + 5/16*sqrt(b*arcs
in(c*x) + a)*a*b*e^(-I*arcsin(c*x))/c^3 - 15/32*I*sqrt(b*arcsin(c*x) + a)*b
^2*e^(-I*arcsin(c*x))/c^3 - 1/24*I*sqrt(b*arcsin(c*x) + a)*a^2*e^(-3*I*arcs
in(c*x))/c^3 - 5/144*sqrt(b*arcsin(c*x) + a)*a*b*e^(-3*I*arcsin(c*x))/c^3 +
5/288*I*sqrt(b*arcsin(c*x) + a)*b^2*e^(-3*I*arcsin(c*x))/c^3

```

**maple [B]** time = 0.19, size = 792, normalized size = 2.21

$$-5\sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{a+b\arcsin(cx)}\cos\left(\frac{3a}{b}\right)S\left(\frac{\sqrt{2}\sqrt{3}\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{\frac{1}{b}}}\right)\sqrt{\frac{1}{b}}b^3+5\sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{a+b\arcsin(cx)}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsin(c\*x))^(5/2),x)

```

[Out] 1/864/c^3/(a+b*arcsin(c*x))^(1/2)*(-5*3^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arcsin(
c*x))^(1/2)*cos(3*a/b)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*a
rcsin(c*x))^(1/2)/b)*(1/b)^(1/2)*b^3+5*3^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arcsin
(c*x))^(1/2)*sin(3*a/b)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(1/b)^(1/2)*(a+b*
arcsin(c*x))^(1/2)/b)*(1/b)^(1/2)*b^3+405*2^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x)
)^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arcsin(c*x))^(1
/2)/b)*(1/b)^(1/2)*b^3-405*2^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(a/b
)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)*(1/b)^(1
/2)*b^3+216*arcsin(c*x)^3*sin((a+b*arcsin(c*x))/b-a/b)*b^3-72*arcsin(c*x)^3
*sin(3*(a+b*arcsin(c*x))/b-3*a/b)*b^3+648*arcsin(c*x)^2*sin((a+b*arcsin(c*x)
)/b-a/b)*a*b^2+540*arcsin(c*x)^2*cos((a+b*arcsin(c*x))/b-a/b)*b^3-216*arcs
in(c*x)^2*sin(3*(a+b*arcsin(c*x))/b-3*a/b)*a*b^2-60*arcsin(c*x)^2*cos(3*(a+
b*arcsin(c*x))/b-3*a/b)*b^3+648*arcsin(c*x)*sin((a+b*arcsin(c*x))/b-a/b)*a^
2*b-810*arcsin(c*x)*sin((a+b*arcsin(c*x))/b-a/b)*b^3+1080*arcsin(c*x)*cos((
a+b*arcsin(c*x))/b-a/b)*a*b^2-216*arcsin(c*x)*sin(3*(a+b*arcsin(c*x))/b-3*a
/b)*a^2*b+30*arcsin(c*x)*sin(3*(a+b*arcsin(c*x))/b-3*a/b)*b^3-120*arcsin(c*
x)*cos(3*(a+b*arcsin(c*x))/b-3*a/b)*a*b^2+216*sin((a+b*arcsin(c*x))/b-a/b)*
a^3-810*sin((a+b*arcsin(c*x))/b-a/b)*a*b^2+540*cos((a+b*arcsin(c*x))/b-a/b)

```

$*a^2*b-72*\sin(3*(a+b*\arcsin(c*x))/b-3*a/b)*a^3+30*\sin(3*(a+b*\arcsin(c*x))/b-3*a/b)*a*b^2-60*\cos(3*(a+b*\arcsin(c*x))/b-3*a/b)*a^2*b)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \arcsin(cx) + a)^{\frac{5}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsin(c\*x))^(5/2),x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(5/2)\*x^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b \arcsin(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*asin(c\*x))^(5/2),x)

[Out] int(x^2\*(a + b\*asin(c\*x))^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \arcsin(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*asin(c\*x))\*\*(5/2),x)

[Out] Integral(x\*\*2\*(a + b\*asin(c\*x))\*\*(5/2), x)

### 3.184 $\int x \left( a + b \sin^{-1}(cx) \right)^{5/2} dx$

**Optimal.** Leaf size=216

$$\frac{15\sqrt{\pi} b^{5/2} \cos\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2} - \frac{15\sqrt{\pi} b^{5/2} \sin\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2} + \frac{15b^2\sqrt{a+b\sin^{-1}(cx)}}{64c^2} - \frac{15}{32}b^2x^2\sqrt{a+b\sin^{-1}(cx)}$$

[Out]  $-1/4*(a+b*\arcsin(c*x))^{(5/2)}/c^2+1/2*x^2*(a+b*\arcsin(c*x))^{(5/2)}-15/128*b^{(5/2)}*\cos(2*a/b)*\text{FresnelC}(2*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/c^2-15/128*b^{(5/2)}*\text{FresnelS}(2*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)}/\text{Pi}^{(1/2)})*\sin(2*a/b)*\text{Pi}^{(1/2)}/c^2+5/8*b*x*(a+b*\arcsin(c*x))^{(3/2)}*(-c^2*x^2+1)^{(1/2)}/c+15/64*b^2*(a+b*\arcsin(c*x))^{(1/2)}/c^2-15/32*b^2*x^2*(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 0.74, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {4629, 4707, 4641, 4723, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\pi} b^{5/2} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{\pi}\sqrt{b}}\right)}{128c^2} - \frac{15\sqrt{\pi} b^{5/2} \sin\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2} + \frac{15b^2\sqrt{a+b\sin^{-1}(cx)}}{64c^2} - \frac{15}{32}b^2x^2\sqrt{a+b\sin^{-1}(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{ArcSin}[c*x])^{(5/2)}, x]$

[Out]  $(15*b^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(64*c^2) - (15*b^2*x^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/32 + (5*b*x*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^{(3/2)})/(8*c) - (a + b*\text{ArcSin}[c*x])^{(5/2)}/(4*c^2) + (x^2*(a + b*\text{ArcSin}[c*x])^{(5/2)})/2 - (15*b^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[(2*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))]/(128*c^2) - (15*b^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[(2*a)/b])/ (128*c^2)$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

#### Rule 3312

$\text{Int}[(c_. + (d_.)*(x_))^{(m_)}*\sin[(e_.) + (f_.)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ (!\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 1]))$

Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4629

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)</sup>\*(x\_)<sup>(m\_)</sup>, x\_Symbol] := Simp[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>n</sup>)/(m + 1), x] - Dist[(b\*c\*n)/(m + 1), Int[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n - 1)</sup>)/Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>], x], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]

Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)</sup>/Sqrt[(d\_) + (e\_.)\*(x\_)<sup>2</sup>], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]

Rule 4707

Int((((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)</sup>\*((f\_.)\*(x\_)<sup>(m\_)</sup>)/Sqrt[(d\_) + (e\_.)\*(x\_)<sup>2</sup>], x\_Symbol] := Simp[(f\*(f\*x)<sup>(m - 1)</sup>\*Sqrt[d + e\*x<sup>2</sup>]\*(a + b\*ArcSin[c\*x])<sup>n</sup>)/(e\*m), x] + (Dist[(f<sup>2</sup>\*(m - 1))/(c<sup>2</sup>\*m), Int[(f\*x)<sup>(m - 2)</sup>\*(a + b\*ArcSin[c\*x])<sup>n</sup>]/Sqrt[d + e\*x<sup>2</sup>], x], x] + Dist[(b\*f\*n\*Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>])/(c\*m\*Sqrt[d + e\*x<sup>2</sup>]), Int[(f\*x)<sup>(m - 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n - 1)</sup>], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && IntegerQ[m]

Rule 4723

Int(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)</sup>\*(x\_)<sup>(m\_)</sup>\*((d\_) + (e\_.)\*(x\_)<sup>2</sup>)<sup>(p\_)</sup>, x\_Symbol] := Dist[d<sup>p</sup>/c<sup>(m + 1)</sup>, Subst[Int[(a + b\*x)<sup>n</sup>\*Sin[x]<sup>m</sup>\*Cos[x]<sup>(2\*p + 1)</sup>, x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

Rubi steps

$$\begin{aligned}
\int x (a + b \sin^{-1}(cx))^{5/2} dx &= \frac{1}{2} x^2 (a + b \sin^{-1}(cx))^{5/2} - \frac{1}{4} (5bc) \int \frac{x^2 (a + b \sin^{-1}(cx))^{3/2}}{\sqrt{1 - c^2 x^2}} dx \\
&= \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c} + \frac{1}{2} x^2 (a + b \sin^{-1}(cx))^{5/2} - \frac{1}{16} (15b^2) \int x \sqrt{a + b \sin^{-1}(cx)} dx \\
&= -\frac{15}{32} b^2 x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} \\
&= -\frac{15}{32} b^2 x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} \\
&= -\frac{15}{32} b^2 x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c} - \frac{(a + b \sin^{-1}(cx))^{3/2}}{4c^2} \\
&= \frac{15b^2 \sqrt{a + b \sin^{-1}(cx)}}{64c^2} - \frac{15}{32} b^2 x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c} \\
&= \frac{15b^2 \sqrt{a + b \sin^{-1}(cx)}}{64c^2} - \frac{15}{32} b^2 x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c} \\
&= \frac{15b^2 \sqrt{a + b \sin^{-1}(cx)}}{64c^2} - \frac{15}{32} b^2 x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c} \\
&= \frac{15b^2 \sqrt{a + b \sin^{-1}(cx)}}{64c^2} - \frac{15}{32} b^2 x^2 \sqrt{a + b \sin^{-1}(cx)} + \frac{5bx\sqrt{1 - c^2 x^2} (a + b \sin^{-1}(cx))^{3/2}}{8c}
\end{aligned}$$

**Mathematica [C]** time = 0.10, size = 141, normalized size = 0.65

$$\frac{e^{-\frac{2ia}{b}} (a + b \sin^{-1}(cx))^{5/2} \left( \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{7}{2}, -\frac{2i(a+b \sin^{-1}(cx))}{b}\right) + e^{\frac{4ia}{b}} \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{7}{2}, \frac{2i(a+b \sin^{-1}(cx))}{b}\right) \right)}{32\sqrt{2} c^2 \left( \frac{(a+b \sin^{-1}(cx))^2}{b^2} \right)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcSin[c\*x])^(5/2),x]

[Out] ((a + b\*ArcSin[c\*x])^(5/2)\*(Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[7/2, ((-2\*I)\*(a + b\*ArcSin[c\*x]))/b] + E^(((4\*I)\*a)/b)\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[7/2, ((2\*I)\*(a + b\*ArcSin[c\*x]))/b]))/(32\*Sqrt[2]\*c^2\*E^(((2\*I)\*a)/b)\*((a + b\*ArcSin[c\*x])^2/b^2)^(3/2))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)



**giac** [C] time = 2.73, size = 1307, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^(5/2),x, algorithm="giac")

[Out]  $\frac{1}{4}I\sqrt{\pi}a^3b^{3/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) - I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b^2+Ib^3/\operatorname{abs}(b))c^2) - 3/8\sqrt{\pi}a^2b^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) - I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b^2+Ib^3/\operatorname{abs}(b))c^2) - 1/4I\sqrt{\pi}a^3b^{3/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) + I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^2-Ib^3/\operatorname{abs}(b))c^2) - 3/8\sqrt{\pi}a^2b^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) + I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^2-Ib^3/\operatorname{abs}(b))c^2) - 1/8\sqrt{b\arcsin(cx)+a}b^2\arcsin(cx)^2e^{2I\arcsin(cx)}/c^2 - 1/8\sqrt{b\arcsin(cx)+a}b^2\arcsin(cx)^2e^{-2I\arcsin(cx)}/c^2 + 3/8\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) - I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b^{3/2}+Ib^{5/2}/\operatorname{abs}(b))c^2) - 9/64I\sqrt{\pi}a^2b^3\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) - I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b^{3/2}+Ib^{5/2}/\operatorname{abs}(b))c^2) + 1/4I\sqrt{\pi}a^3b\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) + I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^{3/2}-Ib^{5/2}/\operatorname{abs}(b))c^2) + 3/8\sqrt{\pi}a^2b^2\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) + I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^{3/2}-Ib^{5/2}/\operatorname{abs}(b))c^2) + 9/64I\sqrt{\pi}a^2b^3\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) + I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b^{3/2}-Ib^{5/2}/\operatorname{abs}(b))c^2) - 1/4I\sqrt{\pi}a^3\sqrt{b}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) - I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b+Ib^2/\operatorname{abs}(b))c^2) + 9/64I\sqrt{\pi}a^2b^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) - I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b+Ib^2/\operatorname{abs}(b))c^2) + 15/256\sqrt{\pi}b^{7/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) - I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{2Ia/b}/((b+Ib^2/\operatorname{abs}(b))c^2) - 9/64I\sqrt{\pi}a^2b^{5/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) + I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b-Ib^2/\operatorname{abs}(b))c^2) + 15/256\sqrt{\pi}b^{7/2}\operatorname{erf}\left(\frac{-\sqrt{b\arcsin(cx)+a}}{\sqrt{b}}\right) + I\sqrt{b\arcsin(cx)+a}\sqrt{b}/\operatorname{abs}(b)e^{-2Ia/b}/((b-Ib^2/\operatorname{abs}(b))c^2) - 1/4\sqrt{b\arcsin(cx)+a}a^2b\arcsin(cx)e^{2I\arcsin(cx)}/c^2 - 5/32I\sqrt{b\arcsin(cx)+a}b^2\arcsin(cx)e^{2I\arcsin(cx)}/c^2 - 1/4\sqrt{b\arcsin(cx)+a}a^2b\arcsin(cx)e^{-2I\arcsin(cx)}/c^2 + 5/32I\sqrt{b\arcsin(cx)+a}b^2\arcsin(cx)e^{-2I\arcsin(cx)}/c^2 - 1/8\sqrt{b\arcsin(cx)+a}a^2e^{2I\arcsin(cx)}/c^2 - 5/32I\sqrt{b\arcsin(cx)+a}a^2b^2e^{2I\arcsin(cx)}/c^2 - 1/8\sqrt{b\arcsin(cx)+a}a^2e^{-2I\arcsin(cx)}/c^2 + 5/32I\sqrt{b\arcsin(cx)+a}a^2b^2e^{-2I\arcsin(cx)}/c^2 + 15/128\sqrt{b\arcsin(cx)+a}b^2e^{2I\arcsin(cx)}/c^2 - 1/8\sqrt{b\arcsin(cx)+a}a^2e^{-2I\arcsin(cx)}/c^2 + 5/32I\sqrt{b\arcsin(cx)+a}a^2b^2e^{-2I\arcsin(cx)}/c^2 + 15/128\sqrt{b\arcsin(cx)+a}b^2e^{-2I\arcsin(cx)}/c^2$

**maple** [B] time = 0.11, size = 394, normalized size = 1.82

$$15\sqrt{\pi}\sqrt{\frac{1}{b}}\sqrt{a+b\arcsin(cx)}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{\frac{1}{b}}}\right)b^3 + 15\sqrt{\pi}\sqrt{\frac{1}{b}}\sqrt{a+b\arcsin(cx)}\sin\left(\frac{2a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsin(c\*x))^(5/2),x)

[Out]  $-1/128/c^2*(15\pi^{1/2}*(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}*\cos(2a/b)*\operatorname{FresnelC}(2/\pi^{1/2}/(1/b)^{1/2}*(a+b\arcsin(cx))^{1/2}/b)*b^3+15\pi^{1/2}*(1/b$

)^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)\*sin(2\*a/b)\*FresnelS(2/Pi^(1/2)/(1/b)^(1/2)\*  
(a+b\*arcsin(c\*x))^(1/2)/b)\*b^3+32\*arcsin(c\*x)^3\*cos(2\*(a+b\*arcsin(c\*x))/b-2  
\*a/b)\*b^3+96\*arcsin(c\*x)^2\*cos(2\*(a+b\*arcsin(c\*x))/b-2\*a/b)\*a\*b^2-40\*arcsin  
(c\*x)^2\*sin(2\*(a+b\*arcsin(c\*x))/b-2\*a/b)\*b^3+96\*arcsin(c\*x)\*cos(2\*(a+b\*arcs  
in(c\*x))/b-2\*a/b)\*a^2\*b-30\*arcsin(c\*x)\*cos(2\*(a+b\*arcsin(c\*x))/b-2\*a/b)\*b^3  
-80\*arcsin(c\*x)\*sin(2\*(a+b\*arcsin(c\*x))/b-2\*a/b)\*a\*b^2+32\*cos(2\*(a+b\*arcsin  
(c\*x))/b-2\*a/b)\*a^3-30\*cos(2\*(a+b\*arcsin(c\*x))/b-2\*a/b)\*a\*b^2-40\*sin(2\*(a+b  
\*arcsin(c\*x))/b-2\*a/b)\*a^2\*b)/(a+b\*arcsin(c\*x))^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \arcsin(cx) + a)^{\frac{5}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsin(c\*x))^(5/2),x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(5/2)\*x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x (a + b \operatorname{asin}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asin(c\*x))^(5/2),x)

[Out] int(x\*(a + b\*asin(c\*x))^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{asin}(cx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asin(c\*x))\*\*(5/2),x)

[Out] Integral(x\*(a + b\*asin(c\*x))\*\*(5/2), x)

### 3.185 $\int (a + b \sin^{-1}(cx))^{5/2} dx$

**Optimal.** Leaf size=179

$$\frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \sin\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c} + \frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{15}{4} b^2 x \sqrt{a + b \sin^{-1}(cx)} + \dots$$

[Out]  $x*(a+b*\arcsin(c*x))^{5/2}+15/8*b^{5/2}*\cos(a/b)*\text{FresnelS}(2^{1/2}/\text{Pi}^{1/2}*(a+b*\arcsin(c*x))^{1/2}/b^{1/2})*2^{1/2}*\text{Pi}^{1/2}/c-15/8*b^{5/2}*\text{FresnelC}(2^{1/2}/\text{Pi}^{1/2}*(a+b*\arcsin(c*x))^{1/2}/b^{1/2})*\sin(a/b)*2^{1/2}*\text{Pi}^{1/2}/c+5/2*b*(a+b*\arcsin(c*x))^{3/2}*(-c^2*x^2+1)^{1/2}/c-15/4*b^2*x*(a+b*\arcsin(c*x))^{1/2}$

**Rubi [A]** time = 0.50, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4619, 4677, 4723, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c} + \frac{15\sqrt{\frac{\pi}{2}} b^{5/2} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{15}{4} b^2 x \sqrt{a + b \sin^{-1}(cx)} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSin}[c*x])^{5/2}, x]$

[Out]  $(-15*b^2*x*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/4 + (5*b*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^{3/2})/(2*c) + x*(a + b*\text{ArcSin}[c*x])^{5/2} + (15*b^{5/2}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a/b]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(4*c) - (15*b^{5/2}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(4*c)$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

#### Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

#### Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}[\{d, e, f\}, x]$

#### Rule 3352

Int[Cos[(d\_.)\*(e\_.) + (f\_.)\*(x\_)]^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4619

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n], x\_Symbol] := Simp[x\*(a + b\*ArcSin[c\*x])^n, x] - Dist[b\*c\*n, Int[(x\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

#### Rule 4677

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^m\*((d\_.) + (e\_.)\*(x\_)^2)^p], x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x])^n)/(2\*e\*(p + 1)), x] + Dist[(b\*n\*d^IntPart[p]\*(d + e\*x^2)^FracPart[p])/(2\*c\*(p + 1)\*(1 - c^2\*x^2)^FracPart[p]), Int[(1 - c^2\*x^2)^(p + 1/2)\*(a + b\*ArcSin[c\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

#### Rule 4723

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\*(x\_)^m\*((d\_.) + (e\_.)\*(x\_)^2)^p], x\_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sin[x]^m\*Cos[x]^(2\*p + 1), x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2\*d + e, 0] && IntegerQ[2\*p] && GtQ[p, -1] && IGtQ[m, 0] && (IntegerQ[p] || GtQ[d, 0])

#### Rubi steps

$$\begin{aligned}
 \int (a + b \sin^{-1}(cx))^{5/2} dx &= x (a + b \sin^{-1}(cx))^{5/2} - \frac{1}{2}(5bc) \int \frac{x (a + b \sin^{-1}(cx))^{3/2}}{\sqrt{1 - c^2x^2}} dx \\
 &= \frac{5b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^{3/2}}{2c} + x (a + b \sin^{-1}(cx))^{5/2} - \frac{1}{4}(15b^2) \int \sqrt{a + b \sin^{-1}(cx)} dx \\
 &= -\frac{15}{4}b^2x\sqrt{a + b \sin^{-1}(cx)} + \frac{5b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^{3/2}}{2c} + x (a + b \sin^{-1}(cx))^{5/2} \\
 &= -\frac{15}{4}b^2x\sqrt{a + b \sin^{-1}(cx)} + \frac{5b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^{3/2}}{2c} + x (a + b \sin^{-1}(cx))^{5/2} \\
 &= -\frac{15}{4}b^2x\sqrt{a + b \sin^{-1}(cx)} + \frac{5b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^{3/2}}{2c} + x (a + b \sin^{-1}(cx))^{5/2} \\
 &= -\frac{15}{4}b^2x\sqrt{a + b \sin^{-1}(cx)} + \frac{5b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^{3/2}}{2c} + x (a + b \sin^{-1}(cx))^{5/2} \\
 &= -\frac{15}{4}b^2x\sqrt{a + b \sin^{-1}(cx)} + \frac{5b\sqrt{1 - c^2x^2} (a + b \sin^{-1}(cx))^{3/2}}{2c} + x (a + b \sin^{-1}(cx))^{5/2}
 \end{aligned}$$

**Mathematica** [C] time = 3.19, size = 379, normalized size = 2.12

$$e^{-\frac{ia}{b}} \left( \frac{i\sqrt{\frac{\pi}{2}}(4a^2+15b^2)\left(-1+e^{\frac{2ia}{b}}\right)\sqrt{a+b\sin^{-1}(cx)}C\left(\sqrt{\frac{1}{b}}\sqrt{\frac{2}{\pi}}\sqrt{a+b\sin^{-1}(cx)}\right)}{\sqrt{\frac{1}{b}}} + \frac{\sqrt{\frac{\pi}{2}}(4a^2+15b^2)\left(1+e^{\frac{2ia}{b}}\right)\sqrt{a+b\sin^{-1}(cx)}S\left(\sqrt{\frac{1}{b}}\sqrt{\frac{2}{\pi}}\sqrt{a+b\sin^{-1}(cx)}\right)}{\sqrt{\frac{1}{b}}} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSin[c*x])^(5/2), x]
```

```
[Out] ((I*(4*a^2 + 15*b^2)*(-1 + E^(((2*I)*a)/b))*Sqrt[Pi/2]*Sqrt[a + b*ArcSin[c*
x]]*FresnelC[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]]])/Sqrt[b^(-1)]
+ ((4*a^2 + 15*b^2)*(1 + E^(((2*I)*a)/b))*Sqrt[Pi/2]*Sqrt[a + b*ArcSin[c*x
]]*FresnelS[Sqrt[b^(-1)]*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]]])/Sqrt[b^(-1)]
+ 2*b*(E^((I*a)/b)*(a + b*ArcSin[c*x])*(-15*b*c*x + 10*a*Sqrt[1 - c^2*x^2]
+ 2*(4*a*c*x + 5*b*Sqrt[1 - c^2*x^2])*ArcSin[c*x] + 4*b*c*x*ArcSin[c*x]^2)
+ 2*a^2*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*ArcSin[c
*x]))/b] + 2*a^2*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[3/2,
(I*(a + b*ArcSin[c*x]))/b]))/(8*c*E^((I*a)/b)*Sqrt[a + b*ArcSin[c*x]])
```

```
fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsin(c*x))^(5/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

```
giac [C] time = 4.07, size = 1179, normalized size = 6.59
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsin(c*x))^(5/2), x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(pi)*a^3*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sq
rt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((
I*b^4/sqrt(abs(b)) + b^3*sqrt(abs(b)))*c) + 1/2*sqrt(2)*sqrt(pi)*a^3*b^3*er
f(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*
arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^4/sqrt(abs(b)) + b^3*sq
rt(abs(b)))*c) + 3/2*I*sqrt(2)*sqrt(pi)*a^2*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*ar
csin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(
b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) - 3/2*I*sqrt(2
)*sqrt(pi)*a^2*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) -
1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sq
rt(abs(b)) + b^2*sqrt(abs(b)))*c) - 3/2*I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2
*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin
(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))
*c) - 15/16*I*sqrt(2)*sqrt(pi)*b^4*erf(-1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) +
a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(I*
a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 3/2*I*sqrt(2)*sqrt(pi)*a^2
*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*s
qrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b
*sqrt(abs(b)))*c) + 15/16*I*sqrt(2)*sqrt(pi)*b^4*erf(1/2*I*sqrt(2)*sqrt(b*a
rcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs
(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 1/2*I*sqrt(
b*arcsin(c*x) + a)*b^2*arcsin(c*x)^2*e^(I*arcsin(c*x))/c + 1/2*I*sqrt(b*arc
sin(c*x) + a)*b^2*arcsin(c*x)^2*e^(-I*arcsin(c*x))/c - sqrt(pi)*a^3*b*erf(-
1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arc
sin(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt
(2)*b*sqrt(abs(b)))*c) - sqrt(pi)*a^3*b*erf(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x
) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e
^(-I*a/b)/((-I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt(2)*b*sqrt(abs(b)))*c) - I*sq
rt(b*arcsin(c*x) + a)*a*b*arcsin(c*x)*e^(I*arcsin(c*x))/c + 5/4*sqrt(b*arcs
```



$$3.186 \quad \int \frac{(a+b \sin^{-1}(cx))^{5/2}}{x} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{(a + b \sin^{-1}(cx))^{5/2}}{x}, x \right)$$

[Out] Unintegrable((a+b\*arcsin(c\*x))^(5/2)/x,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSin[c\*x])^(5/2)/x,x]

[Out] Defer[Int][(a + b\*ArcSin[c\*x])^(5/2)/x, x]

Rubi steps

$$\int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x} dx = \int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x} dx$$

Mathematica [A] time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSin[c\*x])^(5/2)/x,x]

[Out] Integrate[(a + b\*ArcSin[c\*x])^(5/2)/x, x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(5/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{5/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(5/2)/x,x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^(5/2)/x, x)

**maple** [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^(5/2)/x,x)

[Out] int((a+b\*arcsin(c\*x))^(5/2)/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(5/2)/x,x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(5/2)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^(5/2)/x,x)

[Out] int((a + b\*asin(c\*x))^(5/2)/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*(5/2)/x,x)

[Out] Integral((a + b\*asin(c\*x))\*\*(5/2)/x, x)



$$3.187 \quad \int \frac{(a+b \sin^{-1}(cx))^{5/2}}{x^2} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{(a + b \sin^{-1}(cx))^{5/2}}{x^2}, x \right)$$

[Out] Unintegrable((a+b\*arcsin(c\*x))^(5/2)/x^2, x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSin[c\*x])^(5/2)/x^2, x]

[Out] Defer[Int][(a + b\*ArcSin[c\*x])^(5/2)/x^2, x]

Rubi steps

$$\int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x^2} dx = \int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x^2} dx$$

Mathematica [A] time = 10.60, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sin^{-1}(cx))^{5/2}}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSin[c\*x])^(5/2)/x^2, x]

[Out] Integrate[(a + b\*ArcSin[c\*x])^(5/2)/x^2, x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(5/2)/x^2, x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{5/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(5/2)/x^2, x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^(5/2)/x^2, x)

**maple** [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^{\frac{5}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^(5/2)/x^2,x)

[Out] int((a+b\*arcsin(c\*x))^(5/2)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^{\frac{5}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^(5/2)/x^2,x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(5/2)/x^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{5}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^(5/2)/x^2,x)

[Out] int((a + b\*asin(c\*x))^(5/2)/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asin}(cx))^{\frac{5}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*(5/2)/x\*\*2,x)

[Out] Integral((a + b\*asin(c\*x))\*\*(5/2)/x\*\*2, x)

$$3.188 \quad \int \frac{x^2}{\sqrt{a+b \sin^{-1}(cx)}} dx$$

**Optimal.** Leaf size=223

$$\frac{\sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3} - \frac{\sqrt{\frac{\pi}{6}} \cos\left(\frac{3a}{b}\right) C\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3} + \frac{\sqrt{\frac{\pi}{2}} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3} - \frac{\sqrt{\frac{\pi}{6}} \sin\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3}$$

[Out]  $-1/12 \cos(3a/b) \text{FresnelC}(6^{1/2}/\text{Pi}^{1/2} \cdot (a+b \arcsin(cx))^{1/2}/b^{1/2}) \cdot 6^{1/2} \text{Pi}^{1/2}/c^3/b^{1/2} - 1/12 \text{FresnelS}(6^{1/2}/\text{Pi}^{1/2} \cdot (a+b \arcsin(cx))^{1/2}/b^{1/2}) \cdot \sin(3a/b) \cdot 6^{1/2} \text{Pi}^{1/2}/c^3/b^{1/2} + 1/4 \cos(a/b) \text{FresnelC}(2^{1/2}/\text{Pi}^{1/2} \cdot (a+b \arcsin(cx))^{1/2}/b^{1/2}) \cdot 2^{1/2} \text{Pi}^{1/2}/c^3/b^{1/2} + 1/4 \text{FresnelS}(2^{1/2}/\text{Pi}^{1/2} \cdot (a+b \arcsin(cx))^{1/2}/b^{1/2}) \cdot \sin(a/b) \cdot 2^{1/2} \text{Pi}^{1/2}/c^3/b^{1/2}$

**Rubi [A]** time = 0.42, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {4635, 4406, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3} - \frac{\sqrt{\frac{\pi}{6}} \cos\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3} + \frac{\sqrt{\frac{\pi}{2}} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3} - \frac{\sqrt{\frac{\pi}{6}} \sin\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b} c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a + b\*ArcSin[c\*x]], x]

[Out]  $(\text{Sqrt}[\text{Pi}/2] \cdot \text{Cos}[a/b] \cdot \text{FresnelC}[(\text{Sqrt}[2/\text{Pi}] \cdot \text{Sqrt}[a + b \cdot \text{ArcSin}[c \cdot x]])/\text{Sqrt}[b]])/(2 \cdot \text{Sqrt}[b] \cdot c^3) - (\text{Sqrt}[\text{Pi}/6] \cdot \text{Cos}[(3 \cdot a)/b] \cdot \text{FresnelC}[(\text{Sqrt}[6/\text{Pi}] \cdot \text{Sqrt}[a + b \cdot \text{ArcSin}[c \cdot x]])/\text{Sqrt}[b]])/(2 \cdot \text{Sqrt}[b] \cdot c^3) + (\text{Sqrt}[\text{Pi}/2] \cdot \text{FresnelS}[(\text{Sqrt}[2/\text{Pi}] \cdot \text{Sqrt}[a + b \cdot \text{ArcSin}[c \cdot x]])/\text{Sqrt}[b]] \cdot \text{Sin}[a/b])/(2 \cdot \text{Sqrt}[b] \cdot c^3) - (\text{Sqrt}[\text{Pi}/6] \cdot \text{FresnelS}[(\text{Sqrt}[6/\text{Pi}] \cdot \text{Sqrt}[a + b \cdot \text{ArcSin}[c \cdot x]])/\text{Sqrt}[b]] \cdot \text{Sin}[(3 \cdot a)/b])/(2 \cdot \text{Sqrt}[b] \cdot c^3)$

**Rule 3304**

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

**Rule 3305**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

**Rule 3306**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

**Rule 3351**

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]



**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 1.86, size = 317, normalized size = 1.42

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{\sqrt{6} \sqrt{b \arcsin(cx)+a}}{2\sqrt{b}} - \frac{i \sqrt{6} \sqrt{b \arcsin(cx)+a} \sqrt{b}}{2|b|}\right) e^{\left(\frac{3ia}{b}\right)} - \sqrt{\pi} \operatorname{erf}\left(-\frac{i \sqrt{2} \sqrt{b \arcsin(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arcsin(cx)+a} \sqrt{|b|}}{2b}\right) e^{\left(\frac{3ia}{b}\right)}}{4 \left(\sqrt{6} \sqrt{b} + \frac{i \sqrt{6} b^{\frac{3}{2}}}{|b|}\right) c^3} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{i \sqrt{2} \sqrt{b \arcsin(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2} \sqrt{b \arcsin(cx)+a} \sqrt{|b|}}{2b}\right) e^{\left(\frac{3ia}{b}\right)}}{4 c^3 \left(\frac{i \sqrt{2} b}{\sqrt{|b|}} + \sqrt{2} \sqrt{|b|}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{4} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b \arcsin(cx)+a} / \sqrt{b}\right) / \sqrt{b} - \frac{1}{2} I \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b \arcsin(cx)+a} / \sqrt{b}\right) / \sqrt{b} / \sqrt{|b|} e^{\left(\frac{3ia}{b}\right)} / \left(\sqrt{6} \sqrt{b} + \frac{i \sqrt{6} b^{\frac{3}{2}}}{|b|}\right) c^3 - \frac{1}{4} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} I \sqrt{2} \sqrt{b \arcsin(cx)+a} / \sqrt{|b|}\right) / \sqrt{|b|} - \frac{1}{2} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} I \sqrt{2} \sqrt{b \arcsin(cx)+a} / \sqrt{|b|}\right) / \sqrt{|b|} / \sqrt{|b|} e^{\left(\frac{3ia}{b}\right)} / \left(\frac{i \sqrt{2} b}{\sqrt{|b|}} + \sqrt{2} \sqrt{|b|}\right) c^3 - \frac{1}{4} \sqrt{\pi} \operatorname{erf}\left(\frac{1}{2} I \sqrt{2} \sqrt{b \arcsin(cx)+a} / \sqrt{|b|}\right) / \sqrt{|b|} - \frac{1}{2} \sqrt{\pi} \operatorname{erf}\left(\frac{1}{2} I \sqrt{2} \sqrt{b \arcsin(cx)+a} / \sqrt{|b|}\right) / \sqrt{|b|} / \sqrt{|b|} e^{\left(\frac{3ia}{b}\right)} / \left(\frac{i \sqrt{2} b}{\sqrt{|b|}} + \sqrt{2} \sqrt{|b|}\right) c^3 + \frac{1}{4} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b \arcsin(cx)+a} / \sqrt{b}\right) / \sqrt{b} + \frac{1}{2} I \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b \arcsin(cx)+a} / \sqrt{b}\right) / \sqrt{b} / \sqrt{|b|} e^{\left(\frac{3ia}{b}\right)} / \left(\sqrt{6} \sqrt{b} + \frac{i \sqrt{6} b^{\frac{3}{2}}}{|b|}\right) c^3$

**maple** [A] time = 0.11, size = 168, normalized size = 0.75

$$\frac{\sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{2} \left(-\sqrt{3} \cos\left(\frac{3a}{b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) - \sqrt{3} \sin\left(\frac{3a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) + 3 \cos\left(\frac{a}{b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) - 3 \sin\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*arcsin(c\*x))^(1/2),x)

[Out]  $\frac{1}{12} c^3 \left(\frac{1}{b}\right)^{\frac{1}{2}} \pi^{\frac{1}{2}} 2^{\frac{1}{2}} \left(-3^{\frac{1}{2}} \cos\left(\frac{3a}{b}\right) \operatorname{FresnelC}\left(2^{\frac{1}{2}} \sqrt{\frac{3}{b}} \sqrt{a+b \arcsin(cx)}\right) / \pi^{\frac{1}{2}} 3^{\frac{1}{2}} / \left(\frac{1}{b}\right)^{\frac{1}{2}} \left(a+b \arcsin(cx)\right)^{\frac{1}{2}} / b - 3^{\frac{1}{2}} \sin\left(\frac{3a}{b}\right) \operatorname{FresnelS}\left(2^{\frac{1}{2}} \sqrt{\frac{3}{b}} \sqrt{a+b \arcsin(cx)}\right) / \pi^{\frac{1}{2}} 3^{\frac{1}{2}} / \left(\frac{1}{b}\right)^{\frac{1}{2}} \left(a+b \arcsin(cx)\right)^{\frac{1}{2}} / b + 3 \cos\left(\frac{a}{b}\right) \operatorname{FresnelC}\left(2^{\frac{1}{2}} \sqrt{\frac{3}{b}} \sqrt{a+b \arcsin(cx)}\right) / \pi^{\frac{1}{2}} / \left(\frac{1}{b}\right)^{\frac{1}{2}} \left(a+b \arcsin(cx)\right)^{\frac{1}{2}} / b + 3 \sin\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(2^{\frac{1}{2}} \sqrt{\frac{3}{b}} \sqrt{a+b \arcsin(cx)}\right) / \pi^{\frac{1}{2}} / \left(\frac{1}{b}\right)^{\frac{1}{2}} \left(a+b \arcsin(cx)\right)^{\frac{1}{2}} / b\right)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b \arcsin(cx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(b\*arcsin(c\*x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a + b \sin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*asin(c*x))^(1/2), x)`

[Out] `int(x^2/(a + b*asin(c*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + b \sin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*asin(c*x))**(1/2), x)`

[Out] `Integral(x**2/sqrt(a + b*asin(c*x)), x)`

$$3.189 \quad \int \frac{x}{\sqrt{a+b \sin^{-1}(cx)}} dx$$

**Optimal.** Leaf size=99

$$\frac{\sqrt{\pi} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{2\sqrt{b} c^2} - \frac{\sqrt{\pi} \sin\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{2\sqrt{b} c^2}$$

[Out] 1/2\*cos(2\*a/b)\*FresnelS(2\*(a+b\*arcsin(c\*x))^(1/2)/b^(1/2)/Pi^(1/2))\*Pi^(1/2)/c^2/b^(1/2)-1/2\*FresnelC(2\*(a+b\*arcsin(c\*x))^(1/2)/b^(1/2)/Pi^(1/2))\*sin(2\*a/b)\*Pi^(1/2)/c^2/b^(1/2)

**Rubi [A]** time = 0.18, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {4635, 4406, 12, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{2\sqrt{b} c^2} - \frac{\sqrt{\pi} \sin\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{\pi} \sqrt{b}}\right)}{2\sqrt{b} c^2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b\*ArcSin[c\*x]], x]

[Out] (Sqrt[Pi]\*Cos[(2\*a)/b]\*FresnelS[(2\*Sqrt[a + b\*ArcSin[c\*x]])/(Sqrt[b]\*Sqrt[Pi])])/(2\*Sqrt[b]\*c^2) - (Sqrt[Pi]\*FresnelC[(2\*Sqrt[a + b\*ArcSin[c\*x]])/(Sqrt[b]\*Sqrt[Pi])]\*Sin[(2\*a)/b])/(2\*Sqrt[b]\*c^2)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 3352

```
Int[Cos[(d_.)*(e_.) + (f_.)*(x_)]^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

### Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]]^n*Cos[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 4635

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*Cos[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a + b \sin^{-1}(cx)}} dx &= \frac{\text{Subst}\left(\int \frac{\cos(x) \sin(x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{c^2} \\ &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{c^2} \\ &= \frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2c^2} \\ &= \frac{\cos\left(\frac{2a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2c^2} - \frac{\sin\left(\frac{2a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2c^2} \\ &= \frac{\cos\left(\frac{2a}{b}\right) \text{Subst}\left(\int \sin\left(\frac{2x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{bc^2} - \frac{\sin\left(\frac{2a}{b}\right) \text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{bc^2} \\ &= \frac{\sqrt{\pi} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{2\sqrt{b} c^2} - \frac{\sqrt{\pi} C\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right) \sin\left(\frac{2a}{b}\right)}{2\sqrt{b} c^2} \end{aligned}$$

**Mathematica** [C] time = 0.08, size = 123, normalized size = 1.24

$$\frac{e^{-\frac{2ia}{b}} \left( \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{2i(a+b \sin^{-1}(cx))}{b}\right) + e^{\frac{4ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, \frac{2i(a+b \sin^{-1}(cx))}{b}\right) \right)}{4\sqrt{2} c^2 \sqrt{a + b \sin^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*ArcSin[c*x]], x]
```

```
[Out] -1/4*(Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-2*I)*(a + b*ArcSin[c*x]))/b] + E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((2*I)*(a + b*ArcSin[c*x]))/b])/(Sqrt[2]*c^2*E^(((2*I)*a)/b)*Sqrt[a + b*ArcSin[c*x]])
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ  
rate: implementation incomplete (constant residues)

**giac** [C] time = 2.06, size = 132, normalized size = 1.33

$$\frac{i\sqrt{\pi} \operatorname{erf}\left(-\frac{\sqrt{b\arcsin(cx)+a}}{\sqrt{b}} + \frac{i\sqrt{b\arcsin(cx)+a}\sqrt{b}}{|b|}\right) e^{\left(-\frac{2ia}{b}\right)} + i\sqrt{\pi} \operatorname{erf}\left(-\frac{\sqrt{b\arcsin(cx)+a}}{\sqrt{b}} - \frac{i\sqrt{b\arcsin(cx)+a}\sqrt{b}}{|b|}\right) e^{\left(\frac{2ia}{b}\right)}}{4c^2\left(\sqrt{b} - \frac{ib^{\frac{3}{2}}}{|b|}\right) - 4\sqrt{b}c^2\left(\frac{ib}{|b|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(1/2),x, algorithm="giac")

[Out] 1/4\*I\*sqrt(pi)\*erf(-sqrt(b\*arcsin(c\*x) + a)/sqrt(b) + I\*sqrt(b\*arcsin(c\*x)  
+ a)\*sqrt(b)/abs(b))\*e^(-2\*I\*a/b)/(c^2\*(sqrt(b) - I\*b^(3/2)/abs(b))) - 1/4\*  
I\*sqrt(pi)\*erf(-sqrt(b\*arcsin(c\*x) + a)/sqrt(b) - I\*sqrt(b\*arcsin(c\*x) + a)  
\*sqrt(b)/abs(b))\*e^(2\*I\*a/b)/(sqrt(b)\*c^2\*(I\*b/abs(b) + 1))

**maple** [A] time = 0.06, size = 80, normalized size = 0.81

$$\frac{\sqrt{\pi} \sqrt{\frac{1}{b}} \left( \sin\left(\frac{2a}{b}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) - \cos\left(\frac{2a}{b}\right) \operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) \right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arcsin(c\*x))^(1/2),x)

[Out] -1/2\*Pi^(1/2)\*(1/b)^(1/2)\*(sin(2\*a/b)\*FresnelC(2/Pi^(1/2)/(1/b)^(1/2)\*(a+b\*  
arcsin(c\*x))^(1/2)/b)-cos(2\*a/b)\*FresnelS(2/Pi^(1/2)/(1/b)^(1/2)\*(a+b\*arcsi  
n(c\*x))^(1/2)/b))/c^2

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b \arcsin(cx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(b\*arcsin(c\*x) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a + b \operatorname{asin}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*asin(c\*x))^(1/2),x)

[Out] int(x/(a + b\*asin(c\*x))^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b \operatorname{asin}(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asin(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*asin(c*x)), x)
```

$$3.190 \quad \int \frac{1}{\sqrt{a+b \sin^{-1}(cx)}} dx$$

**Optimal.** Leaf size=101

$$\frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{bc}} + \frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{bc}}$$

[Out]  $\cos(a/b)*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(cx))^{(1/2)}/b^{(1/2)})*2^{(1/2)}$   
 $*\text{Pi}^{(1/2)}/c/b^{(1/2)}+\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(cx))^{(1/2)}/b^{(1/2)})$   
 $*\sin(a/b)*2^{(1/2)}*\text{Pi}^{(1/2)}/c/b^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4623, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{bc}} + \frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{bc}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b\*ArcSin[c\*x]], x]

[Out]  $(\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a/b]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(\text{Sqrt}[b]*c) + (\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(\text{Sqrt}[b]*c)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

## Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[1/(b\*c), Subst[Int[x^n\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

## Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + b \sin^{-1}(cx)}} dx &= \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a-x}{b}\right)}{\sqrt{x}} dx, x, a + b \sin^{-1}(cx)\right)}{bc} \\ &= \frac{\cos\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a + b \sin^{-1}(cx)\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a + b \sin^{-1}(cx)\right)}{bc} \\ &= \frac{\left(2 \cos\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{bc} + \frac{\left(2 \sin\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{bc} \\ &= \frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} + \frac{\sqrt{2\pi} S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{\sqrt{b}c} \end{aligned}$$

**Mathematica** [C] time = 0.09, size = 121, normalized size = 1.20

$$\frac{ie^{-\frac{ia}{b}} \left( e^{\frac{2ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, \frac{i(a+b \sin^{-1}(cx))}{b}\right) - \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{i(a+b \sin^{-1}(cx))}{b}\right) \right)}{2c\sqrt{a + b \sin^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[a + b\*ArcSin[c\*x]], x]

[Out] ((I/2)\*(-(Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b])\*Gamma[1/2, ((-I)\*(a + b\*ArcSin[c\*x]))/b]) + E^(((2\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b])\*Gamma[1/2, (I\*(a + b\*ArcSin[c\*x]))/b])/(c\*E^((I\*a)/b)\*Sqrt[a + b\*ArcSin[c\*x]])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [C] time = 1.46, size = 159, normalized size = 1.57

$$\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arcsin(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{|b|}}{2b}\right) e^{\left(\frac{ia}{b}\right)}}{c\left(\frac{i\sqrt{2}b}{\sqrt{|b|}} + \sqrt{2}\sqrt{|b|}\right)} - \frac{\sqrt{\pi} \operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{b\arcsin(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b\arcsin(cx)+a}\sqrt{|b|}}{2b}\right) e^{\left(-\frac{ia}{b}\right)}}{c\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}} + \sqrt{2}\sqrt{|b|}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(1/2), x, algorithm="giac")

```
[Out] -sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - sqrt(pi)*erf(1/2*I*sqrt(2)*sqrt(b*arcsin(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcsin(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b))))
```

**maple** [A] time = 0.04, size = 83, normalized size = 0.82

$$\frac{\sqrt{2} \sqrt{\pi} \sqrt{\frac{1}{b}} \left( \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) + \sin\left(\frac{a}{b}\right) \text{S}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) \right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsin(c*x))^(1/2), x)
```

```
[Out] 2^(1/2)*Pi^(1/2)*(1/b)^(1/2)*(cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)+sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b))/c
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \arcsin(cx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsin(c*x))^(1/2), x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsin(c*x) + a), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asin(c*x))^(1/2), x)
```

```
[Out] int(1/(a + b*asin(c*x))^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asin(c*x))**(1/2), x)
```

```
[Out] Integral(1/sqrt(a + b*asin(c*x)), x)
```

$$3.191 \quad \int \frac{1}{x\sqrt{a+b\sin^{-1}(cx)}} dx$$

**Optimal.** Leaf size=19

$$\text{Int}\left(\frac{1}{x\sqrt{a+b\sin^{-1}(cx)}}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*arcsin(c\*x))^(1/2), x)

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x\sqrt{a+b\sin^{-1}(cx)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*Sqrt[a + b\*ArcSin[c\*x]]), x]

[Out] Defer[Int][1/(x\*Sqrt[a + b\*ArcSin[c\*x]]), x]

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\sin^{-1}(cx)}} dx = \int \frac{1}{x\sqrt{a+b\sin^{-1}(cx)}} dx$$

**Mathematica [A]** time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\sin^{-1}(cx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*Sqrt[a + b\*ArcSin[c\*x]]), x]

[Out] Integrate[1/(x\*Sqrt[a + b\*ArcSin[c\*x]]), x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\arcsin(cx) + a}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(b\*arcsin(c\*x) + a)\*x), x)

**maple** [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsin(c\*x))^(1/2),x)

[Out] int(1/x/(a+b\*arcsin(c\*x))^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\arcsin(cx)+a}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*arcsin(c\*x)+a)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\sqrt{a+b\arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a+b\*asin(c\*x))^(1/2)),x)

[Out] int(1/(x\*(a+b\*asin(c\*x))^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*asin(c\*x))\*\*(1/2),x)

[Out] Integral(1/(x\*sqrt(a+b\*asin(c\*x))), x)

$$3.192 \quad \int \frac{1}{x^2 \sqrt{a+b \sin^{-1}(cx)}} dx$$

**Optimal.** Leaf size=19

$$\text{Int}\left(\frac{1}{x^2 \sqrt{a+b \sin^{-1}(cx)}}, x\right)$$

[Out] Unintegrable(1/x^2/(a+b\*arcsin(c\*x))^(1/2), x)

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 \sqrt{a+b \sin^{-1}(cx)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*Sqrt[a + b\*ArcSin[c\*x]]), x]

[Out] Defer[Int][1/(x^2\*Sqrt[a + b\*ArcSin[c\*x]]), x]

Rubi steps

$$\int \frac{1}{x^2 \sqrt{a+b \sin^{-1}(cx)}} dx = \int \frac{1}{x^2 \sqrt{a+b \sin^{-1}(cx)}} dx$$

**Mathematica [A]** time = 11.75, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a+b \sin^{-1}(cx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*Sqrt[a + b\*ArcSin[c\*x]]), x]

[Out] Integrate[1/(x^2\*Sqrt[a + b\*ArcSin[c\*x]]), x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \arcsin(cx) + a} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*arcsin(c\*x) + a)\*x^2), x, algorithm="giac")

[Out] integrate(1/(sqrt(b\*arcsin(c\*x) + a)\*x^2), x)



**maple** [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + b \arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsin(c\*x))^(1/2), x)

[Out] int(1/x^2/(a+b\*arcsin(c\*x))^(1/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \arcsin(cx) + a} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*arcsin(c\*x) + a)\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \sqrt{a + b \arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*asin(c\*x))^(1/2)), x)

[Out] int(1/(x^2\*(a + b\*asin(c\*x))^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + b \arcsin(cx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asin(c\*x))\*\*(1/2), x)

[Out] Integral(1/(x\*\*2\*sqrt(a + b\*asin(c\*x))), x)

$$3.193 \quad \int \frac{x^2}{(a+b \sin^{-1}(cx))^{3/2}} dx$$

**Optimal.** Leaf size=250

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}} \sin\left(\frac{3a}{b}\right) C\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} + \frac{\sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}$$

[Out]  $-1/2*\cos(a/b)*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(3/2)}/c^{3+1/2}*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(a/b)*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(3/2)}/c^{3+1/2}*\cos(3*a/b)*\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/b^{(3/2)}/c^{3-1/2}*\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(3*a/b)*6^{(1/2)}*\text{Pi}^{(1/2)}/b^{(3/2)}/c^{3-2*x^2*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 0.42, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4631, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}} \sin\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} + \frac{\sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*ArcSin[c\*x])^(3/2), x]

[Out]  $(-2*x^2*\text{Sqrt}[1 - c^2*x^2])/(b*c*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) - (\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a/b]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(b^{(3/2)}*c^3) + (\text{Sqrt}[(3*\text{Pi})/2]*\text{Cos}[(3*a)/b]*\text{FresnelS}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(b^{(3/2)}*c^3) + (\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(b^{(3/2)}*c^3) - (\text{Sqrt}[(3*\text{Pi})/2]*\text{FresnelC}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(3*a)/b])/(b^{(3/2)}*c^3)$

Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

### Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)</sup>\*(x\_)<sup>(m\_)</sup>, x\_Symbol] := Simp[(x<sup>m</sup>\*Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>]\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>)/(b\*c\*(n + 1)), x] - Dist[1/(b\*c<sup>(m + 1)</sup>\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)<sup>(n + 1)</sup>, Sin[x]<sup>(m - 1)</sup>\*(m - (m + 1)\*Sin[x]<sup>2</sup>), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a + b \sin^{-1}(cx))^{3/2}} dx &= -\frac{2x^2\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} + \frac{2 \operatorname{Subst}\left(\int \left(-\frac{\sin(x)}{4\sqrt{a+bx}} + \frac{3\sin(3x)}{4\sqrt{a+bx}}\right) dx, x, \sin^{-1}(cx)\right)}{bc^3} \\ &= -\frac{2x^2\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} - \frac{\operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2bc^3} + \frac{3 \operatorname{Subst}\left(\int \frac{\sin(3x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2bc^3} \\ &= -\frac{2x^2\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{2bc^3} + \frac{\left(3 \cos\left(\frac{3a}{b}\right)\right)}{2bc^3} \\ &= -\frac{2x^2\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{b^2c^3} + \frac{\left(3 \cos\left(\frac{3a}{b}\right)\right)}{b^2c^3} \\ &= -\frac{2x^2\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} - \frac{\sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} + \frac{\sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} \end{aligned}$$

**Mathematica [C]** time = 0.47, size = 343, normalized size = 1.37

$$e^{-\frac{3i(a+b\sin^{-1}(cx))}{b}} \left( -e^{\frac{3ia}{b}+2i\sin^{-1}(cx)} - e^{\frac{3ia}{b}+4i\sin^{-1}(cx)} + e^{\frac{3i(a+2b\sin^{-1}(cx))}{b}} + e^{\frac{2ia}{b}+3i\sin^{-1}(cx)} \sqrt{-\frac{i(a+b\sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{i(a+b\sin^{-1}(cx))}{b}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x<sup>2</sup>/(a + b\*ArcSin[c\*x])<sup>(3/2)</sup>, x]

[Out] (E<sup>((3\*I)\*a)/b</sup> - E<sup>((3\*I)\*a)/b + (2\*I)\*ArcSin[c\*x]</sup>) - E<sup>((3\*I)\*a)/b + (4\*I)\*ArcSin[c\*x]</sup> + E<sup>((3\*I)\*(a + 2\*b\*ArcSin[c\*x]))/b</sup> + E<sup>((2\*I)\*a)/b + (3\*I)\*ArcSin[c\*x]\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, ((-I)\*(a + b\*ArcSin[c\*x]))/b]</sup> + E<sup>((4\*I)\*a)/b + (3\*I)\*ArcSin[c\*x]\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, (I\*(a + b\*ArcSin[c\*x]))/b]</sup> - Sqrt[3]\*E<sup>((3\*I)\*ArcSin[c\*x]\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, ((-3\*I)\*(a + b\*ArcSin[c\*x]))/b]</sup> - Sqrt[3]\*E<sup>((3\*I)\*((2\*a)/b + ArcSin[c\*x]))\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, ((3\*I)\*(a + b\*ArcSin[c\*x]))/b]</sup>)/(4\*b\*c<sup>3</sup>\*E<sup>((3\*I)\*(a + b\*ArcSin[c\*x]))/b</sup>\*Sqrt[a + b\*ArcSin[c\*x]])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ  
rate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="giac")

[Out] integrate(x^2/(b\*arcsin(c\*x) + a)^(3/2), x)

**maple** [A] time = 0.12, size = 295, normalized size = 1.18

$$-\sqrt{3} \sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a + b \arcsin(cx)} \cos\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{2} \sqrt{3} \sqrt{a + b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) + \sqrt{3} \sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a + b \arcsin(cx)} \sin\left(\frac{3a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*arcsin(c\*x))^(3/2),x)

[Out] 
$$-1/2/c^3/b/(a+b*arcsin(c*x))^{1/2}*(-3^{1/2}*(1/b)^{1/2}*Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\cos(3*a/b)*FresnelS(2^{1/2}/Pi^{1/2}*3^{1/2}/(1/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)+3^{1/2}*(1/b)^{1/2}*Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\sin(3*a/b)*FresnelC(2^{1/2}/Pi^{1/2}*3^{1/2}/(1/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)+(1/b)^{1/2}*Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\cos(a/b)*FresnelS(2^{1/2}/Pi^{1/2}/(1/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)-(1/b)^{1/2}*Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\sin(a/b)*FresnelC(2^{1/2}/Pi^{1/2}/(1/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)+\cos((a+b*arcsin(c*x))/b-a/b)-\cos(3*(a+b*arcsin(c*x))/b-3*a/b)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="maxima")

[Out] integrate(x^2/(b\*arcsin(c\*x) + a)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + b \operatorname{asin}(cx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*asin(c\*x))^(3/2),x)

```
[Out] int(x^2/(a + b*asin(c*x))^(3/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^2}{(a + b \operatorname{asin}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asin(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*asin(c*x))**(3/2), x)
```

$$3.194 \quad \int \frac{x}{(a+b \sin^{-1}(cx))^{3/2}} dx$$

**Optimal.** Leaf size=130

$$\frac{2\sqrt{\pi} \cos\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{b^{3/2}c^2} + \frac{2\sqrt{\pi} \sin\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{b^{3/2}c^2} - \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b \sin^{-1}(cx)}}$$

[Out]  $2*\cos(2*a/b)*\text{FresnelC}(2*(a+b*\arcsin(c*x))^{1/2}/b^{1/2}/\text{Pi}^{1/2})*\text{Pi}^{1/2}/b^{3/2}/c^2+2*\text{FresnelS}(2*(a+b*\arcsin(c*x))^{1/2}/b^{1/2}/\text{Pi}^{1/2})*\sin(2*a/b)*\text{Pi}^{1/2}/b^{3/2}/c^2-2*x*(-c^2*x^2+1)^{1/2}/b/c/(a+b*\arcsin(c*x))^{1/2}$

**Rubi [A]** time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {4631, 3306, 3305, 3351, 3304, 3352}

$$\frac{2\sqrt{\pi} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{\pi} \sqrt{b}}\right)}{b^{3/2}c^2} + \frac{2\sqrt{\pi} \sin\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{b^{3/2}c^2} - \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b \sin^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*ArcSin[c\*x])^(3/2), x]

[Out]  $(-2*x*\text{Sqrt}[1 - c^2*x^2])/(b*c*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) + (2*\text{Sqrt}[\text{Pi}]*\text{Cos}[(2*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))/(b^{3/2}*c^2) + (2*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[(2*a)/b])/(b^{3/2}*c^2)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4631

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(x^m\*Sqrt[1 - c^2\*x^2]\*(a + b\*ArcSin[c\*x])^(n + 1))/(b\*c\*(n + 1)), x] - Dist[1/(b\*c^(m + 1)\*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b\*x)^(n + 1), Sin[x]^(m - 1)\*(m - (m + 1)\*Sin[x]^2), x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x}{(a + b \sin^{-1}(cx))^{3/2}} dx &= -\frac{2x\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} + \frac{2 \operatorname{Subst}\left(\int \frac{\cos(2x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{bc^2} \\ &= -\frac{2x\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} + \frac{\left(2 \cos\left(\frac{2a}{b}\right)\right) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{bc^2} + \frac{\left(2 \sin\left(\frac{2a}{b}\right)\right) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{bc^2} \\ &= -\frac{2x\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} + \frac{\left(4 \cos\left(\frac{2a}{b}\right)\right) \operatorname{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right) dx, x, \sqrt{a + b \sin^{-1}(cx)}\right)}{b^2c^2} \\ &= -\frac{2x\sqrt{1 - c^2x^2}}{bc\sqrt{a + b \sin^{-1}(cx)}} + \frac{2\sqrt{\pi} \cos\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{b^{3/2}c^2} + \frac{2\sqrt{\pi} S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{b^{3/2}c^2} \end{aligned}$$

**Mathematica [C]** time = 0.15, size = 155, normalized size = 1.19

$$\frac{ie^{-\frac{2ia}{b}} \left( 2ie^{\frac{2ia}{b}} \sin(2 \sin^{-1}(cx)) - \sqrt{2} \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{2i(a+b \sin^{-1}(cx))}{b}\right) + \sqrt{2} e^{\frac{4ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, \frac{2i(a+b \sin^{-1}(cx))}{b}\right) \right)}{2bc^2\sqrt{a + b \sin^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(a + b\*ArcSin[c\*x])^(3/2), x]

[Out] ((I/2)\*(-(Sqrt[2]\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, ((-2\*I)\*(a + b\*ArcSin[c\*x]))/b]) + Sqrt[2]\*E^(((4\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, ((2\*I)\*(a + b\*ArcSin[c\*x]))/b] + (2\*I)\*E^(((2\*I)\*a)/b)\*Sin[2\*ArcSin[c\*x]])/(b\*c^2\*E^(((2\*I)\*a)/b)\*Sqrt[a + b\*ArcSin[c\*x]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="giac")

[Out] integrate(x/(b\*arcsin(c\*x) + a)^(3/2), x)

**maple** [A] time = 0.07, size = 143, normalized size = 1.10

$$\frac{-2\sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) - 2\sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{2a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right)}{c^2 b \sqrt{a+b \arcsin(cx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arcsin(c\*x))^(3/2),x)

[Out]  $-1/c^2/b/(a+b*\arcsin(c*x))^{1/2}*(-2*(1/b)^{1/2}*Pi^{1/2}*(a+b*\arcsin(c*x))^{1/2}*\cos(2*a/b)*\text{FresnelC}(2/Pi^{1/2}/(1/b)^{1/2}*(a+b*\arcsin(c*x))^{1/2}/b)-2*(1/b)^{1/2}*Pi^{1/2}*(a+b*\arcsin(c*x))^{1/2}*\sin(2*a/b)*\text{FresnelS}(2/Pi^{1/2}/(1/b)^{1/2}*(a+b*\arcsin(c*x))^{1/2}/b)+\sin(2*(a+b*\arcsin(c*x))/b-2*a/b))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="maxima")

[Out] integrate(x/(b\*arcsin(c\*x) + a)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \arcsin(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*asin(c\*x))^(3/2),x)

[Out] int(x/(a + b\*asin(c\*x))^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \arcsin(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*asin(c\*x))\*\*(3/2),x)

[Out] Integral(x/(a + b\*asin(c\*x))\*\*(3/2), x)



$$3.195 \quad \int \frac{1}{(a+b \sin^{-1}(cx))^{3/2}} dx$$

**Optimal.** Leaf size=137

$$\frac{2\sqrt{2\pi} \sin\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{2\pi} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b \sin^{-1}(cx)}}$$

[Out]  $-2*\cos(a/b)*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(3/2)}/c+2*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(a/b)*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(3/2)}/c-2*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 0.29, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {4621, 4723, 3306, 3305, 3351, 3304, 3352}

$$\frac{2\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{2\pi} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b \sin^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSin}[c*x])^{(-3/2)}, x]$

[Out]  $(-2*\text{Sqrt}[1 - c^2*x^2])/(b*c*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) - (2*\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a/b]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(b^{(3/2)}*c) + (2*\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(b^{(3/2)}*c)$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

#### Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

#### Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

#### Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rule 4621

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[(Sqrt[1 - c^
2*x^2]*(a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] + Dist[c/(b*(n + 1)),
Int[(x*(a + b*ArcSin[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a,
b, c}, x] && LtQ[n, -1]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sin[x]^m*C
os[x]^(2*p + 1), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] &&
EqQ[c^2*d + e, 0] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \sin^{-1}(cx))^{3/2}} dx &= -\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\sin^{-1}(cx)}} - \frac{(2c) \int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\sin^{-1}(cx)}} dx}{b} \\ &= -\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\sin^{-1}(cx)}} - \frac{2 \operatorname{Subst}\left(\int \frac{\sin(x)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{bc} \\ &= -\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\sin^{-1}(cx)}} - \frac{(2 \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{\sqrt{a+bx}} dx, x, \sin^{-1}(cx)\right)}{bc} + \frac{(2 \sin\left(\frac{a}{b}\right))}{b} \\ &= -\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\sin^{-1}(cx)}} - \frac{(4 \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \sin\left(\frac{x^2}{b}\right) dx, x, \sqrt{a+b\sin^{-1}(cx)}\right)}{b^2c} + \frac{(4 \sin\left(\frac{a}{b}\right))}{b} \\ &= -\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\sin^{-1}(cx)}} - \frac{2\sqrt{2\pi} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{2\sqrt{2\pi} C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} \end{aligned}$$

**Mathematica [C]** time = 0.33, size = 167, normalized size = 1.22

$$\frac{e^{-\frac{i(a+b\sin^{-1}(cx))}{b}} \left( e^{i\sin^{-1}(cx)} \sqrt{-\frac{i(a+b\sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{i(a+b\sin^{-1}(cx))}{b}\right) + e^{\frac{ia}{b}} \left( e^{\frac{i(a+b\sin^{-1}(cx))}{b}} \sqrt{\frac{i(a+b\sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, \frac{i(a+b\sin^{-1}(cx))}{b}\right) \right) \right)}{bc\sqrt{a+b\sin^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSin[c*x])^(-3/2), x]
```

```
[Out] (E^(I*ArcSin[c*x])*Sqrt[(-I)*(a + b*ArcSin[c*x])/b]*Gamma[1/2, ((-I)*(a +
b*ArcSin[c*x])/b] + E^((I*a)/b)*(-1 - E^((2*I)*ArcSin[c*x]) + E^((I*(a +
b*ArcSin[c*x])/b)*Sqrt[(I*(a + b*ArcSin[c*x])/b]*Gamma[1/2, (I*(a + b*Arc
Sin[c*x])/b))]/(b*c*E^((I*(a + b*ArcSin[c*x])/b)*Sqrt[a + b*ArcSin[c*x]]))
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^(-3/2), x)

**maple** [A] time = 0.07, size = 149, normalized size = 1.09

$$\frac{2 \left( \sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a + b \arcsin(cx)} \cos\left(\frac{a}{b}\right) S\left(\frac{\sqrt{2} \sqrt{a + b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) - \sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a + b \arcsin(cx)} \sin\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{a + b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) \right)}{cb \sqrt{a + b \arcsin(cx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arcsin(c\*x))^(3/2),x)

[Out] -2/c/b\*((1/b)^(1/2)\*Pi^(1/2)\*2^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)\*cos(a/b)\*FresnelS(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)/b)-(1/b)^(1/2)\*Pi^(1/2)\*2^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)\*sin(a/b)\*FresnelC(2^(1/2)/Pi^(1/2)/(1/b)^(1/2)\*(a+b\*arcsin(c\*x))^(1/2)/b)+cos((a+b\*arcsin(c\*x))/b-a/b)/(a+b\*arcsin(c\*x))^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(-3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*asin(c\*x))^(3/2),x)

[Out] int(1/(a + b\*asin(c\*x))^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*asin(c\*x))\*\*(3/2),x)

[Out] Integral((a + b\*asin(c\*x))\*\*(-3/2), x)

$$3.196 \quad \int \frac{1}{x(a+b \sin^{-1}(cx))^{3/2}} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{1}{x(a+b \sin^{-1}(cx))^{3/2}}, x \right)$$

[Out] Unintegrable(1/x/(a+b\*arcsin(c\*x))^(3/2), x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSin[c\*x])^(3/2)), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSin[c\*x])^(3/2)), x]

Rubi steps

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^{3/2}} dx = \int \frac{1}{x(a+b \sin^{-1}(cx))^{3/2}} dx$$

Mathematica [A] time = 3.81, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^(3/2)), x]

[Out] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^(3/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(3/2), x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vect  
 eur & l) Error: Bad Argument Value

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \arcsin(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsin(c\*x))^(3/2),x)

[Out] int(1/x/(a+b\*arcsin(c\*x))^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^(3/2)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x(a + b \operatorname{asin}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*asin(c\*x))^(3/2)),x)

[Out] int(1/(x\*(a + b\*asin(c\*x))^(3/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{asin}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*asin(c\*x))\*\*(3/2),x)

[Out] Integral(1/(x\*(a + b\*asin(c\*x))\*\*(3/2)), x)

$$3.197 \quad \int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{3/2}} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{1}{x^2 (a + b \sin^{-1}(cx))^{3/2}}, x \right)$$

[Out] Unintegrable(1/x^2/(a+b\*arcsin(c\*x))^(3/2), x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*ArcSin[c\*x])^(3/2)), x]

[Out] Defer[Int][1/(x^2\*(a + b\*ArcSin[c\*x])^(3/2)), x]

Rubi steps

$$\int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{3/2}} dx = \int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{3/2}} dx$$

Mathematica [A] time = 11.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^(3/2)), x]

[Out] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^(3/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(3/2), x, algorithm="giac")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^(3/2)\*x^2), x)

**maple** [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \arcsin(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsin(c\*x))^(3/2), x)

[Out] int(1/x^2/(a+b\*arcsin(c\*x))^(3/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(3/2), x, algorithm="maxima")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^(3/2)\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*asin(c\*x))^(3/2)), x)

[Out] int(1/(x^2\*(a + b\*asin(c\*x))^(3/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asin(c\*x))\*\*(3/2), x)

[Out] Integral(1/(x\*\*2\*(a + b\*asin(c\*x))\*\*(3/2)), x)

$$3.198 \quad \int \frac{x^2}{(a+b \sin^{-1}(cx))^{5/2}} dx$$

**Optimal.** Leaf size=291

$$\frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{\sqrt{6\pi} \cos\left(\frac{3a}{b}\right) C\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3} - \frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{\sqrt{6\pi} \sin\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3}$$

[Out]  $-1/3*\cos(a/b)*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(5/2)}/c^3-1/3*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(a/b)*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(5/2)}/c^3+\cos(3*a/b)*\text{FresnelC}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*6^{(1/2)}*\text{Pi}^{(1/2)}/b^{(5/2)}/c^3+\text{FresnelS}(6^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(3*a/b)*6^{(1/2)}*\text{Pi}^{(1/2)}/b^{(5/2)}/c^3-2/3*x^2*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))^{(3/2)}-8/3*x/b^2/c^2/(a+b*\arcsin(c*x))^{(1/2)}+4*x^3/b^2/(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 1.01, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {4633, 4719, 4635, 4406, 3306, 3305, 3351, 3304, 3352, 4623}

$$\frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{\sqrt{6\pi} \cos\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3} - \frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{\sqrt{6\pi} \sin\left(\frac{3a}{b}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*ArcSin[c\*x])^(5/2), x]

[Out]  $(-2*x^2*\text{Sqrt}[1 - c^2*x^2])/(3*b*c*(a + b*\text{ArcSin}[c*x])^{(3/2)}) - (8*x)/(3*b^2*c^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) + (4*x^3)/(b^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) - (\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a/b]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(3*b^{(5/2)}*c^3) + (\text{Sqrt}[6*\text{Pi}]*\text{Cos}[(3*a)/b]*\text{FresnelC}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(b^{(5/2)}*c^3) - (\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(3*b^{(5/2)}*c^3) + (\text{Sqrt}[6*\text{Pi}]*\text{FresnelS}[(\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(3*a)/b])/(b^{(5/2)}*c^3)$

#### Rule 3304

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3305

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[(f\*x^2)/d], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3306

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d,



$e, f\}, x]$  && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] :> Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]<sup>(p\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]<sup>(n\_.)</sup>, x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[a + b\*x]<sup>n</sup>\*Cos[a + b\*x]<sup>p</sup>, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]</sup></sup>

#### Rule 4623

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)</sup>, x\_Symbol] :> Dist[1/(b\*c), Subst[Int[x<sup>n</sup>\*Cos[a/b - x/b], x], x, a + b\*ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x]

#### Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)\*(x\_)<sup>(m\_)</sup>, x\_Symbol] :> Simp[(x<sup>m</sup>\*Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>]\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>)/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>]/Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x<sup>(m - 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>]/Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>], x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]</sup>

#### Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)\*(x\_)<sup>(m\_)</sup>, x\_Symbol] :> Dist[1/c<sup>(m + 1)</sup>, Subst[Int[(a + b\*x)<sup>n</sup>\*Sin[x]<sup>m</sup>\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]</sup>

#### Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_)\*((f\_.)\*(x\_))<sup>(m\_)</sup>)/Sqrt[(d\_ + (e\_.)\*(x\_))<sup>2</sup>], x\_Symbol] :> Simp[((f\*x)<sup>m</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>)/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)<sup>(m - 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]</sup>

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + b \sin^{-1}(cx))^{5/2}} dx &= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} + \frac{4 \int \frac{x}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3bc} - \frac{(2c) \int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))}}{b} \\
&= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{12 \int \frac{x}{\sqrt{a + b \sin^{-1}(cx)}}}{b^2} \\
&= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8 \text{Subst}}{b^2} \\
&= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{12 \text{Subst}}{b^2} \\
&= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{3 \text{Subst}}{b^2} \\
&= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8\sqrt{2\pi} c}{b^2} \\
&= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8\sqrt{2\pi} c}{b^2} \\
&= -\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{4x^3}{b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{\sqrt{2\pi} c}{b^2}
\end{aligned}$$

**Mathematica [C]** time = 1.88, size = 370, normalized size = 1.27

$$e^{3i \sin^{-1}(cx)} (6ia + 6ib \sin^{-1}(cx) + b) - ie^{i \sin^{-1}(cx)} (2a + 2b \sin^{-1}(cx) - ib) - 2be^{-\frac{ia}{b}} \left( -\frac{i(a+b \sin^{-1}(cx))}{b} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{i(a+b \sin^{-1}(cx))}{b}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(a + b\*ArcSin[c\*x])^(5/2), x]

[Out] (((-6\*I)\*a)/E^((3\*I)\*ArcSin[c\*x]) + (b\*(1 - (6\*I)\*ArcSin[c\*x]))/E^((3\*I)\*ArcSin[c\*x]) + E^((3\*I)\*ArcSin[c\*x])\*((6\*I)\*a + b + (6\*I)\*b\*ArcSin[c\*x]) - I\*E^(I\*ArcSin[c\*x])\*(2\*a - I\*b + 2\*b\*ArcSin[c\*x]) - (2\*b\*((-I)\*(a + b\*ArcSin[c\*x]))/b)^(3/2)\*Gamma[1/2, ((-I)\*(a + b\*ArcSin[c\*x]))/b])/E^((I\*a)/b) + (I\*(2\*a + I\*b + 2\*b\*ArcSin[c\*x]) + (2\*I)\*b\*E^((I\*(a + b\*ArcSin[c\*x]))/b))\*((I\*(a + b\*ArcSin[c\*x]))/b)^(3/2)\*Gamma[1/2, (I\*(a + b\*ArcSin[c\*x]))/b])/E^(I\*ArcSin[c\*x]) + (6\*Sqrt[3]\*b\*((-I)\*(a + b\*ArcSin[c\*x]))/b)^(3/2)\*Gamma[1/2, ((-3\*I)\*(a + b\*ArcSin[c\*x]))/b])/E^(((3\*I)\*a)/b) + 6\*Sqrt[3]\*b\*E^(((3\*I)\*a)/b)\*((I\*(a + b\*ArcSin[c\*x]))/b)^(3/2)\*Gamma[1/2, ((3\*I)\*(a + b\*ArcSin[c\*x]))/b])/((12\*b^2\*c^3\*(a + b\*ArcSin[c\*x])^(3/2))

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ  
rate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \arcsin(cx) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="giac")

[Out] integrate(x^2/(b\*arcsin(c\*x) + a)^(5/2), x)

**maple** [B] time = 0.15, size = 660, normalized size = 2.27

$$6 \arcsin(cx) \sqrt{\pi} \sqrt{2} \sqrt{\frac{1}{b}} \sqrt{a + b \arcsin(cx)} \sqrt{3} \cos\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{3} \sqrt{a + b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) b + 6 \arcsin(cx) \sqrt{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*arcsin(c\*x))^(5/2),x)

[Out]  $\frac{1}{6} c^3 b^2 (6 \arcsin(cx) \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} 3^{1/2} \cos(3a/b) \text{FresnelC}(2^{1/2} \pi^{1/2} 3^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) + 6 \arcsin(cx) \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} 3^{1/2} \sin(3a/b) \text{FresnelS}(2^{1/2} \pi^{1/2} 3^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) - 2 \arcsin(cx) \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \cos(a/b) \text{FresnelC}(2^{1/2} \pi^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) - 2 \arcsin(cx) \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \sin(a/b) \text{FresnelS}(2^{1/2} \pi^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) + 6 \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} 3^{1/2} \cos(3a/b) \text{FresnelC}(2^{1/2} \pi^{1/2} 3^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) + 6 \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} 3^{1/2} \sin(3a/b) \text{FresnelS}(2^{1/2} \pi^{1/2} 3^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) - 2 \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \cos(a/b) \text{FresnelC}(2^{1/2} \pi^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) - 2 \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \sin(a/b) \text{FresnelS}(2^{1/2} \pi^{1/2} / (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} / b) + 2 \arcsin(cx) \sin((a+b \arcsin(cx))/b - a/b) - 6 \arcsin(cx) \sin(3(a+b \arcsin(cx))/b - 3a/b) - b \cos((a+b \arcsin(cx))/b - a/b) + 2 \sin((a+b \arcsin(cx))/b - a/b) + a \cos(3(a+b \arcsin(cx))/b - 3a/b) - 6 \sin(3(a+b \arcsin(cx))/b - 3a/b) a) / (a+b \arcsin(cx))^{3/2}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \arcsin(cx) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="maxima")

[Out] integrate(x^2/(b\*arcsin(c\*x) + a)^(5/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + b \operatorname{asin}(cx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*asin(c\*x))^(5/2),x)

[Out] int(x^2/(a + b\*asin(c\*x))^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{asin}(cx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*asin(c\*x))\*\*(5/2),x)

[Out] Integral(x\*\*2/(a + b\*asin(c\*x))\*\*(5/2), x)

$$3.199 \quad \int \frac{x}{(a+b \sin^{-1}(cx))^{5/2}} dx$$

**Optimal.** Leaf size=180

$$\frac{8\sqrt{\pi} \sin\left(\frac{2a}{b}\right) C\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{3b^{5/2}c^2} - \frac{8\sqrt{\pi} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{3b^{5/2}c^2} - \frac{4}{3b^2c^2\sqrt{a+b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b \sin^{-1}(cx)}}$$

[Out]  $-8/3*\cos(2*a/b)*\text{FresnelS}(2*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/b^{(5/2)}/c^2+8/3*\text{FresnelC}(2*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)}/\text{Pi}^{(1/2)})*\sin(2*a/b)*\text{Pi}^{(1/2)}/b^{(5/2)}/c^2-2/3*x*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))^{(3/2)}-4/3/b^2/c^2/(a+b*\arcsin(c*x))^{(1/2)}+8/3*x^2/b^2/(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 0.51, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$ , Rules used = {4633, 4719, 4635, 4406, 12, 3306, 3305, 3351, 3304, 3352, 4641}

$$\frac{8\sqrt{\pi} \sin\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{\pi} \sqrt{b}}\right)}{3b^{5/2}c^2} - \frac{8\sqrt{\pi} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b} \sqrt{\pi}}\right)}{3b^{5/2}c^2} - \frac{4}{3b^2c^2\sqrt{a+b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b \sin^{-1}(cx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/(a + b*\text{ArcSin}[c*x])^{(5/2)}, x]$

[Out]  $(-2*x*\text{Sqrt}[1 - c^2*x^2])/(3*b*c*(a + b*\text{ArcSin}[c*x])^{(3/2)}) - 4/(3*b^2*c^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) + (8*x^2)/(3*b^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) - (8*\text{Sqrt}[\text{Pi}]*\text{Cos}[(2*a)/b]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])])/(3*b^{(5/2)}*c^2) + (8*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[(2*a)/b])/(3*b^{(5/2)}*c^2)$

#### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 3351

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3352

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)])/(f\*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]<sup>(p\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]<sup>(n\_.)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[a + b\*x]<sup>n</sup>\*Cos[a + b\*x]<sup>p</sup>, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]</sup></sup>

Rule 4633

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)\*(x\_)</sup><sup>(m\_.)</sup>, x\_Symbol] := Simp[(x<sup>m</sup>\*Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>]\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>)/(b\*c\*(n + 1)), x] + (Dist[(c\*(m + 1))/(b\*(n + 1)), Int[(x<sup>(m + 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>]/Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>], x], x] - Dist[m/(b\*c\*(n + 1)), Int[(x<sup>(m - 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>]/Sqrt[1 - c<sup>2</sup>\*x<sup>2</sup>], x], x) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]

Rule 4635

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)\*(x\_)</sup><sup>(m\_.)</sup>, x\_Symbol] := Dist[1/c<sup>(m + 1)</sup>, Subst[Int[(a + b\*x)<sup>n</sup>\*Sin[x]<sup>m</sup>\*Cos[x], x], x, ArcSin[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 4641

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_)<sup>2</sup>], x\_Symbol] := Simp[(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>/(b\*c\*Sqrt[d]\*(n + 1)), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]</sup>

Rule 4719

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))<sup>(n\_.)\*((f\_.)\*(x\_))<sup>(m\_.)</sup>)/Sqrt[(d\_.) + (e\_.)\*(x\_)<sup>2</sup>], x\_Symbol] := Simp[((f\*x)<sup>m</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>)/(b\*c\*Sqrt[d]\*(n + 1)), x] - Dist[(f\*m)/(b\*c\*Sqrt[d]\*(n + 1)), Int[(f\*x)<sup>(m - 1)</sup>\*(a + b\*ArcSin[c\*x])<sup>(n + 1)</sup>, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c<sup>2</sup>\*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]</sup>

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + b \sin^{-1}(cx))^{5/2}} dx &= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} + \frac{2 \int \frac{1}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3bc} - \frac{(4c) \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{16 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{16 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{16 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{8 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{8 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{8 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{8 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b} \\
&= -\frac{2x\sqrt{1-c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a + b \sin^{-1}(cx)}} + \frac{8x^2}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{8 \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \sin^{-1}(cx))^{3/2}} dx}{3b}
\end{aligned}$$

**Mathematica [C]** time = 1.36, size = 173, normalized size = 0.96

$$\frac{b \sin(2 \sin^{-1}(cx)) + 2(a + b \sin^{-1}(cx)) \left( -\sqrt{2} e^{-\frac{2ia}{b}} \sqrt{-\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{2i(a+b \sin^{-1}(cx))}{b}\right) - \sqrt{2} e^{\frac{2ia}{b}} \sqrt{\frac{i(a+b \sin^{-1}(cx))}{b}} \Gamma\left(\frac{1}{2}, \frac{2i(a+b \sin^{-1}(cx))}{b}\right) \right)}{3b^2c^2(a + b \sin^{-1}(cx))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(a + b\*ArcSin[c\*x])^(5/2), x]

[Out] -1/3\*(2\*(a + b\*ArcSin[c\*x])\*(E^((-2\*I)\*ArcSin[c\*x]) + E^((2\*I)\*ArcSin[c\*x]) - (Sqrt[2]\*Sqrt[((-I)\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, ((-2\*I)\*(a + b\*ArcSin[c\*x]))/b])/E^(((2\*I)\*a)/b) - Sqrt[2]\*E^(((2\*I)\*a)/b)\*Sqrt[(I\*(a + b\*ArcSin[c\*x]))/b]\*Gamma[1/2, ((2\*I)\*(a + b\*ArcSin[c\*x]))/b]) + b\*Sin[2\*ArcSin[c\*x]])/(b^2\*c^2\*(a + b\*ArcSin[c\*x])^(3/2))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \arcsin(cx) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="giac")

[Out] integrate(x/(b\*arcsin(c\*x) + a)^(5/2), x)

**maple** [B] time = 0.09, size = 311, normalized size = 1.73

$$8 \arcsin(cx) \sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{a + b \arcsin(cx)} \cos\left(\frac{2a}{b}\right) S\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b}} b}\right) b - 8 \arcsin(cx) \sqrt{\frac{1}{b}} \sqrt{\pi} \sqrt{a + b \arcsin(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arcsin(c\*x))^(5/2),x)

[Out]  $-1/3/c^2/b^2*(8*\arcsin(c*x)*(1/b)^{(1/2)}*Pi^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}*\cos(2*a/b)*\text{FresnelS}(2/Pi^{(1/2)}/(1/b)^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b)*b-8*\arcsin(c*x)*(1/b)^{(1/2)}*Pi^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}*\sin(2*a/b)*\text{FresnelC}(2/Pi^{(1/2)}/(1/b)^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b)*b+8*(1/b)^{(1/2)}*Pi^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}*\cos(2*a/b)*\text{FresnelS}(2/Pi^{(1/2)}/(1/b)^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b)*a-8*(1/b)^{(1/2)}*Pi^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}*\sin(2*a/b)*\text{FresnelC}(2/Pi^{(1/2)}/(1/b)^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b)*a+4*\arcsin(c*x)*\cos(2*(a+b*\arcsin(c*x))/b-2*a/b)*b+\sin(2*(a+b*\arcsin(c*x))/b-2*a/b)*b+4*\cos(2*(a+b*\arcsin(c*x))/b-2*a/b)*a)/(a+b*\arcsin(c*x))^{(3/2)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \arcsin(cx) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="maxima")

[Out] integrate(x/(b\*arcsin(c\*x) + a)^(5/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \arcsin(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*asin(c\*x))^(5/2),x)

[Out] int(x/(a + b\*asin(c\*x))^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \arcsin(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*asin(c\*x))\*\*(5/2),x)

[Out] Integral(x/(a + b\*asin(c\*x))\*\*(5/2), x)



$$3.200 \quad \int \frac{1}{(a+b \sin^{-1}(cx))^{5/2}} dx$$

**Optimal.** Leaf size=163

$$\frac{4\sqrt{2\pi} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} - \frac{4\sqrt{2\pi} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{4x}{3b^2\sqrt{a+b \sin^{-1}(cx)}} - \frac{2\sqrt{1-c^2}}{3bc(a+b \sin^{-1}(cx))}$$

[Out]  $-4/3*\cos(a/b)*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(5/2)}/c-4/3*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*(a+b*\arcsin(c*x))^{(1/2)}/b^{(1/2)})*\sin(a/b)*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(5/2)}/c-2/3*(-c^2*x^2+1)^{(1/2)}/b/c/(a+b*\arcsin(c*x))^{(3/2)}+4/3*x/b^2/(a+b*\arcsin(c*x))^{(1/2)}$

**Rubi [A]** time = 0.28, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {4621, 4719, 4623, 3306, 3305, 3351, 3304, 3352}

$$\frac{4\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} - \frac{4\sqrt{2\pi} \sin\left(\frac{a}{b}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{4x}{3b^2\sqrt{a+b \sin^{-1}(cx)}} - \frac{2\sqrt{1-c^2}}{3bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSin}[c*x])^{(-5/2)}, x]$

[Out]  $(-2*\text{Sqrt}[1 - c^2*x^2])/(3*b*c*(a + b*\text{ArcSin}[c*x])^{(3/2)}) + (4*x)/(3*b^2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) - (4*\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a/b]*\text{FresnelC}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/(3*b^{(5/2)*c}) - (4*\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[(\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[a/b])/(3*b^{(5/2)*c})$

#### Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

#### Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

#### Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

#### Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}[\{d, e, f\}, x]$

#### Rule 3352

```
Int[Cos[(d_.)*(e_.) + (f_.)*(x_)]^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

### Rule 4621

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_], x_Symbol] := Simp[(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] + Dist[c/(b*(n + 1)), Int[(x*(a + b*ArcSin[c*x])^(n + 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

### Rule 4623

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_], x_Symbol] := Dist[1/(b*c), Subst[Int[x^n*Cos[a/b - x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 4719

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_)*((f_.)*(x_))^(m_.)/Sqrt[(d_ + (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^(m*(a + b*ArcSin[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && GtQ[d, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \sin^{-1}(cx))^{5/2}} dx &= -\frac{2\sqrt{1 - c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} - \frac{(2c) \int \frac{x}{\sqrt{1 - c^2x^2}(a + b \sin^{-1}(cx))^{3/2}} dx}{3b} \\ &= -\frac{2\sqrt{1 - c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{4 \int \frac{1}{\sqrt{a + b \sin^{-1}(cx)}} dx}{3b^2} \\ &= -\frac{2\sqrt{1 - c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{4 \operatorname{Subst}\left(\int \frac{\cos\left(\frac{a-x}{b}\right)}{\sqrt{x}} dx, x, a + b \sin^{-1}(cx)\right)}{3b^3c} \\ &= -\frac{2\sqrt{1 - c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{(4 \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a + b \sin^{-1}(cx)\right)}{3b^3c} \\ &= -\frac{2\sqrt{1 - c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{(8 \cos\left(\frac{a}{b}\right)) \operatorname{Subst}\left(\int \cos\left(\frac{x^2}{b}\right) dx, x, a + b \sin^{-1}(cx)\right)}{3b^3c} \\ &= -\frac{2\sqrt{1 - c^2x^2}}{3bc(a + b \sin^{-1}(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a + b \sin^{-1}(cx)}} - \frac{4\sqrt{2\pi} \cos\left(\frac{a}{b}\right) C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a + b \sin^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} \end{aligned}$$

**Mathematica [C]** time = 0.98, size = 214, normalized size = 1.31

$$e^{-\frac{i(a + b \sin^{-1}(cx))}{b}} \left( -2be^{i \sin^{-1}(cx)} \left( -\frac{i(a + b \sin^{-1}(cx))}{b} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{i(a + b \sin^{-1}(cx))}{b}\right) - ie^{\frac{ia}{b}} \left( -2ibe^{-\frac{i(a + b \sin^{-1}(cx))}{b}} \left( \frac{i(a + b \sin^{-1}(cx))}{b} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{i(a + b \sin^{-1}(cx))}{b}\right) \right) \right) / (3b^2c(a + b \sin^{-1}(cx))^{3/2})$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSin[c\*x])^(-5/2), x]

[Out]  $(-2*b*E^{(I*ArcSin[c*x])}*((( -I)*(a + b*ArcSin[c*x]))/b)^{(3/2)}*Gamma[1/2, (( -I)*(a + b*ArcSin[c*x]))/b] - I*E^{((I*a)/b)}*(2*a*(-1 + E^{((2*I)*ArcSin[c*x])}) + b*(-I - 2*ArcSin[c*x] + E^{((2*I)*ArcSin[c*x])}*(-I + 2*ArcSin[c*x]))) - (2*I)*b*E^{((I*(a + b*ArcSin[c*x]))/b)}*((I*(a + b*ArcSin[c*x]))/b)^{(3/2)}*Gamma[1/2, (I*(a + b*ArcSin[c*x]))/b])/(3*b^2*c*E^{((I*(a + b*ArcSin[c*x]))/b)}*(a + b*ArcSin[c*x])^{(3/2)})$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(5/2), x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^(-5/2), x)

**maple** [B] time = 0.09, size = 325, normalized size = 1.99

$$\frac{4 \arcsin(cx) \sqrt{\pi} \sqrt{2} \sqrt{\frac{1}{b}} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b} b}}\right) b}{3} - \frac{4 \arcsin(cx) \sqrt{\pi} \sqrt{2} \sqrt{\frac{1}{b}} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{\frac{1}{b} b}}\right) b}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arcsin(c\*x))^(5/2), x)

[Out]  $\frac{2}{3} \frac{c}{b^2} (-2 \arcsin(cx) \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \cos(a/b) \text{FresnelC}(2^{1/2}/\pi^{1/2}/(1/b)^{1/2} (a+b \arcsin(cx))^{1/2}/b) + b (-2 \arcsin(cx) \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \sin(a/b) \text{FresnelS}(2^{1/2}/\pi^{1/2}/(1/b)^{1/2} (a+b \arcsin(cx))^{1/2}/b) + b (-2 \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \cos(a/b) \text{FresnelC}(2^{1/2}/\pi^{1/2}/(1/b)^{1/2} (a+b \arcsin(cx))^{1/2}/b) + a (-2 \pi^{1/2} 2^{1/2} (1/b)^{1/2} (a+b \arcsin(cx))^{1/2} \sin(a/b) \text{FresnelS}(2^{1/2}/\pi^{1/2}/(1/b)^{1/2} (a+b \arcsin(cx))^{1/2}/b) + a (2 \arcsin(cx) \sin((a+b \arcsin(cx))/b) - a/b) + b \cos((a+b \arcsin(cx))/b) - a/b) + 2 \sin((a+b \arcsin(cx))/b) a) / (a+b \arcsin(cx))^{3/2}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="maxima")

[Out] integrate((b\*arcsin(c\*x) + a)^(-5/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*asin(c\*x))^(5/2),x)

[Out] int(1/(a + b\*asin(c\*x))^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*asin(c\*x))\*\*(5/2),x)

[Out] Integral((a + b\*asin(c\*x))\*\*(-5/2), x)

$$3.201 \quad \int \frac{1}{x(a+b \sin^{-1}(cx))^{5/2}} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{1}{x(a+b \sin^{-1}(cx))^{5/2}}, x \right)$$

[Out] Unintegrable(1/x/(a+b\*arcsin(c\*x))^(5/2), x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSin[c\*x])^(5/2)), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSin[c\*x])^(5/2)), x]

Rubi steps

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^{5/2}} dx = \int \frac{1}{x(a+b \sin^{-1}(cx))^{5/2}} dx$$

Mathematica [A] time = 3.95, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sin^{-1}(cx))^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^(5/2)), x]

[Out] Integrate[1/(x\*(a + b\*ArcSin[c\*x])^(5/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(5/2), x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vect  
 eur & l) Error: Bad Argument Value

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \arcsin(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsin(c\*x))^(5/2),x)

[Out] int(1/x/(a+b\*arcsin(c\*x))^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^(5/2)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x(a + b \operatorname{asin}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*asin(c\*x))^(5/2)),x)

[Out] int(1/(x\*(a + b\*asin(c\*x))^(5/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{asin}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*asin(c\*x))\*\*(5/2),x)

[Out] Integral(1/(x\*(a + b\*asin(c\*x))\*\*(5/2)), x)

$$3.202 \quad \int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{5/2}} dx$$

**Optimal.** Leaf size=19

$$\text{Int} \left( \frac{1}{x^2 (a + b \sin^{-1}(cx))^{5/2}}, x \right)$$

[Out] Unintegrable(1/x^2/(a+b\*arcsin(c\*x))^(5/2), x)

**Rubi [A]** time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*ArcSin[c\*x])^(5/2)), x]

[Out] Defer[Int][1/(x^2\*(a + b\*ArcSin[c\*x])^(5/2)), x]

Rubi steps

$$\int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{5/2}} dx = \int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{5/2}} dx$$

**Mathematica [A]** time = 11.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \sin^{-1}(cx))^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^(5/2)), x]

[Out] Integrate[1/(x^2\*(a + b\*ArcSin[c\*x])^(5/2)), x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(5/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(5/2), x, algorithm="giac")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^(5/2)\*x^2), x)

**maple** [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \arcsin(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsin(c\*x))^(5/2),x)

[Out] int(1/x^2/(a+b\*arcsin(c\*x))^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \arcsin(cx) + a)^{\frac{5}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsin(c\*x))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsin(c\*x) + a)^(5/2)\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*asin(c\*x))^(5/2)),x)

[Out] int(1/(x^2\*(a + b\*asin(c\*x))^(5/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asin}(cx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asin(c\*x))\*\*(5/2),x)

[Out] Integral(1/(x\*\*2\*(a + b\*asin(c\*x))\*\*(5/2)), x)



### 3.203 $\int (dx)^{5/2} (a + b \sin^{-1}(cx)) dx$

**Optimal.** Leaf size=120

$$\frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))}{7d} - \frac{20bd^{5/2} F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{147c^{7/2}} + \frac{4b\sqrt{1-c^2x^2} (dx)^{5/2}}{49c} + \frac{20bd^2\sqrt{1-c^2x^2} \sqrt{dx}}{147c^3}$$

[Out]  $2/7*(d*x)^{(7/2)}*(a+b*\arcsin(c*x))/d-20/147*b*d^{(5/2)}*EllipticF(c^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)},I)/c^{(7/2)}+4/49*b*(d*x)^{(5/2)}*(-c^2*x^2+1)^{(1/2)}/c+20/147*b*d^2*(d*x)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c^3$

**Rubi [A]** time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4627, 321, 329, 221}

$$\frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))}{7d} + \frac{20bd^2\sqrt{1-c^2x^2} \sqrt{dx}}{147c^3} - \frac{20bd^{5/2} F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{147c^{7/2}} + \frac{4b\sqrt{1-c^2x^2} (dx)^{5/2}}{49c}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(5/2)\*(a + b\*ArcSin[c\*x]), x]

[Out]  $(20*b*d^2*\text{Sqrt}[d*x]*\text{Sqrt}[1 - c^2*x^2])/(147*c^3) + (4*b*(d*x)^{(5/2)}*\text{Sqrt}[1 - c^2*x^2])/(49*c) + (2*(d*x)^{(7/2)}*(a + b*\text{ArcSin}[c*x]))/(7*d) - (20*b*d^{(5/2)}*EllipticF[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])], -1])/(147*c^{(7/2)})$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int (dx)^{5/2} (a + b \sin^{-1}(cx)) dx &= \frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))}{7d} - \frac{(2bc) \int \frac{(dx)^{7/2}}{\sqrt{1-c^2x^2}} dx}{7d} \\
&= \frac{4b(dx)^{5/2} \sqrt{1-c^2x^2}}{49c} + \frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))}{7d} - \frac{(10bd) \int \frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}} dx}{49c} \\
&= \frac{20bd^2 \sqrt{dx} \sqrt{1-c^2x^2}}{147c^3} + \frac{4b(dx)^{5/2} \sqrt{1-c^2x^2}}{49c} + \frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))}{7d} - \frac{(10bd)^2 \int \frac{(dx)^{1/2}}{\sqrt{1-c^2x^2}} dx}{49c} \\
&= \frac{20bd^2 \sqrt{dx} \sqrt{1-c^2x^2}}{147c^3} + \frac{4b(dx)^{5/2} \sqrt{1-c^2x^2}}{49c} + \frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))}{7d} - \frac{(10bd)^3 \int \frac{(dx)^{-1/2}}{\sqrt{1-c^2x^2}} dx}{49c} \\
&= \frac{20bd^2 \sqrt{dx} \sqrt{1-c^2x^2}}{147c^3} + \frac{4b(dx)^{5/2} \sqrt{1-c^2x^2}}{49c} + \frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))}{7d} - \frac{20bd^3 \int \frac{(dx)^{-3/2}}{\sqrt{1-c^2x^2}} dx}{49c}
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 100, normalized size = 0.83

$$\frac{2d^2 \sqrt{dx} \left( 21ac^3 x^3 + 21bc^3 x^3 \sin^{-1}(cx) - 10b {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^2 x^2\right) + 6bc^2 x^2 \sqrt{1-c^2 x^2} + 10b \sqrt{1-c^2 x^2} \right)}{147c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*ArcSin[c\*x]),x]

[Out] (2\*d^2\*Sqrt[d\*x]\*(21\*a\*c^3\*x^3 + 10\*b\*Sqrt[1 - c^2\*x^2] + 6\*b\*c^2\*x^2\*Sqrt[1 - c^2\*x^2] + 21\*b\*c^3\*x^3\*ArcSin[c\*x] - 10\*b\*Hypergeometric2F1[1/4, 1/2, 5/4, c^2\*x^2]))/(147\*c^3)

**fricas [F]** time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bd^2x^2 \arcsin(cx) + ad^2x^2\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*arcsin(c\*x)),x, algorithm="fricas")

[Out] integral((b\*d^2\*x^2\*arcsin(c\*x) + a\*d^2\*x^2)\*sqrt(d\*x), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}} (b \arcsin(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*arcsin(c\*x)),x, algorithm="giac")

[Out] integrate((d\*x)^(5/2)\*(b\*arcsin(c\*x) + a), x)

**maple [A]** time = 0.04, size = 144, normalized size = 1.20

$$\frac{2(dx)^{\frac{7}{2}} a}{7} + 2b \left( \frac{(dx)^{\frac{7}{2}} \arcsin(cx)}{7} - \frac{2c \left( -\frac{d^2(dx)^{\frac{5}{2}} \sqrt{-c^2x^2+1}}{7c^2} - \frac{5d^4 \sqrt{dx} \sqrt{-c^2x^2+1}}{21c^4} + \frac{5d^4 \sqrt{-cx+1} \sqrt{cx+1} \text{EllipticF}\left(\sqrt{dx} \sqrt{\frac{c}{d}}, i\right)}{21c^4 \sqrt{\frac{c}{d}} \sqrt{-c^2x^2+1}} \right)}{7d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(5/2)*(a+b*arcsin(c*x)),x)`

[Out]  $2/d*(1/7*(d*x)^(7/2)*a+b*(1/7*(d*x)^(7/2)*arcsin(c*x)-2/7*c/d*(-1/7/c^2*d^2*(d*x)^(5/2)*(-c^2*x^2+1)^(1/2)-5/21/c^4*d^4*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)+5/21/c^4*d^4/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{7}bd^{\frac{5}{2}}x^{\frac{7}{2}}\arctan\left(cx,\sqrt{cx+1}\sqrt{-cx+1}\right)+\frac{2}{7}\left(ad^2x^{\frac{7}{2}}+7bcd^2\int\frac{\sqrt{cx+1}\sqrt{-cx+1}x^{\frac{7}{2}}}{7(c^2x^2-1)}dx\right)\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

[Out]  $2/7*b*d^(5/2)*x^(7/2)*arctan2(c*x, \sqrt{c*x + 1}*\sqrt{-c*x + 1}) + 2/7*(a*d^2*x^(7/2) + 7*b*c*d^2*integrate(1/7*\sqrt{c*x + 1}*\sqrt{-c*x + 1}*x^(7/2)/(c^2*x^2 - 1), x))*\sqrt{d}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asin}(cx)) (dx)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))*(d*x)^(5/2),x)`

[Out] `int((a + b*asin(c*x))*(d*x)^(5/2), x)`

**sympy** [A] time = 104.62, size = 82, normalized size = 0.68

$$a \left( \begin{cases} 0 & \text{for } d = 0 \\ \frac{2(dx)^{\frac{7}{2}}}{7d} & \text{otherwise} \end{cases} \right) - bc \left( \begin{cases} 0 & \text{for } d = 0 \\ \frac{d^{\frac{5}{2}}x^{\frac{9}{2}}\Gamma(\frac{9}{4}){}_2F_1\left(\frac{1}{2}, \frac{9}{4} \middle| \frac{13}{4} \right)c^2x^2e^{2i\pi}}{7\Gamma(\frac{13}{4})} & \text{otherwise} \end{cases} \right) + b \left( \begin{cases} 0 & \text{for } d = 0 \\ \frac{2(dx)^{\frac{7}{2}}}{7d} & \text{otherwise} \end{cases} \right) \operatorname{asin}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(5/2)*(a+b*asin(c*x)),x)`

[Out]  $a*\text{Piecewise}((0, \text{Eq}(d, 0)), (2*(d*x)**(7/2)/(7*d), \text{True})) - b*c*\text{Piecewise}((0, \text{Eq}(d, 0)), (d**(5/2)*x**(9/2)*\text{gamma}(9/4)*\text{hyper}((1/2, 9/4), (13/4,)), c**2*x**2*\text{exp\_polar}(2*I*pi))/(7*\text{gamma}(13/4)), \text{True})) + b*\text{Piecewise}((0, \text{Eq}(d, 0)), (2*(d*x)**(7/2)/(7*d), \text{True}))*\text{asin}(c*x)$

### 3.204 $\int (dx)^{3/2} (a + b \sin^{-1}(cx)) dx$

**Optimal.** Leaf size=124

$$\frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} + \frac{12bd^{3/2} F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{25c^{5/2}} - \frac{12bd^{3/2} E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{25c^{5/2}} + \frac{4b\sqrt{1-c^2x^2} (dx)^{3/2}}{25c}$$

[Out]  $2/5*(d*x)^{(5/2)*(a+b*\arcsin(c*x))/d-12/25*b*d^{(3/2)*\text{EllipticE}(c^{(1/2)}*(d*x)^{(1/2)/d^{(1/2)},I)/c^{(5/2)+12/25*b*d^{(3/2)*\text{EllipticF}(c^{(1/2)}*(d*x)^{(1/2)/d^{(1/2)},I)/c^{(5/2)+4/25*b*(d*x)^{(3/2)*(-c^2*x^2+1)^{(1/2)/c}}$

**Rubi [A]** time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {4627, 321, 329, 307, 221, 1199, 424}

$$\frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} + \frac{12bd^{3/2} F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{25c^{5/2}} - \frac{12bd^{3/2} E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{25c^{5/2}} + \frac{4b\sqrt{1-c^2x^2} (dx)^{3/2}}{25c}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(3/2)\*(a + b\*ArcSin[c\*x]),x]

[Out]  $(4*b*(d*x)^{(3/2)*\text{Sqrt}[1 - c^2*x^2]}/(25*c) + (2*(d*x)^{(5/2)*(a + b*\text{ArcSin}[c*x]))/ (5*d) - (12*b*d^{(3/2)*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]], -1]}/(25*c^{(5/2)}) + (12*b*d^{(3/2)*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]], -1]}/(25*c^{(5/2)})$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 307

Int[(x\_)^2/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := With[{q = Rt[-(b/a), 2]}, -Dist[q^(-1), Int[1/Sqrt[a + b\*x^4], x], x] + Dist[1/q, Int[(1 + q\*x^2)/Sqrt[a + b\*x^4], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c)

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

### Rule 1199

Int[((d\_) + (e\_)\*(x\_)^2)/Sqrt[(a\_) + (c\_)\*(x\_)^4], x\_Symbol] := Dist[d/Sqrt[a], Int[Sqrt[1 + (e\*x^2)/d]/Sqrt[1 - (e\*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c^n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int (dx)^{3/2} (a + b \sin^{-1}(cx)) dx &= \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} - \frac{(2bc) \int \frac{(dx)^{5/2}}{\sqrt{1-c^2x^2}} dx}{5d} \\ &= \frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} - \frac{(6bd) \int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}} dx}{25c} \\ &= \frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} - \frac{(12bd) \text{Subst} \left( \int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx \right)}{25c} \\ &= \frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} + \frac{(12bd) \text{Subst} \left( \int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx \right)}{25c^2} \\ &= \frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} + \frac{12bd^{3/2}F \left( \sin^{-1} \left( \frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}} \right) \right)}{25c^{5/2}} \\ &= \frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))}{5d} - \frac{12bd^{3/2}E \left( \sin^{-1} \left( \frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}} \right) \right)}{25c^{5/2}} \end{aligned}$$

**Mathematica [C]** time = 0.03, size = 66, normalized size = 0.53

$$\frac{2(dx)^{3/2} \left( 5acx - 2b {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^2x^2 \right) + 2b\sqrt{1-c^2x^2} + 5bcx \sin^{-1}(cx) \right)}{25c}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*ArcSin[c\*x]), x]

[Out] (2\*(d\*x)^(3/2)\*(5\*a\*c\*x + 2\*b\*Sqrt[1 - c^2\*x^2] + 5\*b\*c\*x\*ArcSin[c\*x] - 2\*b\*Hypergeometric2F1[1/2, 3/4, 7/4, c^2\*x^2]))/(25\*c)

**fricas [F]** time = 2.00, size = 0, normalized size = 0.00

$$\text{integral} \left( (bdx \arcsin(cx) + adx)\sqrt{dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arcsin(c\*x)),x, algorithm="fricas")

[Out] integral((b\*d\*x\*arcsin(c\*x) + a\*d\*x)\*sqrt(d\*x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} (b \arcsin(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arcsin(c\*x)),x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)\*(b\*arcsin(c\*x) + a), x)

**maple** [A] time = 0.01, size = 138, normalized size = 1.11

$$\frac{2(dx)^{\frac{5}{2}}a}{5} + 2b \left( \frac{(dx)^{\frac{5}{2}} \arcsin(cx)}{5} - \frac{2c \left( -\frac{d^2(dx)^{\frac{3}{2}} \sqrt{-c^2x^2+1}}{5c^2} - \frac{3d^3 \sqrt{-cx+1} \sqrt{cx+1} \left( \text{EllipticF}\left(\sqrt{\frac{dx}{d}} \sqrt{\frac{c}{d}}, i\right) - \text{EllipticE}\left(\sqrt{\frac{dx}{d}} \sqrt{\frac{c}{d}}, i\right) \right)}{5c^3 \sqrt{\frac{c}{d}} \sqrt{-c^2x^2+1}} \right)}{5d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*(a+b\*arcsin(c\*x)),x)

[Out] 2/d\*(1/5\*(d\*x)^(5/2)\*a+b\*(1/5\*(d\*x)^(5/2)\*arcsin(c\*x)-2/5\*c/d\*(-1/5/c^2\*d^2\*(d\*x)^(3/2)\*(-c^2\*x^2+1)^(1/2)-3/5/c^3\*d^3/(c/d)^(1/2)\*(-c\*x+1)^(1/2)\*(c\*x+1)^(1/2)/(-c^2\*x^2+1)^(1/2)\*(EllipticF((d\*x)^(1/2)\*(c/d)^(1/2),I)-EllipticE((d\*x)^(1/2)\*(c/d)^(1/2),I))))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{5} b d^{\frac{3}{2}} x^{\frac{5}{2}} \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right) + \frac{2}{5} \left( a d x^{\frac{5}{2}} + 5 b c d \int \frac{\sqrt{cx+1} \sqrt{-cx+1} x^{\frac{5}{2}}}{5(c^2x^2-1)} dx \right) \sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] 2/5\*b\*d^(3/2)\*x^(5/2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + 2/5\*(a\*d\*x^(5/2) + 5\*b\*c\*d\*integrate(1/5\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^(5/2)/(c^2\*x^2 - 1), x))\*sqrt(d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asin}(cx)) (dx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))\*(d\*x)^(3/2),x)

[Out] int((a + b\*asin(c\*x))\*(d\*x)^(3/2), x)

sympy [A] time = 17.70, size = 82, normalized size = 0.66

$$a \left( \begin{cases} 0 & \text{for } d = 0 \\ \frac{2(dx)^{\frac{5}{2}}}{5d} & \text{otherwise} \end{cases} \right) - bc \left( \begin{cases} 0 & \text{for } d = 0 \\ \frac{d^{\frac{3}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \middle| \frac{11}{4}, c^2 x^2 e^{2i\pi}\right)}{5\Gamma\left(\frac{11}{4}\right)} & \text{otherwise} \end{cases} \right) + b \left( \begin{cases} 0 & \text{for } d = 0 \\ \frac{2(dx)^{\frac{5}{2}}}{5d} & \text{otherwise} \end{cases} \right) \operatorname{asin}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*asin(c*x)),x)
```

```
[Out] a*Piecewise((0, Eq(d, 0)), (2*(d*x)**(5/2)/(5*d), True)) - b*c*Piecewise((0, Eq(d, 0)), (d**(3/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c**2*x**2*exp_polar(2*I*pi))/(5*gamma(11/4)), True)) + b*Piecewise((0, Eq(d, 0)), (2*(d*x)**(5/2)/(5*d), True))*asin(c*x)
```

### 3.205 $\int \sqrt{dx} (a + b \sin^{-1}(cx)) dx$

Optimal. Leaf size=88

$$\frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))}{3d} - \frac{4b\sqrt{d} F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{9c^{3/2}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{dx}}{9c}$$

[Out]  $2/3*(d*x)^{(3/2)}*(a+b*\arcsin(c*x))/d-4/9*b*\text{EllipticF}(c^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)}, I)*d^{(1/2)}/c^{(3/2)}+4/9*b*(d*x)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c$

**Rubi [A]** time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4627, 321, 329, 221}

$$\frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))}{3d} + \frac{4b\sqrt{1-c^2x^2}\sqrt{dx}}{9c} - \frac{4b\sqrt{d} F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right) \middle| -1\right)}{9c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*(a + b\*ArcSin[c\*x]), x]

[Out]  $(4*b*\text{Sqrt}[d*x]*\text{Sqrt}[1 - c^2*x^2])/(9*c) + (2*(d*x)^{(3/2)}*(a + b*\text{ArcSin}[c*x]))/(3*d) - (4*b*\text{Sqrt}[d]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])], -1])/(9*c^{(3/2)})$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a+(b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcSin[c\*x])^(n-1))/Sqrt[1-c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps



$$\begin{aligned}
\int \sqrt{dx} (a + b \sin^{-1}(cx)) dx &= \frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))}{3d} - \frac{(2bc) \int \frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}} dx}{3d} \\
&= \frac{4b\sqrt{dx} \sqrt{1-c^2x^2}}{9c} + \frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))}{3d} - \frac{(2bd) \int \frac{1}{\sqrt{dx} \sqrt{1-c^2x^2}} dx}{9c} \\
&= \frac{4b\sqrt{dx} \sqrt{1-c^2x^2}}{9c} + \frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))}{3d} - \frac{(4b) \text{Subst} \left( \int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx} \right)}{9c} \\
&= \frac{4b\sqrt{dx} \sqrt{1-c^2x^2}}{9c} + \frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))}{3d} - \frac{4b\sqrt{d} F \left( \sin^{-1} \left( \frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}} \right) \right) - 1}{9c^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 66, normalized size = 0.75

$$\frac{2\sqrt{dx} \left( 3acx - 2b {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^2x^2 \right) \right) + 2b\sqrt{1-c^2x^2} + 3bcx \sin^{-1}(cx)}{9c}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*ArcSin[c\*x]),x]

[Out] (2\*Sqrt[d\*x]\*(3\*a\*c\*x + 2\*b\*Sqrt[1 - c^2\*x^2] + 3\*b\*c\*x\*ArcSin[c\*x] - 2\*b\*Hypergeometric2F1[1/4, 1/2, 5/4, c^2\*x^2]))/(9\*c)

**fricas [F]** time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left( \sqrt{dx} (b \arcsin(cx) + a), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arcsin(c\*x)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*(b\*arcsin(c\*x) + a), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx} (b \arcsin(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arcsin(c\*x)),x, algorithm="giac")

[Out] integrate(sqrt(d\*x)\*(b\*arcsin(c\*x) + a), x)

**maple [A]** time = 0.01, size = 119, normalized size = 1.35

$$\frac{\frac{2(dx)^{\frac{3}{2}} a}{3} + 2b \left( \frac{(dx)^{\frac{3}{2}} \arcsin(cx)}{3} - \frac{2c \left( -\frac{d^2 \sqrt{dx} \sqrt{-c^2x^2+1}}{3c^2} + \frac{d^2 \sqrt{-cx+1} \sqrt{cx+1} \text{EllipticF} \left( \sqrt{dx} \sqrt{\frac{c}{d}}, i \right)}{3c^2 \sqrt{\frac{c}{d}} \sqrt{-c^2x^2+1}} \right)}{3d} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*(a+b\*arcsin(c\*x)),x)

[Out]  $2/d*(1/3*(d*x)^{(3/2)}*a+b*(1/3*(d*x)^{(3/2)}*\arcsin(c*x)-2/3*c/d*(-1/3/c^2*d^2*(d*x)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}+1/3/c^2*d^2/(c/d)^{(1/2)}*(-c*x+1)^{(1/2)}*(c*x+1)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}*EllipticF((d*x)^{(1/2)}*(c/d)^{(1/2)},I))))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3}b\sqrt{d}x^{\frac{3}{2}}\arctan\left(cx,\sqrt{cx+1}\sqrt{-cx+1}\right)+\frac{2}{3}\left(3bc\int\frac{\sqrt{cx+1}\sqrt{-cx+1}x^{\frac{3}{2}}}{3(c^2x^2-1)}dx+ax^{\frac{3}{2}}\right)\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

[Out]  $2/3*b*\sqrt{d}*x^{(3/2)}*\arctan2(c*x,\sqrt{c*x+1}*\sqrt{-c*x+1})+2/3*(3*b*c*\int(1/3*\sqrt{c*x+1}*\sqrt{-c*x+1}*x^{(3/2)}/(c^2*x^2-1),x)+a*x^{(3/2)})*\sqrt{d}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asin}(cx)) \sqrt{dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))*(d*x)^(1/2),x)`

[Out] `int((a + b*asin(c*x))*(d*x)^(1/2), x)`

**sympy** [A] time = 2.64, size = 76, normalized size = 0.86

$$\frac{2a(dx)^{\frac{3}{2}}}{3d} - \frac{bc(dx)^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{9}{4} \middle| c^2x^2e^{2i\pi}\right)}{3d^2\Gamma\left(\frac{9}{4}\right)} + \frac{2b(dx)^{\frac{3}{2}}\operatorname{asin}(cx)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(1/2)*(a+b*asin(c*x)),x)`

[Out]  $2*a*(d*x)**(3/2)/(3*d) - b*c*(d*x)**(5/2)*\gamma(5/4)*\operatorname{hyper}((1/2, 5/4), (9/4, ), c**2*x**2*\exp\_polar(2*I*pi))/(3*d**2*\gamma(9/4)) + 2*b*(d*x)**(3/2)*\operatorname{asin}(c*x)/(3*d)$

$$3.206 \quad \int \frac{a+b \sin^{-1}(cx)}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=89

$$\frac{2\sqrt{dx} (a + b \sin^{-1}(cx))}{d} + \frac{4bF\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{\sqrt{c}\sqrt{d}} - \frac{4bE\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{\sqrt{c}\sqrt{d}}$$

[Out]  $-4*b*EllipticE(c^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)}, I)/c^{(1/2)}/d^{(1/2)}+4*b*EllipticF(c^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)}, I)/c^{(1/2)}/d^{(1/2)}+2*(a+b*arcsin(c*x))*(d*x)^{(1/2)}/d$

**Rubi [A]** time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4627, 329, 307, 221, 1199, 424}

$$\frac{2\sqrt{dx} (a + b \sin^{-1}(cx))}{d} + \frac{4bF\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{\sqrt{c}\sqrt{d}} - \frac{4bE\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])/Sqrt[d\*x], x]

[Out]  $(2*\text{Sqrt}[d*x]*(a + b*\text{ArcSin}[c*x]))/d - (4*b*EllipticE[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])], -1])/(\text{Sqrt}[c]*\text{Sqrt}[d]) + (4*b*EllipticF[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])], -1])/(\text{Sqrt}[c]*\text{Sqrt}[d])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 307

Int[(x\_)^2/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := With[{q = Rt[-(b/a), 2]}, -Dist[q^(-1), Int[1/Sqrt[a + b\*x^4], x], x] + Dist[1/q, Int[(1 + q\*x^2)/Sqrt[a + b\*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 1199

Int[((d\_) + (e\_.)\*(x\_)^2)/Sqrt[(a\_) + (c\_.)\*(x\_)^4], x\_Symbol] := Dist[d/Sqrt[a], Int[Sqrt[1 + (e\*x^2)/d]/Sqrt[1 - (e\*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

## Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

## Rubi steps

$$\begin{aligned} \int \frac{a + b \sin^{-1}(cx)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx} (a + b \sin^{-1}(cx))}{d} - \frac{(2bc) \int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}} dx}{d} \\ &= \frac{2\sqrt{dx} (a + b \sin^{-1}(cx))}{d} - \frac{(4bc) \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx} \right)}{d^2} \\ &= \frac{2\sqrt{dx} (a + b \sin^{-1}(cx))}{d} + \frac{(4b) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx} \right)}{d} - \frac{(4b) \operatorname{Subst} \left( \int \frac{1+\frac{cx^2}{d}}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx} \right)}{d} \\ &= \frac{2\sqrt{dx} (a + b \sin^{-1}(cx))}{d} + \frac{4bF \left( \sin^{-1} \left( \frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}} \right) \middle| -1 \right)}{\sqrt{c} \sqrt{d}} - \frac{(4b) \operatorname{Subst} \left( \int \frac{\sqrt{1+\frac{cx^2}{d}}}{\sqrt{1-\frac{cx^2}{d}}} dx, x, \sqrt{dx} \right)}{d} \\ &= \frac{2\sqrt{dx} (a + b \sin^{-1}(cx))}{d} - \frac{4bE \left( \sin^{-1} \left( \frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}} \right) \middle| -1 \right)}{\sqrt{c} \sqrt{d}} + \frac{4bF \left( \sin^{-1} \left( \frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}} \right) \middle| -1 \right)}{\sqrt{c} \sqrt{d}} \end{aligned}$$

**Mathematica** [C] time = 0.01, size = 45, normalized size = 0.51

$$\frac{2x \left( 3(a + b \sin^{-1}(cx)) - 2bcx {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^2x^2 \right) \right)}{3\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSin[c*x])/Sqrt[d*x], x]
```

```
[Out] (2*x*(3*(a + b*ArcSin[c*x]) - 2*b*c*x*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2]))/(3*Sqrt[d*x])
```

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{dx} (b \arcsin(cx) + a)}{dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsin(c*x))/(d*x)^(1/2), x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b*arcsin(c*x) + a)/(d*x), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \arcsin(cx) + a}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/(d\*x)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)/sqrt(d\*x), x)

**maple** [A] time = 0.01, size = 98, normalized size = 1.10

$$\frac{2a\sqrt{dx} + 2b \left( \sqrt{dx} \arcsin(cx) + \frac{2\sqrt{-cx+1} \sqrt{cx+1} \left( \text{EllipticF}\left(\sqrt{dx} \sqrt{\frac{c}{d}}, i\right) - \text{EllipticE}\left(\sqrt{dx} \sqrt{\frac{c}{d}}, i\right) \right)}{\sqrt{\frac{c}{d}} \sqrt{-c^2x^2+1}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))/(d\*x)^(1/2),x)

[Out] 2/d\*(a\*(d\*x)^(1/2)+b\*((d\*x)^(1/2)\*arcsin(c\*x)+2/(c/d)^(1/2)\*(-c\*x+1)^(1/2)\*(c\*x+1)^(1/2)/(-c^2\*x^2+1)^(1/2)\*(EllipticF((d\*x)^(1/2)\*(c/d)^(1/2),I)-EllipticE((d\*x)^(1/2)\*(c/d)^(1/2),I))))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( b\sqrt{d} \sqrt{x} \arctan \left( cx, \sqrt{cx+1} \sqrt{-cx+1} \right) + \left( bc \int \frac{\sqrt{-cx+1} \sqrt{x}}{\sqrt{cx+1} cx - \sqrt{cx+1}} dx + a\sqrt{x} \right) \sqrt{d} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/(d\*x)^(1/2),x, algorithm="maxima")

[Out] 2\*(b\*sqrt(d)\*sqrt(x)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + (b\*c\*d\*integrate(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(x)/(c^2\*d\*x^2 - d), x) + a\*sqrt(x))\*sqrt(d))/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asin}(cx)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))/(d\*x)^(1/2),x)

[Out] int((a + b\*asin(c\*x))/(d\*x)^(1/2), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))/(d\*x)\*\*(1/2),x)

[Out] Exception raised: TypeError

$$3.207 \quad \int \frac{a+b \sin^{-1}(cx)}{(dx)^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{4b\sqrt{c}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{d^{3/2}} - \frac{2(a+b\sin^{-1}(cx))}{d\sqrt{dx}}$$

[Out] 4\*b\*EllipticF(c^(1/2)\*(d\*x)^(1/2)/d^(1/2),1)\*c^(1/2)/d^(3/2)-2\*(a+b\*arcsin(c\*x))/d/(d\*x)^(1/2)

**Rubi [A]** time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {4627, 329, 221}

$$\frac{4b\sqrt{c}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{d^{3/2}} - \frac{2(a+b\sin^{-1}(cx))}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])/(d\*x)^(3/2), x]

[Out] (-2\*(a + b\*ArcSin[c\*x]))/(d\*Sqrt[d\*x]) + (4\*b\*Sqrt[c]\*EllipticF[ArcSin[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]], -1])/d^(3/2)

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sin^{-1}(cx)}{(dx)^{3/2}} dx &= -\frac{2(a + b \sin^{-1}(cx))}{d\sqrt{dx}} + \frac{(2bc) \int \frac{1}{\sqrt{dx} \sqrt{1-c^2x^2}} dx}{d} \\
&= -\frac{2(a + b \sin^{-1}(cx))}{d\sqrt{dx}} + \frac{(4bc) \text{Subst} \left( \int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx} \right)}{d^2} \\
&= -\frac{2(a + b \sin^{-1}(cx))}{d\sqrt{dx}} + \frac{4b\sqrt{c} F \left( \sin^{-1} \left( \frac{\sqrt{c} \sqrt{dx}}{\sqrt{d}} \right) \middle| -1 \right)}{d^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 40, normalized size = 0.73

$$-\frac{2x \left( a - 2bcx {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^2x^2 \right) + b \sin^{-1}(cx) \right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])/(d\*x)^(3/2), x]

[Out] (-2\*x\*(a + b\*ArcSin[c\*x] - 2\*b\*c\*x\*Hypergeometric2F1[1/4, 1/2, 5/4, c^2\*x^2]))/(d\*x)^(3/2)

**fricas [F]** time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx} (b \arcsin(cx) + a)}{d^2x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/(d\*x)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*(b\*arcsin(c\*x) + a)/(d^2\*x^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \arcsin(cx) + a}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/(d\*x)^(3/2), x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)/(d\*x)^(3/2), x)

**maple [A]** time = 0.01, size = 85, normalized size = 1.55

$$\frac{-\frac{2a}{\sqrt{dx}} + 2b \left( -\frac{\arcsin(cx)}{\sqrt{dx}} + \frac{2c\sqrt{-cx+1}\sqrt{cx+1} \text{EllipticF} \left( \sqrt{dx} \sqrt{\frac{c}{d}} \middle| i \right)}{d\sqrt{\frac{c}{d}} \sqrt{-c^2x^2+1}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))/(d\*x)^(3/2), x)

[Out]  $2/d*(-a/(d*x)^{(1/2)}+b*(-1/(d*x)^{(1/2)}*\arcsin(c*x)+2*c/d/(c/d)^{(1/2)*(-c*x+1)^{(1/2)}*(c*x+1)^{(1/2)/(-c^2*x^2+1)^{(1/2)}*EllipticF((d*x)^{(1/2)}*(c/d)^{(1/2), I}))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( b\sqrt{d}\sqrt{x} \arctan \left( cx, \sqrt{cx+1} \sqrt{-cx+1} \right) + \left( bc\sqrt{x} \int \frac{\sqrt{-cx+1}}{\sqrt{cx+1}cx^2-\sqrt{cx+1}\sqrt{x}} dx + a \right) \sqrt{d}\sqrt{x} \right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsin(c*x))/(d*x)^(3/2),x, algorithm="maxima")`

[Out]  $-2*(b*\sqrt{d}*\sqrt{x}*\arctan2(c*x, \sqrt{c*x+1}*\sqrt{-c*x+1})) + (b*c*d^2*\sqrt{x}*integrate(\sqrt{c*x+1}*\sqrt{-c*x+1}*\sqrt{x}/(c^2*d^2*x^3 - d^2*x), x) + a)*\sqrt{d}*\sqrt{x})/(d^2*x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asin}(cx)}{(dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))/(d*x)^(3/2),x)`

[Out] `int((a + b*asin(c*x))/(d*x)^(3/2), x)`

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asin(c*x))/(d*x)**(3/2),x)`

[Out] Exception raised: TypeError



### 3.208 $\int \frac{a+b \sin^{-1}(cx)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=125

$$\frac{2(a+b \sin^{-1}(cx))}{3d(dx)^{3/2}} + \frac{4bc^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}} - \frac{4bc^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}} - \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}}$$

[Out]  $-2/3*(a+b*\arcsin(c*x))/d/(d*x)^{(3/2)}-4/3*b*c^{(3/2)}*EllipticE(c^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)},I)/d^{(5/2)}+4/3*b*c^{(3/2)}*EllipticF(c^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)},I)/d^{(5/2)}-4/3*b*c*(-c^2*x^2+1)^{(1/2)}/d^2/(d*x)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {4627, 325, 329, 307, 221, 1199, 424}

$$\frac{2(a+b \sin^{-1}(cx))}{3d(dx)^{3/2}} - \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} + \frac{4bc^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}} - \frac{4bc^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSin}[c*x])/(d*x)^{(5/2)}, x]$

[Out]  $(-4*b*c*\text{Sqrt}[1 - c^2*x^2])/(3*d^2*\text{Sqrt}[d*x]) - (2*(a + b*\text{ArcSin}[c*x]))/(3*d*(d*x)^{(3/2)}) - (4*b*c^{(3/2)}*EllipticE[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/(\text{Sqrt}[d]), -1])/(3*d^{(5/2)}) + (4*b*c^{(3/2)}*EllipticF[\text{ArcSin}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/(\text{Sqrt}[d]), -1])/(3*d^{(5/2)})$

#### Rule 221

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^4], x\_Symbol] \rightarrow \text{Simp}[EllipticF[\text{ArcSin}[(\text{Rt}[-b, 4]*x)/\text{Rt}[a, 4]], -1]/(\text{Rt}[a, 4]*\text{Rt}[-b, 4]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[b/a] \ \&\& \ \text{GtQ}[a, 0]$

#### Rule 307

$\text{Int}[(x_)^2/\text{Sqrt}[(a_) + (b_.)*(x_)^4], x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(b/a), 2]\}, -\text{Dist}[q^{(-1)}, \text{Int}[1/\text{Sqrt}[a + b*x^4], x], x] + \text{Dist}[1/q, \text{Int}[(1 + q*x^2)/\text{Sqrt}[a + b*x^4], x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[b/a]$

#### Rule 325

$\text{Int}[(c_.)*(x_)^{(m_)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c^{(m+1)}), x] - \text{Dist}[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), \text{Int}[(c*x)^{(m+n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 329

$\text{Int}[(c_.)*(x_)^{(m_)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)}))/c^n]^p, x], x, (c*x)^{(1/k)}], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 424

$\text{Int}[\text{Sqrt}[(a_) + (b_.)*(x_)^2]/\text{Sqrt}[(c_) + (d_.)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*EllipticE[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c)$

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

### Rule 1199

Int[((d\_) + (e\_)\*(x\_)^2)/Sqrt[(a\_) + (c\_)\*(x\_)^4], x\_Symbol] := Dist[d/Sqrt[a], Int[Sqrt[1 + (e\*x^2)/d]/Sqrt[1 - (e\*x^2)/d], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

### Rule 4627

Int[((a\_) + ArcSin[(c\_)\*(x\_)])\*(b\_)^(n\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \sin^{-1}(cx)}{(dx)^{5/2}} dx &= -\frac{2(a + b \sin^{-1}(cx))}{3d(dx)^{3/2}} + \frac{(2bc) \int \frac{1}{(dx)^{3/2} \sqrt{1-c^2x^2}} dx}{3d} \\
 &= -\frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a + b \sin^{-1}(cx))}{3d(dx)^{3/2}} - \frac{(2bc^3) \int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}} dx}{3d^3} \\
 &= -\frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a + b \sin^{-1}(cx))}{3d(dx)^{3/2}} - \frac{(4bc^3) \text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx}\right)}{3d^4} \\
 &= -\frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a + b \sin^{-1}(cx))}{3d(dx)^{3/2}} + \frac{(4bc^2) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx}\right)}{3d^3} - \frac{(4bc^2) \text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx}\right)}{3d^4} \\
 &= -\frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a + b \sin^{-1}(cx))}{3d(dx)^{3/2}} + \frac{4bc^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}} - \frac{(4bc^2) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}} dx, x, \sqrt{dx}\right)}{3d^3} \\
 &= -\frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a + b \sin^{-1}(cx))}{3d(dx)^{3/2}} - \frac{4bc^{3/2}E\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}} + \frac{4bc^{3/2}F\left(\sin^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.02, size = 42, normalized size = 0.34

$$\frac{2x\left(a + 2bcx {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; c^2x^2\right) + b \sin^{-1}(cx)\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])/(d\*x)^(5/2), x]

[Out] (-2\*x\*(a + b\*ArcSin[c\*x] + 2\*b\*c\*x\*Hypergeometric2F1[-1/4, 1/2, 3/4, c^2\*x^2]))/(3\*(d\*x)^(5/2))

**fricas [F]** time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}(b \arcsin(cx) + a)}{d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/(d\*x)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*(b\*arcsin(c\*x) + a)/(d^3\*x^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \arcsin(cx) + a}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/(d\*x)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)/(d\*x)^(5/2), x)

**maple** [A] time = 0.01, size = 129, normalized size = 1.03

$$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}} + 2b \left( -\frac{\arcsin(cx)}{3(dx)^{\frac{3}{2}}} + \frac{2c \left( -\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}} + \frac{c\sqrt{-cx+1}\sqrt{cx+1} \left( \text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}}, i\right) - \text{EllipticE}\left(\sqrt{dx}\sqrt{\frac{c}{d}}, i\right) \right)}{d\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}} \right)}{3d} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))/(d\*x)^(5/2), x)

[Out] 2/d\*(-1/3\*a/(d\*x)^(3/2)+b\*(-1/3/(d\*x)^(3/2)\*arcsin(c\*x)+2/3\*c/d\*(-(-c^2\*x^2+1)^(1/2)/(d\*x)^(1/2)+c/d/(c/d)^(1/2)\*(-c\*x+1)^(1/2)\*(c\*x+1)^(1/2)/(-c^2\*x^2+1)^(1/2)\*(EllipticF((d\*x)^(1/2)\*(c/d)^(1/2),I)-EllipticE((d\*x)^(1/2)\*(c/d)^(1/2),I)))))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( b\sqrt{d}x^{\frac{3}{2}} \arctan \left( cx, \sqrt{cx+1} \sqrt{-cx+1} \right) + \left( bcd^3x^{\frac{5}{2}} \int \frac{\sqrt{cx+1} \sqrt{-cx+1} \sqrt{x}}{c^2d^3x^4-d^3x^2} dx + ax \right) \sqrt{d} \sqrt{x} \right)}{3d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))/(d\*x)^(5/2),x, algorithm="maxima")

[Out] -2/3\*(b\*sqrt(d)\*x^(3/2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + (3\*b\*c\*d^3\*x^(5/2)\*integrate(1/3\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(x)/(c^2\*d^3\*x^4 - d^3\*x^2), x) + a\*x)\*sqrt(d)\*sqrt(x))/(d^3\*x^3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asin}(cx)}{(dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))/(d\*x)^(5/2), x)

[Out] int((a + b\*asin(c\*x))/(d\*x)^(5/2), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asin(c*x))/(d*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

### 3.209 $\int (dx)^{5/2} (a + b \sin^{-1}(cx))^2 dx$

**Optimal.** Leaf size=109

$$\frac{16b^2c^2(dx)^{11/2} {}_3F_2\left(1, \frac{11}{4}, \frac{11}{4}; \frac{13}{4}, \frac{15}{4}; c^2x^2\right)}{693d^3} - \frac{8bc(dx)^{9/2} {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; c^2x^2\right)(a + b \sin^{-1}(cx))}{63d^2} + \frac{2(dx)^{7/2}(a + b \sin^{-1}(cx))}{7d}$$

[Out]  $2/7*(d*x)^{(7/2)}*(a+b*\arcsin(c*x))^{2/d}-8/63*b*c*(d*x)^{(9/2)}*(a+b*\arcsin(c*x))$   
 $*\text{hypergeom}([1/2, 9/4], [13/4], c^2*x^2)/d^2+16/693*b^2*c^2*(d*x)^{(11/2)}*\text{Hypere}$   
 $\text{geometricPFQ}([1, 11/4, 11/4], [13/4, 15/4], c^2*x^2)/d^3$

**Rubi [A]** time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {4627, 4711}

$$\frac{16b^2c^2(dx)^{11/2} {}_3F_2\left(1, \frac{11}{4}, \frac{11}{4}; \frac{13}{4}, \frac{15}{4}; c^2x^2\right)}{693d^3} - \frac{8bc(dx)^{9/2} {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; c^2x^2\right)(a + b \sin^{-1}(cx))}{63d^2} + \frac{2(dx)^{7/2}(a + b \sin^{-1}(cx))}{7d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(5/2)}*(a + b*\text{ArcSin}[c*x])^2, x]$

[Out]  $(2*(d*x)^{(7/2)}*(a + b*\text{ArcSin}[c*x])^2)/(7*d) - (8*b*c*(d*x)^{(9/2)}*(a + b*\text{ArcSin}[c*x])*\text{Hypergeometric2F1}[1/2, 9/4, 13/4, c^2*x^2])/(63*d^2) + (16*b^2*c^2*(d*x)^{(11/2)}*\text{HypergeometricPFQ}[\{1, 11/4, 11/4\}, \{13/4, 15/4\}, c^2*x^2])/(693*d^3)$

#### Rule 4627

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*(d*x)^m, x]$   
 $\text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcSin}[c*x])^n/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(d*x)^{m+1}*(a + b*\text{ArcSin}[c*x])^{n-1}]/\text{Sqrt}[1 - c^2*x^2], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 4711

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^m*(f*x)^n/\text{Sqrt}[d + e*x^2], x]$   
 $\text{Simp}[(f*x)^{m+1}*(a + b*\text{ArcSin}[c*x])*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2]/(\text{Sqrt}[d]*f*(m+1)), x] - \text{Simp}[(b*c*(f*x)^{m+2}*\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2*x^2])/\text{Sqrt}[d]*f^2*(m+1)*(m+2)), x] /;$   
 $\text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[d, 0] \ \&\& \ \text{IntegerQ}[m]$

#### Rubi steps

$$\int (dx)^{5/2} (a + b \sin^{-1}(cx))^2 dx = \frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))^2}{7d} - \frac{(4bc) \int \frac{(dx)^{7/2} (a + b \sin^{-1}(cx))}{\sqrt{1-c^2x^2}} dx}{7d}$$

$$= \frac{2(dx)^{7/2} (a + b \sin^{-1}(cx))^2}{7d} - \frac{8bc(dx)^{9/2} (a + b \sin^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; c^2x^2\right)}{63d^2}$$

**Mathematica [A]** time = 0.06, size = 90, normalized size = 0.83

$$\frac{2}{693}x(dx)^{5/2} \left( 8b^2c^2x^2 {}_3F_2\left(1, \frac{11}{4}, \frac{11}{4}; \frac{13}{4}, \frac{15}{4}; c^2x^2\right) + 11(a + b \sin^{-1}(cx)) \left( 9(a + b \sin^{-1}(cx)) - 4bcx {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; c^2x^2\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*ArcSin[c\*x])^2,x]

[Out] (2\*x\*(d\*x)^(5/2)\*(11\*(a + b\*ArcSin[c\*x])\*(9\*(a + b\*ArcSin[c\*x]) - 4\*b\*c\*x\*Hypergeometric2F1[1/2, 9/4, 13/4, c^2\*x^2]) + 8\*b^2\*c^2\*x^2\*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, c^2\*x^2]))/693

**fricas** [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 d^2 x^2 \arcsin(cx)^2 + 2abd^2 x^2 \arcsin(cx) + a^2 d^2 x^2\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral((b^2\*d^2\*x^2\*arcsin(c\*x)^2 + 2\*a\*b\*d^2\*x^2\*arcsin(c\*x) + a^2\*d^2\*x^2)\*sqrt(d\*x), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}} (a + b \arcsin(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)\*(a+b\*arcsin(c\*x))^2,x)

[Out] int((d\*x)^(5/2)\*(a+b\*arcsin(c\*x))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{7} b^2 d^{\frac{5}{2}} x^{\frac{7}{2}} \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^2 + \frac{1}{42} a^2 c^2 d^{\frac{5}{2}} \left( \frac{4\left(3c^2 x^{\frac{7}{2}} + 7x^{\frac{3}{2}}\right)}{c^4} + \frac{42 \arctan\left(\sqrt{c} \sqrt{x}\right)}{c^{\frac{11}{2}}} + \frac{21 \log\left(\frac{c\sqrt{x}-\sqrt{c}}{c\sqrt{x}+\sqrt{c}}\right)}{c^{\frac{11}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] 2/7\*b^2\*d^(5/2)\*x^(7/2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 1/42\*a^2\*c^2\*d^(5/2)\*(4\*(3\*c^2\*x^(7/2) + 7\*x^(3/2))/c^4 + 42\*arctan(sqrt(c)\*sqrt(x))/c^(11/2) + 21\*log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(11/2)) + 14\*a\*b\*c^2\*d^(5/2)\*integrate(1/7\*x^(9/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) + 4\*b^2\*c\*d^(5/2)\*integrate(1/7\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^(7/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) - 1/6\*a^2\*d^(5/2)\*(4\*x^(3/2)/c^2 + 6\*arctan(sqrt(c)\*sqrt(x))/c^(7/2) + 3\*log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(7/2)) - 14\*a\*b\*d^(5/2)\*integrate(1/7\*x^(5/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asin}(c x))^2 (d x)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))^2*(d*x)^(5/2), x)`

[Out] `int((a + b*asin(c*x))^2*(d*x)^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(5/2)*(a+b*asin(c*x))**2, x)`

[Out] Timed out

### 3.210 $\int (dx)^{3/2} (a + b \sin^{-1}(cx))^2 dx$

**Optimal.** Leaf size=109

$$\frac{16b^2c^2(dx)^{9/2} {}_3F_2\left(1, \frac{9}{4}, \frac{9}{4}; \frac{11}{4}, \frac{13}{4}; c^2x^2\right)}{315d^3} - \frac{8bc(dx)^{7/2} {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; c^2x^2\right)(a + b \sin^{-1}(cx))}{35d^2} + \frac{2(dx)^{5/2}(a + b \sin^{-1}(cx))}{5d}$$

[Out]  $2/5*(d*x)^{(5/2)}*(a+b*\arcsin(c*x))^{2/d}-8/35*b*c*(d*x)^{(7/2)}*(a+b*\arcsin(c*x))$   
 $*\text{hypergeom}([1/2, 7/4], [11/4], c^2*x^2)/d^2+16/315*b^2*c^2*(d*x)^{(9/2)}*\text{Hyper}$   
 $\text{geometricPFQ}([1, 9/4, 9/4], [11/4, 13/4], c^2*x^2)/d^3$

**Rubi [A]** time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {4627, 4711}

$$\frac{16b^2c^2(dx)^{9/2} {}_3F_2\left(1, \frac{9}{4}, \frac{9}{4}; \frac{11}{4}, \frac{13}{4}; c^2x^2\right)}{315d^3} - \frac{8bc(dx)^{7/2} {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; c^2x^2\right)(a + b \sin^{-1}(cx))}{35d^2} + \frac{2(dx)^{5/2}(a + b \sin^{-1}(cx))}{5d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(3/2)}*(a + b*\text{ArcSin}[c*x])^2, x]$

[Out]  $(2*(d*x)^{(5/2)}*(a + b*\text{ArcSin}[c*x])^2)/(5*d) - (8*b*c*(d*x)^{(7/2)}*(a + b*\text{Arc}$   
 $\text{Sin}[c*x])*\text{Hypergeometric2F1}[1/2, 7/4, 11/4, c^2*x^2])/(35*d^2) + (16*b^2*c^2$   
 $* (d*x)^{(9/2)}*\text{HypergeometricPFQ}[\{1, 9/4, 9/4\}, \{11/4, 13/4\}, c^2*x^2])/(315$   
 $*d^3)$

#### Rule 4627

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*(d*x)^m, x]$   
 $\text{:= Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^n/(d*(m+1)), x] - \text{Dist}[(b*c^n$   
 $)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{n-1}/\text{Sqrt}[1 - c^2$   
 $*x^2], x], x] /;$  FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4711

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^m/\text{Sqrt}[d + e*x], x]$   
 $\text{:= Simp}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])*\text{Hypergeomet}$   
 $\text{ric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2])/( \text{Sqrt}[d]*f*(m+1)), x] - \text{Simp}$   
 $[(b*c*(f*x)^{(m+2)}*\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+$   
 $m/2\}, c^2*x^2])/( \text{Sqrt}[d]*f^2*(m+1)*(m+2)), x] /;$  FreeQ[{a, b, c, d, e,  
 f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]

#### Rubi steps

$$\int (dx)^{3/2} (a + b \sin^{-1}(cx))^2 dx = \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))^2}{5d} - \frac{(4bc) \int \frac{(dx)^{5/2} (a + b \sin^{-1}(cx))}{\sqrt{1-c^2x^2}} dx}{5d}$$

$$= \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))^2}{5d} - \frac{8bc(dx)^{7/2} (a + b \sin^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; c^2x^2\right)}{35d^2} +$$

**Mathematica [A]** time = 0.07, size = 90, normalized size = 0.83

$$\frac{2}{315}x(dx)^{3/2} \left( 8b^2c^2x^2 {}_3F_2\left(1, \frac{9}{4}, \frac{9}{4}; \frac{11}{4}, \frac{13}{4}; c^2x^2\right) + 9(a + b \sin^{-1}(cx)) \left( 7(a + b \sin^{-1}(cx)) - 4bcx {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; c^2x^2\right) \right) \right)$$



Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])^2,x]

[Out] (2\*x\*(d\*x)^(3/2)\*(9\*(a + b\*ArcSin[c\*x])\*(7\*(a + b\*ArcSin[c\*x]) - 4\*b\*c\*x\*Hypergeometric2F1[1/2, 7/4, 11/4, c^2\*x^2]) + 8\*b^2\*c^2\*x^2\*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, c^2\*x^2]))/315

**fricas** [F] time = 1.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 dx \arcsin(cx)^2 + 2 ab dx \arcsin(cx) + a^2 dx\right) \sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral((b^2\*d\*x\*arcsin(c\*x)^2 + 2\*a\*b\*d\*x\*arcsin(c\*x) + a^2\*d\*x)\*sqrt(d\*x), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [F] time = 0.17, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} (a + b \arcsin(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^2,x)

[Out] int((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{5} b^2 d^{\frac{3}{2}} x^{\frac{5}{2}} \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^2 + \frac{1}{10} a^2 c^2 d^{\frac{3}{2}} \left( \frac{4\left(c^2 x^{\frac{5}{2}} + 5\sqrt{x}\right)}{c^4} - \frac{10 \arctan\left(\sqrt{c}\sqrt{x}\right)}{c^{\frac{9}{2}}} + \frac{5 \log\left(\frac{c\sqrt{x}-1}{c\sqrt{x}+1}\right)}{c^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] 2/5\*b^2\*d^(3/2)\*x^(5/2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 1/10\*a^2\*c^2\*d^(3/2)\*(4\*(c^2\*x^(5/2) + 5\*sqrt(x))/c^4 - 10\*arctan(sqrt(c)\*sqrt(x))/c^(9/2) + 5\*log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(9/2)) + 10\*a\*b\*c^2\*d^(3/2)\*integrate(1/5\*x^(7/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) + 4\*b^2\*c\*d^(3/2)\*integrate(1/5\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^(5/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) - 1/2\*a^2\*d^(3/2)\*(4\*sqrt(x)/c^2 - 2\*arctan(sqrt(c)\*sqrt(x))/c^(5/2) + log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(5/2)) - 10\*a\*b\*d^(3/2)\*integrate(1/5\*x^(3/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asin}(cx))^2 (dx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asin(c*x))^2*(d*x)^(3/2), x)`

[Out] `int((a + b*asin(c*x))^2*(d*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} (a + b \operatorname{asin}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(3/2)*(a+b*asin(c*x))**2, x)`

[Out] `Integral((d*x)**(3/2)*(a + b*asin(c*x))**2, x)`

### 3.211 $\int \sqrt{dx} (a + b \sin^{-1}(cx))^2 dx$

**Optimal.** Leaf size=109

$$\frac{16b^2c^2(dx)^{7/2} {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; c^2x^2\right)}{105d^3} - \frac{8bc(dx)^{5/2} {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{9}{4}; c^2x^2\right)(a + b \sin^{-1}(cx))}{15d^2} + \frac{2(dx)^{3/2}(a + b \sin^{-1}(cx))}{3d}$$

[Out]  $2/3*(d*x)^{(3/2)}*(a+b*\arcsin(c*x))^{2/d}-8/15*b*c*(d*x)^{(5/2)}*(a+b*\arcsin(c*x))$   
 $*\text{hypergeom}([1/2, 5/4], [9/4], c^2*x^2)/d^2+16/105*b^2*c^2*(d*x)^{(7/2)}*\text{Hyperg}$   
 $\text{eometricPFQ}([1, 7/4, 7/4], [9/4, 11/4], c^2*x^2)/d^3$

**Rubi [A]** time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {4627, 4711}

$$\frac{16b^2c^2(dx)^{7/2} {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; c^2x^2\right)}{105d^3} - \frac{8bc(dx)^{5/2} {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{9}{4}; c^2x^2\right)(a + b \sin^{-1}(cx))}{15d^2} + \frac{2(dx)^{3/2}(a + b \sin^{-1}(cx))}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])^2,x]

[Out]  $(2*(d*x)^{(3/2)}*(a + b*\text{ArcSin}[c*x])^2)/(3*d) - (8*b*c*(d*x)^{(5/2)}*(a + b*\text{ArcSin}[c*x])*$   
 $\text{Hypergeometric2F1}[1/2, 5/4, 9/4, c^2*x^2])/(15*d^2) + (16*b^2*c^2*(d*x)^{(7/2)}*$   
 $\text{HypergeometricPFQ}[\{1, 7/4, 7/4\}, \{9/4, 11/4\}, c^2*x^2])/(105*d^3)$

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol]  
 $\rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^n/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2], x], x] /;$  FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4711

Int((((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol]  
 $\rightarrow \text{Simp}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2])/(Sqrt[d]*f*(m+1)), x] - \text{Simp}[(b*c*(f*x)^{(m+2)}*\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2*x^2])/(Sqrt[d]*f^2*(m+1)*(m+2)), x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]

#### Rubi steps

$$\int \sqrt{dx} (a + b \sin^{-1}(cx))^2 dx = \frac{2(dx)^{3/2}(a + b \sin^{-1}(cx))^2}{3d} - \frac{(4bc) \int \frac{(dx)^{3/2}(a + b \sin^{-1}(cx))}{\sqrt{1-c^2x^2}} dx}{3d}$$

$$= \frac{2(dx)^{3/2}(a + b \sin^{-1}(cx))^2}{3d} - \frac{8bc(dx)^{5/2}(a + b \sin^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{9}{4}; c^2x^2\right)}{15d^2} +$$

**Mathematica [A]** time = 0.05, size = 90, normalized size = 0.83

$$\frac{2}{105}x\sqrt{dx} \left(8b^2c^2x^2 {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; c^2x^2\right) + 7(a + b \sin^{-1}(cx)) \left(5(a + b \sin^{-1}(cx)) - 4bcx {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{9}{4}; c^2x^2\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])^2,x]

[Out] (2\*x\*Sqrt[d\*x]\*(7\*(a + b\*ArcSin[c\*x])\*(5\*(a + b\*ArcSin[c\*x]) - 4\*b\*c\*x\*Hypergeometric2F1[1/2, 5/4, 9/4, c^2\*x^2]) + 8\*b^2\*c^2\*x^2\*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2\*x^2]))/105

**fricas** [F] time = 1.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral((b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2)\*sqrt(d\*x), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \sqrt{dx} (a + b \arcsin(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^2,x)

[Out] int((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3} b^2 \sqrt{d} x^{\frac{3}{2}} \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^2 + \frac{1}{6} a^2 c^2 \sqrt{d} \left( \frac{4x^{\frac{3}{2}}}{c^2} + \frac{6 \arctan(\sqrt{c} \sqrt{x})}{c^2} + \frac{3 \log\left(\frac{c\sqrt{x}-\sqrt{c}}{c\sqrt{x}+\sqrt{c}}\right)}{c^2} \right) + 6 abc^2 \sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] 2/3\*b^2\*sqrt(d)\*x^(3/2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^2 + 1/6\*a^2\*c^2\*sqrt(d)\*(4\*x^(3/2)/c^2 + 6\*arctan(sqrt(c)\*sqrt(x))/c^(7/2) + 3\*log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(7/2)) + 6\*a\*b\*c^2\*sqrt(d)\*integrate(1/3\*x^(5/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) + 4\*b^2\*c\*sqrt(d)\*integrate(1/3\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^(3/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) - 1/2\*a^2\*sqrt(d)\*(2\*arctan(sqrt(c)\*sqrt(x))/c^(3/2) + log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(3/2)) - 6\*a\*b\*sqrt(d)\*integrate(1/3\*sqrt(x)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \arcsin(cx))^2 \sqrt{dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(c*x))^2*(d*x)^(1/2), x)
```

```
[Out] int((a + b*asin(c*x))^2*(d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx} (a + b \operatorname{asin}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*asin(c*x))**2, x)
```

```
[Out] Integral(sqrt(d*x)*(a + b*asin(c*x))**2, x)
```

$$3.212 \quad \int \frac{(a+b \sin^{-1}(cx))^2}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=107

$$\frac{16b^2c^2(dx)^{5/2} {}_3F_2\left(1, \frac{5}{4}, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; c^2x^2\right)}{15d^3} - \frac{8bc(dx)^{3/2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^2x^2\right)(a+b \sin^{-1}(cx))}{3d^2} + \frac{2\sqrt{dx} (a+b \sin^{-1}(cx))^2}{d}$$

[Out]  $-8/3*b*c*(d*x)^{(3/2)}*(a+b*\arcsin(c*x))*\text{hypergeom}([1/2, 3/4], [7/4], c^2*x^2)/d^2+16/15*b^2*c^2*(d*x)^{(5/2)}*\text{HypergeometricPFQ}([1, 5/4, 5/4], [7/4, 9/4], c^2*x^2)/d^3+2*(a+b*\arcsin(c*x))^2*(d*x)^{(1/2)}/d$

**Rubi [A]** time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {4627, 4711}

$$\frac{16b^2c^2(dx)^{5/2} {}_3F_2\left(1, \frac{5}{4}, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; c^2x^2\right)}{15d^3} - \frac{8bc(dx)^{3/2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^2x^2\right)(a+b \sin^{-1}(cx))}{3d^2} + \frac{2\sqrt{dx} (a+b \sin^{-1}(cx))^2}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^2/Sqrt[d\*x], x]

[Out]  $(2*\text{Sqrt}[d*x]*(a + b*\text{ArcSin}[c*x])^2)/d - (8*b*c*(d*x)^{(3/2)}*(a + b*\text{ArcSin}[c*x])*\text{Hypergeometric2F1}[1/2, 3/4, 7/4, c^2*x^2])/(3*d^2) + (16*b^2*c^2*(d*x)^{(5/2)}*\text{HypergeometricPFQ}[\{1, 5/4, 5/4\}, \{7/4, 9/4\}, c^2*x^2])/(15*d^3)$

#### Rule 4627

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 4711

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*ArcSin[c\*x])\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/(Sqrt[d]\*f\*(m+1)), x] - Simp[(b\*c\*(f\*x)^(m+2)\*HypergeometricPFQ[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2\*x^2])/(Sqrt[d]\*f^2\*(m+1)\*(m+2)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \sin^{-1}(cx))^2}{\sqrt{dx}} dx &= \frac{2\sqrt{dx} (a+b \sin^{-1}(cx))^2}{d} - \frac{(4bc) \int \frac{\sqrt{dx} (a+b \sin^{-1}(cx))}{\sqrt{1-c^2x^2}} dx}{d} \\ &= \frac{2\sqrt{dx} (a+b \sin^{-1}(cx))^2}{d} - \frac{8bc(dx)^{3/2} (a+b \sin^{-1}(cx)) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^2x^2\right)}{3d^2} + \frac{16b^2c^2(dx)^{5/2} {}_3F_2\left(1, \frac{5}{4}, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; c^2x^2\right)}{15d^3} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 90, normalized size = 0.84

$$\frac{2x \left( 8b^2c^2x^2 {}_3F_2\left(1, \frac{5}{4}, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; c^2x^2\right) + 5(a+b \sin^{-1}(cx)) \left( 3(a+b \sin^{-1}(cx)) - 4bcx {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; c^2x^2\right) \right) \right)}{15\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^2/Sqrt[d\*x], x]

[Out] (2\*x\*(5\*(a + b\*ArcSin[c\*x])\*(3\*(a + b\*ArcSin[c\*x]) - 4\*b\*c\*x\*Hypergeometric2F1[1/2, 3/4, 7/4, c^2\*x^2]) + 8\*b^2\*c^2\*x^2\*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2\*x^2]))/(15\*Sqrt[d\*x])

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b^2 \arcsin(cx))^2 + 2ab \arcsin(cx) + a^2}{\sqrt{dx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(1/2), x, algorithm="fricas")

[Out] integral((b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2)\*sqrt(d\*x)/(d\*x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^2}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^2/sqrt(d\*x), x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^2/(d\*x)^(1/2), x)

[Out] int((a+b\*arcsin(c\*x))^2/(d\*x)^(1/2), x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(1/2), x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^2/(d\*x)^(1/2), x)

[Out] int((a + b\*asin(c\*x))^2/(d\*x)^(1/2), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*2/(d\*x)\*\*(1/2),x)

[Out] Exception raised: TypeError



$$3.213 \quad \int \frac{(a+b \sin^{-1}(cx))^2}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=105

$$\frac{16b^2c^2(dx)^{3/2} {}_3F_2\left(\frac{3}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{7}{4}; c^2x^2\right)}{3d^3} + \frac{8bc\sqrt{dx} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^2x^2\right)(a+b \sin^{-1}(cx))}{d^2} - \frac{2(a+b \sin^{-1}(cx))^2}{d\sqrt{dx}}$$

[Out]  $-16/3*b^2*c^2*(d*x)^{(3/2)}*HypergeometricPFQ([3/4, 3/4, 1], [5/4, 7/4], c^2*x^2)/d^3-2*(a+b*\arcsin(c*x))^2/d/(d*x)^{(1/2)}+8*b*c*(a+b*\arcsin(c*x))*hypergeom([1/4, 1/2], [5/4], c^2*x^2)*(d*x)^{(1/2)}/d^2$

**Rubi [A]** time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {4627, 4711}

$$\frac{16b^2c^2(dx)^{3/2} {}_3F_2\left(\frac{3}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{7}{4}; c^2x^2\right)}{3d^3} + \frac{8bc\sqrt{dx} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^2x^2\right)(a+b \sin^{-1}(cx))}{d^2} - \frac{2(a+b \sin^{-1}(cx))^2}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^2/(d\*x)^(3/2), x]

[Out]  $(-2*(a + b*\text{ArcSin}[c*x])^2)/(d*\text{Sqrt}[d*x]) + (8*b*c*\text{Sqrt}[d*x]*(a + b*\text{ArcSin}[c*x])*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2])/d^2 - (16*b^2*c^2*(d*x)^{(3/2)}*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(3*d^3)$

**Rule 4627**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)]^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcSin[c\*x])^(n-1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

**Rule 4711**

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))\*((f\_.)\*(x\_))^(m\_)]/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*ArcSin[c\*x])\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/(Sqrt[d]\*f\*(m+1)), x] - Simp[(b\*c\*(f\*x)^(m+2)\*HypergeometricPFQ[{1, 1+m/2, 1+m/2}, {3/2+m/2, 2+m/2}, c^2\*x^2])/(Sqrt[d]\*f^2\*(m+1)\*(m+2)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]

**Rubi steps**

$$\begin{aligned} \int \frac{(a+b \sin^{-1}(cx))^2}{(dx)^{3/2}} dx &= -\frac{2(a+b \sin^{-1}(cx))^2}{d\sqrt{dx}} + \frac{(4bc) \int \frac{a+b \sin^{-1}(cx)}{\sqrt{dx} \sqrt{1-c^2x^2}} dx}{d} \\ &= -\frac{2(a+b \sin^{-1}(cx))^2}{d\sqrt{dx}} + \frac{8bc\sqrt{dx} (a+b \sin^{-1}(cx)) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^2x^2\right)}{d^2} - \frac{16b^2c^2(dx)^{3/2}}{3d^3} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 87, normalized size = 0.83

$$\frac{2x \left( 8b^2c^2x^2 {}_3F_2\left(\frac{3}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{7}{4}; c^2x^2\right) + 3(a+b \sin^{-1}(cx)) \left( a - 4bcx {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; c^2x^2\right) + b \sin^{-1}(cx) \right) \right)}{3(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^2/(d\*x)^(3/2), x]

[Out] (-2\*x\*(3\*(a + b\*ArcSin[c\*x])\*(a + b\*ArcSin[c\*x] - 4\*b\*c\*x\*Hypergeometric2F1[1/4, 1/2, 5/4, c^2\*x^2]) + 8\*b^2\*c^2\*x^2\*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2\*x^2]))/(3\*(d\*x)^(3/2))

**fricas** [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b^2 \arcsin(cx))^2 + 2ab \arcsin(cx) + a^2}{d^2 x^2} \sqrt{dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(3/2), x, algorithm="fricas")

[Out] integral((b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2)\*sqrt(d\*x)/(d^2\*x^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^2}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(3/2), x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^2/(d\*x)^(3/2), x)

**maple** [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^2}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^2/(d\*x)^(3/2), x)

[Out] int((a+b\*arcsin(c\*x))^2/(d\*x)^(3/2), x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(3/2), x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \arcsin(cx))^2}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^2/(d\*x)^(3/2), x)

[Out] int((a + b\*asin(c\*x))^2/(d\*x)^(3/2), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*2/(d\*x)\*\*(3/2),x)

[Out] Exception raised: TypeError

$$3.214 \quad \int \frac{(a+b \sin^{-1}(cx))^2}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=109

$$\frac{16b^2c^2\sqrt{dx} {}_3F_2\left(\frac{1}{4}, \frac{1}{4}, 1; \frac{3}{4}, \frac{5}{4}; c^2x^2\right)}{3d^3} - \frac{8bc {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; c^2x^2\right)(a+b \sin^{-1}(cx))}{3d^2\sqrt{dx}} - \frac{2(a+b \sin^{-1}(cx))^2}{3d(dx)^{3/2}}$$

[Out]  $-2/3*(a+b*\arcsin(c*x))^2/d/(d*x)^{(3/2)}-8/3*b*c*(a+b*\arcsin(c*x))*\text{hypergeom}([-1/4, 1/2], [3/4], c^2*x^2)/d^2/(d*x)^{(1/2)}+16/3*b^2*c^2*\text{HypergeometricPFQ}([1/4, 1/4, 1], [3/4, 5/4], c^2*x^2)*(d*x)^{(1/2)}/d^3$

**Rubi [A]** time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {4627, 4711}

$$\frac{16b^2c^2\sqrt{dx} {}_3F_2\left(\frac{1}{4}, \frac{1}{4}, 1; \frac{3}{4}, \frac{5}{4}; c^2x^2\right)}{3d^3} - \frac{8bc {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; c^2x^2\right)(a+b \sin^{-1}(cx))}{3d^2\sqrt{dx}} - \frac{2(a+b \sin^{-1}(cx))^2}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSin[c\*x])^2/(d\*x)^(5/2), x]

[Out]  $(-2*(a + b*\text{ArcSin}[c*x])^2)/(3*d*(d*x)^{(3/2)}) - (8*b*c*(a + b*\text{ArcSin}[c*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, c^2*x^2])/(3*d^2*\text{Sqrt}[d*x]) + (16*b^2*c^2*\text{Sqrt}[d*x]*HypergeometricPFQ[\{1/4, 1/4, 1\}, \{3/4, 5/4\}, c^2*x^2])/(3*d^3)$

**Rule 4627**

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

**Rule 4711**

Int[(((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.))/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*ArcSin[c\*x])\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2\*x^2])/(Sqrt[d]\*f\*(m + 1)), x] - Simp[(b\*c\*(f\*x)^(m + 2)\*HypergeometricPFQ[\{1, 1 + m/2, 1 + m/2\}, \{3/2 + m/2, 2 + m/2\}, c^2\*x^2])/(Sqrt[d]\*f^2\*(m + 1)\*(m + 2)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2\*d + e, 0] && GtQ[d, 0] && !IntegerQ[m]

**Rubi steps**

$$\begin{aligned} \int \frac{(a+b \sin^{-1}(cx))^2}{(dx)^{5/2}} dx &= -\frac{2(a+b \sin^{-1}(cx))^2}{3d(dx)^{3/2}} + \frac{(4bc) \int \frac{a+b \sin^{-1}(cx)}{(dx)^{3/2}\sqrt{1-c^2x^2}} dx}{3d} \\ &= -\frac{2(a+b \sin^{-1}(cx))^2}{3d(dx)^{3/2}} - \frac{8bc(a+b \sin^{-1}(cx)) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; c^2x^2\right)}{3d^2\sqrt{dx}} + \frac{16b^2c^2\sqrt{dx} {}_3F_2\left(\frac{1}{4}, \frac{1}{4}, 1; \frac{3}{4}, \frac{5}{4}; c^2x^2\right)}{3d^3} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 87, normalized size = 0.80

$$\frac{x\left(16b^2c^2x^2 {}_3F_2\left(\frac{1}{4}, \frac{1}{4}, 1; \frac{3}{4}, \frac{5}{4}; c^2x^2\right) - 2(a+b \sin^{-1}(cx))\left(a+4bcx {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; c^2x^2\right) + b \sin^{-1}(cx)\right)\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSin[c\*x])^2/(d\*x)^(5/2), x]

[Out] (x\*(-2\*(a + b\*ArcSin[c\*x])\*(a + b\*ArcSin[c\*x] + 4\*b\*c\*x\*Hypergeometric2F1[-1/4, 1/2, 3/4, c^2\*x^2]) + 16\*b^2\*c^2\*x^2\*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, c^2\*x^2]))/(3\*(d\*x)^(5/2))

**fricas** [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b^2 \arcsin(cx))^2 + 2ab \arcsin(cx) + a^2}{d^3 x^3} \sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(5/2), x, algorithm="fricas")

[Out] integral((b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2)\*sqrt(d\*x)/(d^3\*x^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^2}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(5/2), x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^2/(d\*x)^(5/2), x)

**maple** [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^2}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^2/(d\*x)^(5/2), x)

[Out] int((a+b\*arcsin(c\*x))^2/(d\*x)^(5/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left( 3 a^2 c^2 \sqrt{d} \left( \frac{2 \arctan(\sqrt{c} \sqrt{x})}{\sqrt{c} d^3} - \frac{\log\left(\frac{c \sqrt{x} - \sqrt{c}}{c \sqrt{x} + \sqrt{c}}\right)}{\sqrt{c} d^3} \right) - 12 a b c^2 \sqrt{d} \int \frac{x^2 \arctan\left(\frac{c x}{\sqrt{c x + 1} \sqrt{-c x + 1}}\right)}{c^2 d^3 x^5 - d^3 x^3} dx + 8 b^2 c \sqrt{d} \int \frac{\sqrt{c x + 1} \sqrt{-c x + 1} x^{\frac{3}{2}}}{c^2 d^3} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^2/(d\*x)^(5/2), x, algorithm="maxima")

[Out] -1/6\*((3\*a^2\*c^2\*sqrt(d)\*(2\*arctan(sqrt(c)\*sqrt(x))/(sqrt(c)\*d^3) - log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/(sqrt(c)\*d^3)) - 36\*a\*b\*c^2\*sqrt(d)\*integrate(1/3\*x^(5/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*d^3\*x^5 - d^3\*x^3), x) + 24\*b^2\*c\*sqrt(d)\*integrate(1/3\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^(3/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*d^3\*x^5 - d^3\*x^3), x) - a^2\*sqrt(d)\*(6\*c^(3/2)\*arctan(sqrt(c)\*sqrt(x))/d^3 - 3\*c^(3/2)\*log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/d^3 - 4/(d^3\*x^(3/2))) + 36\*a\*b\*sqrt(d)\*integrate(1/3\*sqrt(x)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*d^3\*x^5 - d^3\*x^3), x)

```
1))))/(c^2*d^3*x^5 - d^3*x^3), x))*d^(5/2)*x^(3/2) + 4*b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2)/(d^(5/2)*x^(3/2))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asin}(c x))^2}{(d x)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(c*x))^2/(d*x)^(5/2), x)
```

```
[Out] int((a + b*asin(c*x))^2/(d*x)^(5/2), x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asin(c*x))**2/(d*x)**(5/2), x)
```

```
[Out] Exception raised: TypeError
```

$$3.215 \quad \int (dx)^{3/2} \left( a + b \sin^{-1}(cx) \right)^3 dx$$

Optimal. Leaf size=69

$$\frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))^3}{5d} - \frac{6bc \operatorname{Int} \left( \frac{(dx)^{5/2} (a + b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2}}, x \right)}{5d}$$

[Out]  $2/5*(d*x)^{(5/2)}*(a+b*\arcsin(c*x))^{3/d}-6/5*b*c*\operatorname{Unintegrable}((d*x)^{(5/2)}*(a+b*\arcsin(c*x))^{2/(-c^2*x^2+1)^{(1/2)},x)/d$

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (dx)^{3/2} \left( a + b \sin^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(d*x)^{(3/2)}*(a + b*\operatorname{ArcSin}[c*x])^3, x]$

[Out]  $(2*(d*x)^{(5/2)}*(a + b*\operatorname{ArcSin}[c*x])^3)/(5*d) - (6*b*c*\operatorname{Defer}[\operatorname{Int}[(d*x)^{(5/2)}*(a + b*\operatorname{ArcSin}[c*x])^2]/\operatorname{Sqrt}[1 - c^2*x^2], x])/(5*d)$

Rubi steps

$$\int (dx)^{3/2} \left( a + b \sin^{-1}(cx) \right)^3 dx = \frac{2(dx)^{5/2} (a + b \sin^{-1}(cx))^3}{5d} - \frac{(6bc) \int \frac{(dx)^{5/2} (a + b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2}} dx}{5d}$$

Mathematica [A] time = 39.37, size = 0, normalized size = 0.00

$$\int (dx)^{3/2} \left( a + b \sin^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(d*x)^{(3/2)}*(a + b*\operatorname{ArcSin}[c*x])^3, x]$

[Out]  $\operatorname{Integrate}[(d*x)^{(3/2)}*(a + b*\operatorname{ArcSin}[c*x])^3, x]$

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \left( b^3 dx \arcsin(cx)^3 + 3ab^2 dx \arcsin(cx)^2 + 3a^2b dx \arcsin(cx) + a^3 dx \right) \sqrt{dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((d*x)^{(3/2)}*(a+b*\arcsin(c*x))^3, x, \text{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b^3*d*x*\arcsin(c*x)^3 + 3*a*b^2*d*x*\arcsin(c*x)^2 + 3*a^2*b*d*x*\arcsin(c*x) + a^3*d*x)*\operatorname{sqrt}(d*x), x)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((d*x)^{(3/2)}*(a+b*\arcsin(c*x))^3, x, \text{algorithm}="giac")$

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vect  
 eur & l) Error: Bad Argument Value

**maple** [A] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} (a + b \arcsin(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^3,x)

[Out] int((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{5} b^3 d^{\frac{3}{2}} x^{\frac{5}{2}} \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^3 + \frac{1}{10} a^3 c^2 d^{\frac{3}{2}} \left( \frac{4\left(c^2 x^{\frac{5}{2}} + 5\sqrt{x}\right)}{c^4} - \frac{10 \arctan\left(\sqrt{c}\sqrt{x}\right)}{c^{\frac{9}{2}}} + \frac{5 \log\left(\frac{c\sqrt{x}-\sqrt{c}}{c\sqrt{x}+\sqrt{c}}\right)}{c^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out] 2/5\*b^3\*d^(3/2)\*x^(5/2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^3 + 1/10  
 \*a^3\*c^2\*d^(3/2)\*(4\*(c^2\*x^(5/2) + 5\*sqrt(x))/c^4 - 10\*arctan(sqrt(c)\*sqrt(x))/c^(9/2) + 5\*log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(9/2)) +  
 15\*a\*b^2\*c^2\*d^(3/2)\*integrate(1/5\*x^(7/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))^2/(c^2\*x^2 - 1), x) + 15\*a^2\*b\*c^2\*d^(3/2)\*integrate(1/5\*x^(7/2)  
 )\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) + 6\*b^3\*c\*d^(3/2)\*integrate(1/5\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^(5/2)\*arctan(c\*x/(sqrt(c  
 \*x + 1)\*sqrt(-c\*x + 1)))^2/(c^2\*x^2 - 1), x) - 1/2\*a^3\*d^(3/2)\*(4\*sqrt(x)/c^2 - 2\*arctan(sqrt(c)\*sqrt(x))/c^(5/2) + log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x)  
 + sqrt(c)))/c^(5/2)) - 15\*a\*b^2\*d^(3/2)\*integrate(1/5\*x^(3/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))^2/(c^2\*x^2 - 1), x) - 15\*a^2\*b\*d^(3/2)\*int  
 egrate(1/5\*x^(3/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asin}(cx))^3 (dx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^3\*(d\*x)^(3/2),x)

[Out] int((a + b\*asin(c\*x))^3\*(d\*x)^(3/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} (a + b \operatorname{asin}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*(a+b\*asin(c\*x))\*\*3,x)

[Out] Integral((d\*x)\*\*(3/2)\*(a + b\*asin(c\*x))\*\*3, x)



$$3.216 \quad \int \sqrt{dx} \left( a + b \sin^{-1}(cx) \right)^3 dx$$

Optimal. Leaf size=67

$$\frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))^3}{3d} - \frac{2bc \operatorname{Int} \left( \frac{(dx)^{3/2} (a + b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2}}, x \right)}{d}$$

[Out]  $2/3*(d*x)^{(3/2)}*(a+b*\arcsin(c*x))^{3/d}-2*b*c*\operatorname{Unintegrable}((d*x)^{(3/2)}*(a+b*\arcsin(c*x))^{2/(-c^2*x^2+1)^{(1/2)},x)/d$

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \sqrt{dx} \left( a + b \sin^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] `Int[Sqrt[d*x]*(a + b*ArcSin[c*x])^3,x]`

[Out]  $(2*(d*x)^{(3/2)}*(a + b*\operatorname{ArcSin}[c*x])^3)/(3*d) - (2*b*c*\operatorname{Defer}[\operatorname{Int}][((d*x)^{(3/2)}*(a + b*\operatorname{ArcSin}[c*x])^2)/\operatorname{Sqrt}[1 - c^2*x^2], x])/d$

Rubi steps

$$\int \sqrt{dx} \left( a + b \sin^{-1}(cx) \right)^3 dx = \frac{2(dx)^{3/2} (a + b \sin^{-1}(cx))^3}{3d} - \frac{(2bc) \int \frac{(dx)^{3/2} (a + b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2}} dx}{d}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] `Integrate[Sqrt[d*x]*(a + b*ArcSin[c*x])^3,x]`

[Out] \$Aborted

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( (b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3) \sqrt{dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*arcsin(c*x))^3,x, algorithm="fricas")`

[Out] `integral((b^3*arcsin(c*x)^3 + 3*a*b^2*arcsin(c*x)^2 + 3*a^2*b*arcsin(c*x) + a^3)*sqrt(d*x), x)`

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*arcsin(c*x))^3,x, algorithm="giac")`

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vect  
 eur & l) Error: Bad Argument Value

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \sqrt{dx} (a + b \arcsin(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^3,x)

[Out] int((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3} b^3 \sqrt{d} x^{\frac{3}{2}} \arctan\left(cx, \sqrt{cx+1} \sqrt{-cx+1}\right)^3 + \frac{1}{6} a^3 c^2 \sqrt{d} \left( \frac{4x^{\frac{3}{2}}}{c^2} + \frac{6 \arctan(\sqrt{c} \sqrt{x})}{c^{\frac{7}{2}}} + \frac{3 \log\left(\frac{c\sqrt{x}-\sqrt{c}}{c\sqrt{x}+\sqrt{c}}\right)}{c^{\frac{7}{2}}} \right) + 3 ab^2 c^2 \sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arcsin(c\*x))^3,x, algorithm="maxima")

[Out] 2/3\*b^3\*sqrt(d)\*x^(3/2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))^3 + 1/6\*a^3\*c^2\*sqrt(d)\*(4\*x^(3/2)/c^2 + 6\*arctan(sqrt(c)\*sqrt(x))/c^(7/2) + 3\*log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(7/2)) + 3\*a\*b^2\*c^2\*sqrt(d)\*integrate(x^(5/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))^2/(c^2\*x^2 - 1), x) + 3\*a^2\*b\*c^2\*sqrt(d)\*integrate(x^(5/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x) + 2\*b^3\*c\*sqrt(d)\*integrate(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^(3/2)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))^2/(c^2\*x^2 - 1), x) - 1/2\*a^3\*sqrt(d)\*(2\*arctan(sqrt(c)\*sqrt(x))/c^(3/2) + log((c\*sqrt(x) - sqrt(c))/(c\*sqrt(x) + sqrt(c)))/c^(3/2)) - 3\*a\*b^2\*sqrt(d)\*integrate(sqrt(x)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))^2/(c^2\*x^2 - 1), x) - 3\*a^2\*b\*sqrt(d)\*integrate(sqrt(x)\*arctan(c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)))/(c^2\*x^2 - 1), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asin}(cx))^3 \sqrt{dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^3\*(d\*x)^(1/2),x)

[Out] int((a + b\*asin(c\*x))^3\*(d\*x)^(1/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx} (a + b \operatorname{asin}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)\*(a+b\*asin(c\*x))\*\*3,x)

[Out] Integral(sqrt(d\*x)\*(a + b\*asin(c\*x))\*\*3, x)

$$3.217 \quad \int \frac{(a+b \sin^{-1}(cx))^3}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=65

$$\frac{2\sqrt{dx} (a+b \sin^{-1}(cx))^3}{d} - \frac{6bc \operatorname{Int}\left(\frac{\sqrt{dx} (a+b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2}}, x\right)}{d}$$

[Out]  $2*(a+b*\arcsin(c*x))^3*(d*x)^{(1/2)}/d-6*b*c*\operatorname{Unintegrable}((a+b*\arcsin(c*x))^2*(d*x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}, x)/d$

**Rubi [A]** time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sin^{-1}(cx))^3}{\sqrt{dx}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSin}[c*x])^3/\operatorname{Sqrt}[d*x], x]$

[Out]  $(2*\operatorname{Sqrt}[d*x]*(a + b*\operatorname{ArcSin}[c*x])^3)/d - (6*b*c*\operatorname{Defer}[\operatorname{Int}[(\operatorname{Sqrt}[d*x]*(a + b*\operatorname{ArcSin}[c*x])^2)/\operatorname{Sqrt}[1 - c^2*x^2], x])/d$

Rubi steps

$$\int \frac{(a+b \sin^{-1}(cx))^3}{\sqrt{dx}} dx = \frac{2\sqrt{dx} (a+b \sin^{-1}(cx))^3}{d} - \frac{(6bc) \int \frac{\sqrt{dx} (a+b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2}} dx}{d}$$

**Mathematica [A]** time = 9.01, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sin^{-1}(cx))^3}{\sqrt{dx}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{ArcSin}[c*x])^3/\operatorname{Sqrt}[d*x], x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{ArcSin}[c*x])^3/\operatorname{Sqrt}[d*x], x]$

**fricas [A]** time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3)\sqrt{dx}}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\arcsin(c*x))^3/(d*x)^{(1/2)}, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b^3*\arcsin(c*x)^3 + 3*a*b^2*\arcsin(c*x)^2 + 3*a^2*b*\arcsin(c*x) + a^3)*\operatorname{sqrt}(d*x)/(d*x), x)$

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^3}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsin(c*x))^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsin(c*x) + a)^3/sqrt(d*x), x)
```

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^3}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsin(c*x))^3/(d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsin(c*x))^3/(d*x)^(1/2),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsin(c*x))^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b \arcsin(cx))^3}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(c*x))^3/(d*x)^(1/2),x)
```

```
[Out] int((a + b*asin(c*x))^3/(d*x)^(1/2), x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asin(c*x))**3/(d*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

$$3.218 \quad \int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=65

$$\frac{6bc \operatorname{Int}\left(\frac{(a+b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2} \sqrt{dx}}, x\right)}{d} - \frac{2(a+b \sin^{-1}(cx))^3}{d\sqrt{dx}}$$

[Out]  $-2*(a+b*\arcsin(c*x))^3/d/(d*x)^{(1/2)}+6*b*c*\operatorname{Unintegrable}((a+b*\arcsin(c*x))^2/(d*x)^{(1/2)/(-c^2*x^2+1)^{(1/2)},x)/d$

**Rubi [A]** time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{3/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSin}[c*x])^3/(d*x)^{(3/2)}, x]$

[Out]  $(-2*(a + b*\operatorname{ArcSin}[c*x])^3)/(d*\operatorname{Sqrt}[d*x]) + (6*b*c*\operatorname{Defer}[\operatorname{Int}[(a + b*\operatorname{ArcSin}[c*x])^2/(\operatorname{Sqrt}[d*x]*\operatorname{Sqrt}[1 - c^2*x^2]), x])/d$

Rubi steps

$$\int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{3/2}} dx = -\frac{2(a+b \sin^{-1}(cx))^3}{d\sqrt{dx}} + \frac{(6bc) \int \frac{(a+b \sin^{-1}(cx))^2}{\sqrt{dx} \sqrt{1-c^2x^2}} dx}{d}$$

**Mathematica [A]** time = 8.44, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{3/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{ArcSin}[c*x])^3/(d*x)^{(3/2)}, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{ArcSin}[c*x])^3/(d*x)^{(3/2)}, x]$

**fricas [A]** time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3)\sqrt{dx}}{d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\arcsin(c*x))^3/(d*x)^{(3/2)}, x, \text{algorithm}=\text{"fricas"})$

[Out]  $\operatorname{integral}((b^3*\arcsin(c*x))^3 + 3*a*b^2*\arcsin(c*x)^2 + 3*a^2*b*\arcsin(c*x) + a^3)*\operatorname{sqrt}(d*x)/(d^2*x^2), x$

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^3}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/(d\*x)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^3/(d\*x)^(3/2), x)

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^3}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^3/(d\*x)^(3/2),x)

[Out] int((a+b\*arcsin(c\*x))^3/(d\*x)^(3/2),x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/(d\*x)^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b \arcsin(cx))^3}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^3/(d\*x)^(3/2),x)

[Out] int((a + b\*asin(c\*x))^3/(d\*x)^(3/2), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*3/(d\*x)\*\*(3/2),x)

[Out] Exception raised: TypeError

$$3.219 \quad \int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=67

$$\frac{2bc \operatorname{Int}\left(\frac{(a+b \sin^{-1}(cx))^2}{\sqrt{1-c^2x^2}(dx)^{3/2}}, x\right)}{d} - \frac{2(a+b \sin^{-1}(cx))^3}{3d(dx)^{3/2}}$$

[Out]  $-2/3*(a+b*\arcsin(c*x))^3/d/(d*x)^{(3/2)}+2*b*c*\operatorname{Unintegrable}((a+b*\arcsin(c*x))^2/(d*x)^{(3/2)/(-c^2*x^2+1)^{(1/2)}, x)/d$

**Rubi [A]** time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{5/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSin}[c*x])^3/(d*x)^{(5/2)}, x]$

[Out]  $(-2*(a + b*\operatorname{ArcSin}[c*x])^3)/(3*d*(d*x)^{(3/2)}) + (2*b*c*\operatorname{Defer}[\operatorname{Int}[(a + b*\operatorname{ArcSin}[c*x])^2/((d*x)^{(3/2})*\operatorname{Sqrt}[1 - c^2*x^2]), x])/d$

Rubi steps

$$\int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{5/2}} dx = -\frac{2(a+b \sin^{-1}(cx))^3}{3d(dx)^{3/2}} + \frac{(2bc) \int \frac{(a+b \sin^{-1}(cx))^2}{(dx)^{3/2}\sqrt{1-c^2x^2}} dx}{d}$$

**Mathematica [A]** time = 14.41, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sin^{-1}(cx))^3}{(dx)^{5/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{ArcSin}[c*x])^3/(d*x)^{(5/2)}, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{ArcSin}[c*x])^3/(d*x)^{(5/2)}, x]$

**fricas [A]** time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b^3 \arcsin(cx)^3 + 3ab^2 \arcsin(cx)^2 + 3a^2b \arcsin(cx) + a^3)\sqrt{dx}}{d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\arcsin(c*x))^3/(d*x)^{(5/2)}, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b^3*\arcsin(c*x)^3 + 3*a*b^2*\arcsin(c*x)^2 + 3*a^2*b*\arcsin(c*x) + a^3)*\operatorname{sqrt}(d*x)/(d^3*x^3), x)$

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \arcsin(cx) + a)^3}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/(d\*x)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsin(c\*x) + a)^3/(d\*x)^(5/2), x)

**maple** [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a + b \arcsin(cx))^3}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsin(c\*x))^3/(d\*x)^(5/2),x)

[Out] int((a+b\*arcsin(c\*x))^3/(d\*x)^(5/2),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsin(c\*x))^3/(d\*x)^(5/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \arcsin(cx))^3}{(dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asin(c\*x))^3/(d\*x)^(5/2),x)

[Out] int((a + b\*asin(c\*x))^3/(d\*x)^(5/2), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asin(c\*x))\*\*3/(d\*x)\*\*(5/2),x)

[Out] Exception raised: TypeError



$$3.220 \quad \int \frac{(dx)^{3/2}}{a+b \sin^{-1}(cx)} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^{3/2}}{a+b \sin^{-1}(cx)}, x\right)$$

[Out] Unintegrable((d\*x)^(3/2)/(a+b\*arcsin(c\*x)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(dx)^{3/2}}{a+b \sin^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^(3/2)/(a + b\*ArcSin[c\*x]), x]

[Out] Defer[Int] [(d\*x)^(3/2)/(a + b\*ArcSin[c\*x]), x]

Rubi steps

$$\int \frac{(dx)^{3/2}}{a+b \sin^{-1}(cx)} dx = \int \frac{(dx)^{3/2}}{a+b \sin^{-1}(cx)} dx$$

Mathematica [A] time = 3.07, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{3/2}}{a+b \sin^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^(3/2)/(a + b\*ArcSin[c\*x]), x]

[Out] Integrate[(d\*x)^(3/2)/(a + b\*ArcSin[c\*x]), x]

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx} dx}{b \arcsin(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d\*x/(b\*arcsin(c\*x) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)/(b\*arcsin(c\*x) + a), x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{a + b \arcsin(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a+b\*arcsin(c\*x)),x)

[Out] int((d\*x)^(3/2)/(a+b\*arcsin(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate((d\*x)^(3/2)/(b\*arcsin(c\*x) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(dx)^{\frac{3}{2}}}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a + b\*asin(c\*x)),x)

[Out] int((d\*x)^(3/2)/(a + b\*asin(c\*x)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)/(a+b\*asin(c\*x)),x)

[Out] Integral((d\*x)\*\*(3/2)/(a + b\*asin(c\*x)), x)

$$3.221 \quad \int \frac{\sqrt{dx}}{a+b \sin^{-1}(cx)} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sqrt{dx}}{a+b \sin^{-1}(cx)}, x\right)$$

[Out] Unintegrable((d\*x)^(1/2)/(a+b\*arcsin(c\*x)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{dx}}{a+b \sin^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[d\*x]/(a + b\*ArcSin[c\*x]), x]

[Out] Defer[Int][Sqrt[d\*x]/(a + b\*ArcSin[c\*x]), x]

Rubi steps

$$\int \frac{\sqrt{dx}}{a+b \sin^{-1}(cx)} dx = \int \frac{\sqrt{dx}}{a+b \sin^{-1}(cx)} dx$$

Mathematica [A] time = 2.90, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{a+b \sin^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[d\*x]/(a + b\*ArcSin[c\*x]), x]

[Out] Integrate[Sqrt[d\*x]/(a + b\*ArcSin[c\*x]), x]

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{b \arcsin(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*arcsin(c\*x) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] integrate(sqrt(d\*x)/(b\*arcsin(c\*x) + a), x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{a + b \arcsin(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a+b\*arcsin(c\*x)),x)

[Out] int((d\*x)^(1/2)/(a+b\*arcsin(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{b \arcsin(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(sqrt(d\*x)/(b\*arcsin(c\*x) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{dx}}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a + b\*asin(c\*x)),x)

[Out] int((d\*x)^(1/2)/(a + b\*asin(c\*x)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{a + b \operatorname{asin}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)/(a+b\*asin(c\*x)),x)

[Out] Integral(sqrt(d\*x)/(a + b\*asin(c\*x)), x)

$$3.222 \quad \int \frac{1}{\sqrt{dx} (a+b \sin^{-1}(cx))} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{\sqrt{dx} (a + b \sin^{-1}(cx))}, x\right)$$

[Out] Unintegrable(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\sqrt{dx} (a + b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])), x]

[Out] Defer[Int][1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])), x]

Rubi steps

$$\int \frac{1}{\sqrt{dx} (a + b \sin^{-1}(cx))} dx = \int \frac{1}{\sqrt{dx} (a + b \sin^{-1}(cx))} dx$$

Mathematica [A] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a + b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])), x]

[Out] Integrate[1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])), x]

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{bdx \arcsin(cx) + adx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d\*x\*arcsin(c\*x) + a\*d\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (b \arcsin(cx) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] integrate(1/(sqrt(d\*x)\*(b\*arcsin(c\*x) + a)), x)

**maple** [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a + b \arcsin(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x)),x)

[Out] int(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (b \arcsin(cx) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(d\*x)\*(b\*arcsin(c\*x) + a)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b \operatorname{asin}(cx)) \sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*asin(c\*x))\*(d\*x)^(1/2)),x)

[Out] int(1/((a + b\*asin(c\*x))\*(d\*x)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a + b \operatorname{asin}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(1/2)/(a+b\*asin(c\*x)),x)

[Out] Integral(1/(sqrt(d\*x)\*(a + b\*asin(c\*x))), x)

$$3.223 \quad \int \frac{1}{(dx)^{3/2}(a+b \sin^{-1}(cx))} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(dx)^{3/2}(a+b \sin^{-1}(cx))}, x\right)$$

[Out] Unintegrable(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(dx)^{3/2}(a+b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])), x]

[Out] Defer[Int][1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])), x]

Rubi steps

$$\int \frac{1}{(dx)^{3/2}(a+b \sin^{-1}(cx))} dx = \int \frac{1}{(dx)^{3/2}(a+b \sin^{-1}(cx))} dx$$

Mathematica [A] time = 4.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{3/2}(a+b \sin^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])), x]

[Out] Integrate[1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])), x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{bd^2x^2 \arcsin(cx) + ad^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d^2\*x^2\*arcsin(c\*x) + a\*d^2\*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}(b \arcsin(cx) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x)), x, algorithm="giac")

[Out] integrate(1/((d\*x)^(3/2)\*(b\*arcsin(c\*x) + a)), x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \arcsin(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x)),x)

[Out] int(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((d\*x)^(3/2)\*(b\*arcsin(c\*x) + a)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b \operatorname{asin}(cx)) (dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*asin(c\*x))\*(d\*x)^(3/2)),x)

[Out] int(1/((a + b\*asin(c\*x))\*(d\*x)^(3/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \operatorname{asin}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(3/2)/(a+b\*asin(c\*x)),x)

[Out] Integral(1/((d\*x)\*\*(3/2)\*(a + b\*asin(c\*x))), x)



$$3.224 \quad \int \frac{(dx)^{3/2}}{(a+b \sin^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=21

$$\text{Int} \left( \frac{(dx)^{3/2}}{(a + b \sin^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable((d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2, x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(dx)^{3/2}}{(a + b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^(3/2)/(a + b\*ArcSin[c\*x])^2, x]

[Out] Defer[Int] [(d\*x)^(3/2)/(a + b\*ArcSin[c\*x])^2, x]

Rubi steps

$$\int \frac{(dx)^{3/2}}{(a + b \sin^{-1}(cx))^2} dx = \int \frac{(dx)^{3/2}}{(a + b \sin^{-1}(cx))^2} dx$$

**Mathematica [A]** time = 6.21, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{3/2}}{(a + b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^(3/2)/(a + b\*ArcSin[c\*x])^2, x]

[Out] Integrate[(d\*x)^(3/2)/(a + b\*ArcSin[c\*x])^2, x]

**fricas [A]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx} dx}{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2, x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d\*x/(b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)/(b\*arcsin(c\*x) + a)^2, x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x)

[Out] int((d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{cx+1}\sqrt{-cx+1}d^{\frac{3}{2}}x^{\frac{3}{2}} - \frac{1}{2}(b^2c \arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}) + abc)\sqrt{d} \int \frac{(5c^2dx^2-3d)\sqrt{cx+1}\sqrt{-cx+1}\sqrt{x}}{abc^3x^2-abc+(b^2c^3x^2-b^2c)\arctan(cx, \sqrt{cx+1}\sqrt{-cx+1})} dx}{b^2c \arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}) + abc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] -(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*d^(3/2)\*x^(3/2) - (b^2\*c\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b\*c)\*sqrt(d)\*integrate(1/2\*(5\*c^2\*d\*x^2 - 3\*d)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(x)/(a\*b\*c^3\*x^2 - a\*b\*c + (b^2\*c^3\*x^2 - b^2\*c)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))), x))/(b^2\*c\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b\*c)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(dx)^{\frac{3}{2}}}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a + b\*asin(c\*x))^2,x)

[Out] int((d\*x)^(3/2)/(a + b\*asin(c\*x))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)/(a+b\*asin(c\*x))\*\*2,x)

[Out] Integral((d\*x)\*\*(3/2)/(a + b\*asin(c\*x))\*\*2, x)

$$3.225 \quad \int \frac{\sqrt{dx}}{(a+b \sin^{-1}(cx))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{\sqrt{dx}}{(a+b \sin^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable((d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2, x)

**Rubi** [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{dx}}{(a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[d\*x]/(a + b\*ArcSin[c\*x])^2, x]

[Out] Defer[Int][Sqrt[d\*x]/(a + b\*ArcSin[c\*x])^2, x]

Rubi steps

$$\int \frac{\sqrt{dx}}{(a+b \sin^{-1}(cx))^2} dx = \int \frac{\sqrt{dx}}{(a+b \sin^{-1}(cx))^2} dx$$

**Mathematica** [A] time = 5.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[d\*x]/(a + b\*ArcSin[c\*x])^2, x]

[Out] Integrate[Sqrt[d\*x]/(a + b\*ArcSin[c\*x])^2, x]

**fricas** [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx}}{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2, x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*arcsin(c\*x)^2 + 2\*a\*b\*arcsin(c\*x) + a^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(b \arcsin(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] integrate(sqrt(d\*x)/(b\*arcsin(c\*x) + a)^2, x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x)

[Out] int((d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{2} (b^2 c \arctan(cx, \sqrt{cx+1} \sqrt{-cx+1}) + abc) \sqrt{d} \int \frac{(3c^2x^2-1)\sqrt{cx+1}\sqrt{-cx+1}\sqrt{x}}{abc^3x^3-abcx+(b^2c^3x^3-b^2cx)\arctan(cx,\sqrt{cx+1}\sqrt{-cx+1})} dx - \sqrt{cx+1}\sqrt{-cx+1}}{b^2c \arctan(cx, \sqrt{cx+1}\sqrt{-cx+1}) + abc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] ((b^2\*c\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b\*c)\*sqrt(d)\*integrate(1/2\*(3\*c^2\*x^2 - 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(x)/(a\*b\*c^3\*x^3 - a\*b\*c\*x + (b^2\*c^3\*x^3 - b^2\*c\*x)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))), x) - sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(d)\*sqrt(x)/(b^2\*c\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b\*c)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{dx}}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a + b\*asin(c\*x))^2,x)

[Out] int((d\*x)^(1/2)/(a + b\*asin(c\*x))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)/(a+b\*asin(c\*x))\*\*2,x)

[Out] Integral(sqrt(d\*x)/(a + b\*asin(c\*x))\*\*2, x)

$$3.226 \quad \int \frac{1}{\sqrt{dx} (a+b \sin^{-1}(cx))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{1}{\sqrt{dx} (a+b \sin^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{\sqrt{dx} (a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])^2),x]

[Out] Defer[Int][1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])^2), x]

Rubi steps

$$\int \frac{1}{\sqrt{dx} (a+b \sin^{-1}(cx))^2} dx = \int \frac{1}{\sqrt{dx} (a+b \sin^{-1}(cx))^2} dx$$

Mathematica [A] time = 9.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])^2),x]

[Out] Integrate[1/(Sqrt[d\*x]\*(a + b\*ArcSin[c\*x])^2), x]

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx}}{b^2 dx \arcsin(cx)^2 + 2 ab dx \arcsin(cx) + a^2 dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*d\*x\*arcsin(c\*x)^2 + 2\*a\*b\*d\*x\*arcsin(c\*x) + a^2\*d\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (b \arcsin(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] integrate(1/(sqrt(d\*x)\*(b\*arcsin(c\*x) + a)^2), x)

**maple** [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x)

[Out] int(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{2} \left( b^2 c d x \arctan \left( c x, \sqrt{c x + 1} \sqrt{-c x + 1} \right) + a b c d x \right) \sqrt{d} \int \frac{(c^2 x^2 + 1) \sqrt{c x + 1} \sqrt{-c x + 1} \sqrt{x}}{a b c^3 d x^4 - a b c d x^2 + (b^2 c^3 d x^4 - b^2 c d x^2) \arctan \left( c x, \sqrt{c x + 1} \sqrt{-c x + 1} \right)} dx - \sqrt{c}}{b^2 c d x \arctan \left( c x, \sqrt{c x + 1} \sqrt{-c x + 1} \right) + a b c d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] ((b^2\*c\*d\*x\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b\*c\*d\*x)\*sqrt(d) \*integrate(1/2\*(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(x)/(a\*b\*c^3\*d\*x^4 - a\*b\*c\*d\*x^2 + (b^2\*c^3\*d\*x^4 - b^2\*c\*d\*x^2)\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1))), x) - sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(d)\*sqrt(x))/(b^2\*c\*d\*x\*arctan2(c\*x, sqrt(c\*x + 1)\*sqrt(-c\*x + 1)) + a\*b\*c\*d\*x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b \operatorname{asin}(c x))^2 \sqrt{d x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*asin(c\*x))^2\*(d\*x)^(1/2)),x)

[Out] int(1/((a + b\*asin(c\*x))^2\*(d\*x)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(1/2)/(a+b\*asin(c\*x))\*\*2,x)

[Out] Integral(1/(sqrt(d\*x)\*(a + b\*asin(c\*x))\*\*2), x)

$$3.227 \quad \int \frac{1}{(dx)^{3/2} (a+b \sin^{-1}(cx))^2} dx$$

Optimal. Leaf size=21

$$\text{Int} \left( \frac{1}{(dx)^{3/2} (a+b \sin^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(dx)^{3/2} (a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])^2),x]

[Out] Defer[Int][1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])^2), x]

Rubi steps

$$\int \frac{1}{(dx)^{3/2} (a+b \sin^{-1}(cx))^2} dx = \int \frac{1}{(dx)^{3/2} (a+b \sin^{-1}(cx))^2} dx$$

Mathematica [A] time = 14.81, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{3/2} (a+b \sin^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])^2),x]

[Out] Integrate[1/((d\*x)^(3/2)\*(a + b\*ArcSin[c\*x])^2), x]

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx}}{b^2 d^2 x^2 \arcsin(cx)^2 + 2abd^2 x^2 \arcsin(cx) + a^2 d^2 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*d^2\*x^2\*arcsin(c\*x)^2 + 2\*a\*b\*d^2\*x^2\*arcsin(c\*x) + a^2\*d^2\*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x)^(3/2)\*(b\*arcsin(c\*x) + a)^2), x)

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x)

[Out] int(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{2} (b^2 c d^2 x^2 \arctan(cx, \sqrt{cx+1} \sqrt{-cx+1}) + abcd^2 x^2) \sqrt{d} \int \frac{(c^2 x^2 - 3) \sqrt{cx+1} \sqrt{-cx+1} \sqrt{x}}{abc^3 d^2 x^5 - abcd^2 x^3 + (b^2 c^3 d^2 x^5 - b^2 cd^2 x^3) \arctan(cx, \sqrt{cx+1} \sqrt{-cx+1})} dx}{b^2 cd^2 x^2 \arctan(cx, \sqrt{cx+1} \sqrt{-cx+1}) + abcd^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*arcsin(c\*x))^2,x, algorithm="maxima")

[Out] -((b^2\*c\*d^2\*x^2\*arctan2(c\*x, sqrt(c\*x + 1))\*sqrt(-c\*x + 1)) + a\*b\*c\*d^2\*x^2)\*sqrt(d)\*integrate(1/2\*(c^2\*x^2 - 3)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(x)/(a\*b\*c^3\*d^2\*x^5 - a\*b\*c\*d^2\*x^3 + (b^2\*c^3\*d^2\*x^5 - b^2\*c\*d^2\*x^3)\*arctan2(c\*x, sqrt(c\*x + 1))\*sqrt(-c\*x + 1))), x) + sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*sqrt(d)\*sqrt(x)/(b^2\*c\*d^2\*x^2\*arctan2(c\*x, sqrt(c\*x + 1))\*sqrt(-c\*x + 1)) + a\*b\*c\*d^2\*x^2)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b \operatorname{asin}(cx))^2 (dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*asin(c\*x))^2\*(d\*x)^(3/2)),x)

[Out] int(1/((a + b\*asin(c\*x))^2\*(d\*x)^(3/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \operatorname{asin}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(3/2)/(a+b\*asin(c\*x))\*\*2,x)

[Out] Integral(1/((d\*x)\*\*(3/2)\*(a + b\*asin(c\*x))\*\*2), x)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]], 2]],
      Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3, ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
  ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,``^``)
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+``) or
    type(expn,``*``)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```



```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```